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**OPTIMAL MONETARY
POLICY IN A NEW
KEYNESIAN MODEL
WITH HABITS IN
CONSUMPTION**

by Campbell Leith,
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and Raffaele Rossi



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by Campbell Leith, Ioana Moldovan
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Abstract

While consumption habits have been utilised as a means of generating a hump shaped output response to monetary policy shocks in sticky-price New Keynesian economies, there is relatively little analysis of the impact of habits (particularly, external habits) on optimal policy. In this paper we consider the implications of external habits for optimal monetary policy, when those habits either exist at the level of the aggregate basket of consumption goods ('superficial' habits) or at the level of individual goods ('deep' habits: see Ravn, Schmitt-Grohe, and Uribe (2006)). External habits generate an additional distortion in the economy, which implies that the flex-price equilibrium will no longer be efficient and that policy faces interesting new trade-offs and potential stabilisation biases. Furthermore, the endogenous mark-up behaviour, which emerges when habits are deep, can also significantly affect the optimal policy response to shocks, as well as dramatically affecting the stabilising properties of standard simple rules.

Keywords: consumption habits, nominal inertia, optimal monetary policy

JEL Classification: E30, E57, E61

Non-Technical Summary

Within the benchmark New Keynesian analysis of monetary policy (see, for example, Woodford (2003)), monetary policy typically influences the economy through the impact of interest rates on a representative household's intertemporal consumption decision. The purely forward-looking consumption dynamics, that such basic intertemporal consumption decisions imply, are unable to capture the hump-shaped output response to changes in monetary policy one typically finds in the data. As a means of accounting for such patterns, some authors have augmented the benchmark model with various forms of habits effects in consumption. The habits effects can either be internal, whereby households' current utility depends on their own aggregate past consumption (see for example, Fuhrer (2000), Christiano, Eichenbaum, and Evans (2005), Leith and Malley (2005)) or external, where instead households' current utility depends upon other households' aggregate past consumption (see, for example, Smets and Wouters (2007)). The latter, commonly defined as "catching up with Joneses" implies that households fail to internalise the externality their own consumption causes on the utility of other households.

More recently, Ravn, Schmitt-Grohe, and Uribe (2006) offer an alternative form of habits behaviour, which they label 'deep'. Deep habits occur at the level of individual goods rather than at the level of an aggregate consumption basket ('superficial' habits). While this distinction does not affect the dynamic description of aggregate demand relative to the case of superficial habits, it does render the individual firms' pricing decisions intertemporal and, in the flexible price economy considered by Ravn, Schmitt-Grohe, and Uribe (2006), can produce a counter-cyclical mark-up which significantly affects the responses of key aggregates to shocks. While the focus of the papers listed above is on the dynamic response of economies which feature some form of habits, they do not consider the implications for optimal policy of such an extension.

This paper extends the benchmark sticky-price New Keynesian economy to include external habits in consumption, where these habits can be either superficial or deep. The focus on external habits implies that there is an externality associated with fluctuations in consumption which implies that the flexible price equilibrium will not usually be efficient, thereby creating an additional trade-off for policy makers, which may give rise to additional stabilization biases if policy is constrained to be time consistent. Such trade-offs will occur whether or not habits are superficial or deep. We also consider the implications for optimal policy of assuming habits are of the deep kind. In addition to examining optimal policy, we also consider how the introduction of habits affects the conduct of policy through simple rules. We find that the introduction of deep habits can induce problems of indeterminacy, as the tightening of monetary policy can induce inflation through variations in mark-up behaviour, such that an interest rate rule which satisfies the Taylor principle (where nominal interest rates rise more than one for one

with increases in inflation above target) may not be sufficient to ensure determinacy of the local equilibrium. We also consider whether or not there is a significant role for the output gap in an optimal simple rule given that our economy contains a major additional externality beyond that associated with nominal inertia.

1 Introduction

Within the benchmark New Keynesian analysis of monetary policy (see, for example, Woodford (2003)), monetary policy typically influences the economy through the impact of interest rates on a representative household's intertemporal consumption decision. It has often been felt that the purely forward-looking consumption dynamics that such basic intertemporal consumption decisions imply, are unable to capture the hump-shaped output response to changes in monetary policy one typically finds in the data. As a means of accounting for such patterns, some authors have augmented the benchmark model with various forms of habits effects in consumption. The habits effects can either be internal (see for example, Fuhrer (2000), Christiano, Eichenbaum, and Evans (2005), Leith and Malley (2005)) or external (see, for example, Smets and Wouters (2007)) the latter reflecting a catching up with the Joneses effect whereby households fail to internalise the externality their own consumption causes on the utility of other households. Both forms of habits behaviour can help the New Keynesian monetary policy model capture the persistence found in the data (see, for example Kozicki and Tinsley (2002)), although the policy implications are likely to be different. More recently, Ravn, Schmitt-Grohe, and Uribe (2006) offer an alternative form of habits behaviour, which they label 'deep'. Deep habits occur at the level of individual goods rather than at the level of an aggregate consumption basket ('superficial' habits). While this distinction does not affect the dynamic description of aggregate consumption behaviour relative to the case of superficial habits, it does render the individual firms' pricing decisions intertemporal and, in the flexible price economy considered by Ravn, Schmitt-Grohe, and Uribe (2006), can produce a counter-cyclical mark-up which significantly affects the responses of key aggregates to shocks. Ravn, Schmitt-Grohe, Uribe, and Uuskula (2008) then extend the analysis of deep habits to a sticky price environment and find that such a model is able to explain, with moderate degrees of price stickiness and plausible policy rules, the prices puzzle and inflation persistence.¹

While the focus of the papers listed above is on the dynamic response of economies which feature some form of habits, they do not consider the implications for optimal policy of such an extension. In contrast, Amato and Laubach (2004) consider optimal monetary policy in a sticky-price New Keynesian economy which has been augmented to include internal (but superficial) habits. Since the form of habits is internal (households care about their consumption relative to their own past consumption, rather than the consumption of other households), there is no additional externality associated with consumption habits themselves, and, given an efficient steady-state, the flexible price equilibrium in the neighbourhood of that steady-state remains efficient. Accordingly, as in the benchmark New Keynesian model, there is no trade-off between output gap and

¹It should be noted that Ravn, Schmitt-Grohe, Uribe, and Uuskula (2008) do not consider the optimal policy or determinacy issues that are the focus of this paper. We also adopt a different mechanism for introducing nominal inertia which has implications for optimal policy, as discussed below.

inflation stabilisation in the face of technology shocks and interesting policy trade-offs require the introduction of additional inefficiencies (such as mark-up shocks or a desire for interest rate smoothing).

In this paper we extend the benchmark sticky-price New Keynesian economy to include *external* habits in consumption, where these habits can be either superficial or deep. The focus on external habits implies that there is an externality associated with fluctuations in consumption which implies that the flexible price equilibrium will not usually be efficient, thereby creating an additional trade-off for policy makers, which may give rise to additional stabilisation biases if policy is constrained to be time consistent. Such trade-offs will occur whether or not habits are superficial or deep. We also consider the implications for optimal policy of assuming habits are of the deep kind. Here the ability of policy to influence the time profile of endogenously determined mark-ups can significantly affect the monetary policy stance and how it differs across discretion and commitment. In addition to examining optimal policy, we also consider how the introduction of habits affects the conduct of policy through simple rules. We find that the introduction of deep habits can induce problems of indeterminacy, as the tightening of monetary policy can induce inflation through variations in mark-up behaviour, such that an interest rate rule which satisfies the Taylor principle (where nominal interest rates rise more than one for one with increases in inflation above target) may not be sufficient to ensure determinacy of the local equilibrium. We also consider whether or not there is a significant role for the output gap in an optimal simple rule given that our economy contains a major additional externality beyond that associated with nominal inertia.

The plan of the paper is as follows: in the next section we outline our model with deep and superficial habits. In section 3 we consider optimal policy under both commitment and discretion, where the policy-maker's objective function is derived from a second order approximation to households' utility. In section 4 we turn to our analysis of simple rules, considering both their determinacy properties and, for rules which can ensure determinacy, their ability to mimic optimal policy. Section 5 concludes.

2 The Model

The economy is comprised of households, two monopolistically competitive production sectors, and the government. There is a continuum of final goods that enter the households' consumption basket, each final good being produced as an aggregate of a continuum of intermediate goods. Households can either form external consumption habits at the level of each final good in their basket, Ravn, Schmitt-Grohe, and Uribe (2006) call this type of habits 'deep', or they can form habits at the level of the consumption basket - 'superficial' habits. Throughout the paper, we use the same terminology. Furthermore, we assume price inertia at the level of intermediate goods producers. We shall derive a

general model, and note when assuming superficial or deep habits alters the behavioural equations.

2.1 Households

The economy is populated by a continuum of households, indexed by k and of measure 1. Households derive utility from consumption of a composite good and disutility from hours spent working.

Deep Habits When habits are of the deep kind, each household's consumption basket, X_t^k , is an aggregate of a continuum of habit-adjusted final goods, indexed by i and of measure 1,

$$X_t^k = \left(\int_0^1 \left(C_{it}^k - \theta C_{it-1} \right)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}},$$

where C_{it}^k is household k 's consumption of good i and $C_{it} \equiv \int_0^1 C_{it}^k dk$ denotes the cross-sectional average consumption of this good. η is the elasticity of substitution between habit-adjusted final goods ($\eta > 1$), while the parameter θ measures the degree of external habit formation in the consumption of each individual good i . Setting θ to 0 returns us to the usual case of no habits.

The composition of the consumption basket is chosen in order to minimize expenditures, and the demand for final goods is

$$C_{it}^k = \left(\frac{P_{it}}{P_t} \right)^{-\eta} X_t^k + \theta C_{it-1}, \quad \forall i$$

where P_t represents the overall price index (or CPI), defined as an average of final goods prices, $P_t \equiv \left(\int_0^1 P_{it}^{1-\eta} di \right)^{1/(1-\eta)}$. Aggregating across households yields the total demand for good i , $i \in [0, 1]$,

$$C_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\eta} X_t + \theta C_{it-1}. \quad (1)$$

Due to the presence of habits, this demand is dynamic in nature, as it depends not only on current period elements but also on the lagged value of consumption. This, in turn, will make the pricing/output decisions of the firms producing these final goods, intertemporal.

Superficial Habits Habits are superficial when they are formed at the level of the aggregate consumption good. Households derive utility from the habit-adjusted composite good X_t^k ,

$$X_t^k = C_t^k - \theta C_{t-1},$$

where household k 's consumption, C_t^k , is an aggregate of a continuum of final goods indexed by $i \in [0, 1]$,

$$C_t^k = \left(\int_0^1 \left(C_{it}^k \right)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}},$$

with η the elasticity of substitution between them and $C_{t-1} \equiv \int_0^1 C_{t-1}^k dk$ the cross-sectional average of consumption.

Households decide the composition of the consumption basket to minimize expenditures and the demand for individual good i is

$$C_{it}^k = \left(\frac{P_{it}}{P_t} \right)^{-\eta} C_t^k = \left(\frac{P_{it}}{P_t} \right)^{-\eta} \left(X_t^k + \theta C_{t-1} \right),$$

where $P_t \equiv \left(\int_0^1 P_{it}^{1-\eta} di \right)^{\frac{1}{1-\eta}}$ is the consumer price index. The overall demand for good i is obtained by aggregating across all households,

$$\begin{aligned} C_{it} &= \int_0^1 C_{it}^k dk \\ &= \left(\frac{P_{it}}{P_t} \right)^{-\eta} C_t. \end{aligned} \quad (2)$$

Unlike the case of deep habits, this demand is *not* dynamic and the final goods producing firms will face a static pricing/output decision.

Remainder of the Household's Problem The remainder of the household's problem is the same irrespective of whether or not habits are deep or superficial. Specifically, households choose the habit-adjusted consumption aggregate, X_t^k , hours worked, N_t^k , and the portfolio allocation, D_{t+1}^k , to maximize expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(X_t^k)^{1-\sigma}}{1-\sigma} - \chi \frac{(N_t^k)^{1+v}}{1+v} \right]$$

subject to the budget constraint

$$\int_0^1 P_{it} C_{it}^k di + E_t Q_{t,t+1} D_{t+1}^k = W_t N_t^k + D_t^k + \Phi_t - T_t \quad (3)$$

and the usual transversality condition. E_t is the mathematical expectation conditional on information available at time t , β is the discount factor ($0 < \beta < 1$), χ the relative weight on disutility from time spent working, and σ and v are the inverses of the intertemporal elasticities of habit-adjusted consumption and work ($\sigma, v > 0$; $\sigma \neq 1$). The household's period- t income includes: wage income from providing labour services to intermediate goods producing firms, $W_t N_t^k$, dividends from the monopolistically competitive firms, Φ_t , and payments on the portfolio of assets, D_t^k . Financial markets are

complete and $Q_{t,t+1}$ is the one-period stochastic discount factor for nominal payoffs. T_t are lump-sum taxes collected by the government. In the maximization problem, households take as given the processes for C_{t-1} , W_t , Φ_t , and T_t , as well as the initial asset position D_{-1}^k .

The first order conditions for labour and habit-adjusted consumption are

$$\frac{\chi (N_t^k)^v}{(X_t^k)^{-\sigma}} = w_t$$

and

$$Q_{t,t+1} = \beta \left(\frac{X_{t+1}^k}{X_t^k} \right)^{-\sigma} \frac{P_t}{P_{t+1}}, \quad (4)$$

where $w_t \equiv \frac{W_t}{P_t}$ is the real wage (see Appendix A for further details). The Euler equation for consumption can be written as

$$1 = \beta E_t \left[\left(\frac{X_{t+1}^k}{X_t^k} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] R_t,$$

where $R_t^{-1} = E_t [Q_{t,t+1}]$ denotes the inverse of the risk-free gross nominal interest rate between periods t and $t + 1$.

2.2 Firms

In this subsection we consider the behaviour of firms. These are split into two kinds: final and intermediate goods producing firms, respectively. In the case of the former, their behaviour depends upon the form of demand curve they face, which is dynamic in the case of deep habits, and static under superficial habits. Intermediate goods firms produce a differentiated intermediate good and are subject to nominal inertia in the form of Calvo (1983) contracts. This structure is adopted for reasons of tractability, allowing us to easily switch between superficial and deep habits. Additionally, combining optimal price setting under both Calvo contracts and dynamic demand curves would undermine the desirable aggregation properties of the Calvo model as each firm given the signal to re-set prices would set a different price dependent on the level of consumption habits their product enjoyed relative to other firms'. By separating the two pricing decisions we avoid reintroducing the history-dependence in price setting the Calvo set-up is designed to avoid.²

²In contrast, Ravn, Schmitt-Grohe, Uribe, and Uuskula (2008) employ quadratic price adjustment costs as in Rotemberg (1982) which allows them to retain the representative firm fiction. However, this mechanism also removes the price dispersion one observes under Calvo (1983), which is intuitively a more convincing means of generating costs of inflation when analysing optimal policy.



2.2.1 Final Goods Producers

We assume that final goods are produced by monopolistically competitive firms as an aggregate of a set of intermediate goods (indexed by j), according to the function

$$Y_{it} = \left(\int_0^1 (Y_{jit})^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad (5)$$

where ε is the constant elasticity of substitution between inputs in production ($\varepsilon > 1$).

Taking as given intermediate goods prices $\{P_{jit}\}_j$ and subject to the available technology (5), firms first choose the amount of intermediate inputs $\{Y_{jit}\}_j$ that minimize production costs $\int_0^1 P_{jit} Y_{jit} dj$. The first order conditions yield the demand functions

$$Y_{jit} = \left(\frac{P_{jit}}{P_{it}^m} \right)^{-\varepsilon} Y_{it}, \quad \forall i, j \quad (6)$$

where $P_{it}^m \equiv \left(\int_0^1 P_{jit}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$ is the aggregate of intermediate goods prices in sector i and represents the nominal marginal cost of producing an additional unit of the final good i . Nominal profits are given by $\Phi_{it} = (P_{it} - P_{it}^m) Y_{it}$. It is important to note that this cost-minimisation problem takes the same form whether firms are faced with consumers whose habits are deep or superficial. However, their pricing decisions will differ across this dimension. We now examine the pricing decision of final goods firms, dependent upon whether habits are deep or superficial.

Deep Habits When habits are deep, firms face the dynamic demand from households, given by expression (1), and their profit maximization problem becomes intertemporal: the choice of price affects market share and future profits. Therefore, firms choose processes for Y_{it} and P_{it} to maximize the present discounted value of expected profits, $E_t \sum_{s=0}^{\infty} Q_{t,t+s} \Phi_{it+s}$, subject to this dynamic demand and the constraint that $C_{it} = Y_{it}$. $Q_{t,t+s}$ is the s -step ahead equivalent of the one-period stochastic discount factor in (4). The first order conditions for Y_{it} and P_{it} are

$$v_{it} = (P_{it} - P_{it}^m) + \theta E_t [Q_{t,t+1} v_{it+1}] \quad (7)$$

and

$$Y_{it} = v_{it} \left[\eta \left(\frac{P_{it}}{P_t} \right)^{-\eta-1} \frac{X_t}{P_t} \right], \quad (8)$$

where the Lagrange multiplier v_{it} represents the shadow price of producing an additional unit of the final good i . This shadow value equals the marginal benefit of additional profits, $(P_{it} - P_{it}^m)$, plus the discounted expected payoff from higher future sales, $\theta E_t [Q_{t,t+1} v_{it+1}]$. Due to the presence of habits in consumption, increasing output by

one unit in the current period leads to an increase in sales of θ in the next period. In the absence of habits, when $\theta = 0$, the intertemporal effects of higher output disappear and the shadow price simply equals time- t profits. The other first order condition in equation (8) says that an increase in price brings additional revenues, Y_{it} , while simultaneously causing a decline in demand, given by the term in square brackets and valued at the shadow value v_{it} .

Superficial Habits Under superficial habits the profit maximization problem of the final goods firms is the typical static problem whereby firms choose the price to maximize current profits, $\Phi_{it} = (P_{it} - P_{it}^m) Y_{it}$, subject to the demand for their good (2) and under the restriction that all demand be satisfied at the chosen price, $C_{it} = Y_{it}$. The optimal price is set at a *constant* markup, $\mu = \frac{\eta}{\eta-1}$, over the marginal cost,

$$P_{it} = \mu P_{it}^m.$$

2.2.2 Intermediate Goods Producers

The intermediate goods sectors consist of a continuum of monopolistically competitive firms indexed by j and of measure 1. Each firm j produces a unique good using only labour as input in the production process

$$Y_{jit} = A_t N_{jit}. \quad (9)$$

Total factor productivity, A_t , affects all firms symmetrically and follows an exogenous stationary process, $\ln A_t = \rho \ln A_{t-1} + \varepsilon_t$, with persistence parameter $\rho \in (0, 1)$ and random shocks $\varepsilon_t \sim iidN(0, \sigma_A^2)$.

Firms choose the amount of labour that minimizes production costs, $(1 - \varkappa) W_t N_{jit}$. The subsidy \varkappa , financed by lump-sum taxes, is designed to ensure that the long-run equilibrium is efficient.³ The minimization problem gives a demand for labour, $N_{jit} = \frac{Y_{jit}}{A_{jit}}$, and a nominal marginal cost, $MC_t = (1 - \varkappa) \frac{W_t}{A_t}$, which is the same across firms. (See Appendix A for more details.) Nominal profits are expressed as $\Phi_{jit} \equiv (P_{jit} - MC_t) Y_{jit}$.

We further assume that intermediate goods producers are subject to the constraints of Calvo (1983)-contracts such that, with fixed probability $(1 - \alpha)$ in each period, a firm can reset its price and with probability α the firm retains the price of the previous period. When a firm can set the price, it does so in order to maximize the present discounted value of profits, $E_t \sum_{s=0}^{\infty} \alpha^s Q_{t,t+s} \Phi_{jit+s}$, and subject to the demand for its own good (6) and the constraint that all demand be satisfied at the chosen price. Profits are discounted by the s -step ahead stochastic discount factor $Q_{t,t+s}$ and by the probability

³In this model, all firms are monopolistically competitive but only intermediate goods producing firms are subsidized. Still, in steady-state the subsidy level is such that all production inefficiencies are eliminated.

of not being able to set prices in future periods.

Optimally, the relative price satisfies the following relationship:

$$\frac{P_{jit}^*}{P_t} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{E_t \sum_{s=0}^{\infty} (\alpha\beta)^s (X_{t+s})^{-\sigma} mc_{t+s} (P_{it+s}^m)^\varepsilon Y_{it+s}}{E_t \sum_{s=0}^{\infty} (\alpha\beta)^s (X_{t+s})^{-\sigma} \left(\frac{P_{t+s}}{P_t} \right)^{-1} (P_{it+s}^m)^\varepsilon Y_{it+s}},$$

where $mc_t = \frac{MC_t}{P_t}$ is the real marginal cost.

P_{it}^m represents the price at the level of sector i and is an average of intermediate goods prices within that sector. With α of firms keeping last period's price and $(1 - \alpha)$ of firms setting a new price, the law of motion of this price index is:

$$(P_{it}^m)^{1-\varepsilon} = \alpha (P_{it-1}^m)^{1-\varepsilon} + (1 - \alpha) (P_{jit}^*)^{1-\varepsilon}.$$

This description of intermediate goods firms is the same irrespective of the nature of habits formation.

2.3 The Government

The government collects lump-sum taxes which it rebates to intermediate goods producing firms as subsidies, which ensure an efficient long-run level of output. There is no government spending per se. The government budget constraint is given by

$$\varkappa W_t N_t = T_t. \quad (10)$$

In this cashless economy, monetary policy is conducted in optimal fashion, with the nominal interest rate being the central bank's policy instrument. However, we also consider the consequences of the central bank adopting more simple forms of policy, such as Taylor-type interest rate rules, and explore how closely these simple policy rules come to the optimal.

2.4 Equilibrium

In the absence of sector-specific shocks or other forms of heterogeneity, final goods producers are symmetric and so are households. However, symmetry does not apply to intermediate goods producers who face randomness in price setting. There is a distribution of intermediate goods prices and aggregate output is therefore determined as (see Appendix B for details on aggregation)

$$Y_t = A_t \frac{N_t}{\Delta_t}. \quad (11)$$

$\Delta_t \equiv \int_0^1 \left(\frac{P_{jt}}{P_t^m}\right)^{-\varepsilon} dj$ is the measure of price dispersion, which can be shown (see Woodford (2003), Chapter 6) to follow an AR(1) process given by

$$\Delta_t = (1 - \alpha) \left(\frac{P_t^*}{P_t^m}\right)^{-\varepsilon} + \alpha (\pi_t^m)^\varepsilon \Delta_{t-1}. \quad (12)$$

Note that we have two measures of aggregate prices, a producer price index P_t^m and the usual consumer price index P_t , and consequently, there will be two measures of inflation. We define: $\pi_t^m \equiv \frac{P_t^m}{P_{t-1}^m}$ and $\pi_t \equiv \frac{P_t}{P_{t-1}}$. Furthermore, the two inflation variables are related according to the following relationship

$$\pi_t = \pi_t^m \frac{\mu_t}{\mu_{t-1}}, \quad (13)$$

where $\mu_t \equiv \frac{P_t}{P_t^m}$ is the markup of final goods producers. In the presence of deep habits, this markup is time-varying. The overall markup in the economy is given by the product of markups in the intermediate goods and final goods sectors and equals the inverse of the real marginal cost, $mc_t^{-1} = \frac{P_t}{MC_t}$. It should be noted that when habits are superficial, the mark-up in the final goods sectors is constant, $\mu_t = \mu$, and there is no longer any wedge between consumer price and producer price inflation.

Finally, the aggregate version of the household's budget constraint (3) combines with the government budget constraint (10) and the definition of aggregate profits ($\Phi_t = P_t Y_t - (1 - \varkappa) W_t N_t$) to obtain the usual aggregate resource constraint,

$$Y_t = C_t. \quad (14)$$

The equilibrium is then characterized by equations (11) - (14), to which we add the monetary policy specification (to be detailed in Sections 3 and 4 below) and the following set of equations:

Consumers:

$$X_t = C_t - \theta C_{t-1} \quad (15)$$

$$\frac{\chi N_t^\nu}{X_t^{-\sigma}} = w_t \quad (16)$$

$$X_t^{-\sigma} = \beta E_t \left[X_{t+1}^{-\sigma} \frac{P_t}{P_{t+1}} \right] R_t \quad (17)$$

Government:

$$\varkappa W_t N_t = T_t \quad (18)$$

Intermediate goods producers:

$$(P_t^m)^{1-\varepsilon} = \alpha (P_{t-1}^m)^{1-\varepsilon} + (1 - \alpha) (P_{jt}^*)^{1-\varepsilon} \quad (19)$$

$$\frac{P_{jt}^*}{P_t} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{K_{1t}}{K_{2t}} \quad (20)$$

$$\text{where} \quad : \quad K_{1t} = X_t^{-\sigma} m c_t \mu_t^{-\varepsilon} Y_t + \alpha \beta E_t [K_{1t+1} \pi_{t+1}^\varepsilon] \quad (21)$$

$$: \quad K_{2t} = X_t^{-\sigma} \mu_t^{-\varepsilon} Y_t + \alpha \beta E_t [K_{2t+1} \pi_{t+1}^{\varepsilon-1}] \quad (22)$$

$$m c_t = (1 - \varkappa) \frac{w_t}{A_t} \quad (23)$$

$$\ln A_t = \rho \ln A_{t-1} + \varepsilon_t \quad (24)$$

Final goods producers:

The differences in the two economies when habits are deep rather than superficial emerges in the behaviour of the final goods firms. As noted above, when habits are superficial they simply adopt a constant mark-up over the price of the bundle of intermediate goods,

$$\mu_t = \mu. \quad (25)$$

In contrast, when habits are deep and final goods firms face a dynamic demand curve for their product, the endogenous time varying mark-up is described by the following two equations (note that we have written the shadow price of producing final goods in real terms, i.e. $\omega_t \equiv \frac{v_t}{P_t}$),

$$\omega_t = \left(1 - \frac{1}{\mu_t} \right) + \theta \beta E_t \left[\left(\frac{X_{t+1}}{X_t} \right)^{-\sigma} \omega_{t+1} \right] \quad (26)$$

$$Y_t = \eta \omega_t X_t. \quad (27)$$

2.5 Solution Method and Model Calibration

In the absence of a closed-form solution, the model's equilibrium conditions are log-linearized around the efficient deterministic steady state. The efficiency of the steady state, obtained through the subsidy allocated to intermediate goods producers, allows us to obtain an accurate expression for welfare involving only second-order terms.

In order to solve the model, we must select numerical values for some key structural parameters. Table 1 reports our choices, which are similar to those of other studies using a New Keynesian economy with habits in consumption. The model is calibrated to a quarterly frequency. We assume zero average inflation ($\pi = 1$) and an annual real rate of interest of 4%, which together imply a discount factor β of 0.9902. The risk aversion parameter σ is set at 2.0, while v equals 0.25,⁴ and the relative weight on labour in the utility function χ is assumed to be 3.0. Consistent with the empirical

⁴ v is the inverse of the Frisch labour supply elasticity. While micro estimates of this elasticity are rather small, they tend not to fit well in macro models. Here, we follow the macroeconomic literature and choose a larger value of 4.0.

evidence, the level of price inertia α is set at 0.75 and the degree of market power is 1.21, split approximately equally between the two monopolistically competitive sectors of our economy. The steady state value of the markup in the final goods sector is given as, $\mu = \left[1 - \frac{(1-\theta\beta)}{(1-\theta)\eta}\right]^{-1}$, and depends on both the elasticity of substitution between final goods η and the degree of habit formation θ . However, the impact of θ on the markup μ is minimal and we therefore set $\eta = \varepsilon = 11$. For the habits formation parameter θ , we use a benchmark value of 0.65, which falls within the range of estimates identified in the literature⁵. However, we allow θ to vary in the $[0, 1)$ interval as we conduct a sensitivity analysis of our results. Technology shocks are assumed persistent with persistence parameter $\rho = 0.9$ and standard deviation $\sigma_A = 0.009$. Finally, we set the subsidy rate \varkappa so as to ensure an efficient steady state, $\varkappa = 1 - \frac{1}{1-\theta\beta} \left(\frac{1}{\mu} \frac{\varepsilon-1}{\varepsilon}\right)$.⁶

2.6 Log-linear Representation

Upon log-linearizing and combining the relevant equilibrium conditions, we obtain a system of equations which characterize the dynamics of the economy in the neighbourhood of the efficient steady state. Firstly, we have the IS curve in terms of habit-adjusted consumption,

$$\widehat{X}_t = E_t \widehat{X}_{t+1} - \frac{1}{\sigma} \widehat{R}_t + \frac{1}{\sigma} E_t \widehat{\pi}_{t+1}, \quad (28)$$

and the New Keynesian Phillips Curve (NKPC) written in terms of producer price inflation,

$$\widehat{\pi}_t^m = \beta E_t \widehat{\pi}_{t+1}^m + \kappa (\widehat{m}c_t + \widehat{\mu}_t), \quad (29)$$

where $\kappa \equiv \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}$ and it should be remembered that the markup, μ_t , is constant under superficial habits such that, $\widehat{\mu}_t = 0$. In contrast, when habits are deep, the dynamic equation describing changes in the markup can be written as,

$$\widehat{\mu}_t = \mu\omega (\widehat{\omega}_t - \theta\beta E_t \widehat{\omega}_{t+1}) - \mu\omega\theta\beta\sigma \left(\widehat{X}_t - E_t \widehat{X}_{t+1}\right), \quad (30)$$

where the shadow value of producing another unit of a final good subject to deep habits is given by,

$$\widehat{\omega}_t = \widehat{Y}_t - \widehat{X}_t. \quad (31)$$

⁵Macro-based estimates of habits formation of the superficial type range from 0.59 as in Smets and Wouters (2003) to very high values of 0.98 as reported by Bouakez, Cardia, and Ruge-Murcia (2005). For the deep type of habits, Ravn, Schmitt-Grohe, and Uribe (2006) and Ravn, Schmitt-Grohe, Uribe, and Uuskula (2008) give a value of 0.86 and 0.85, respectively. Micro-based estimates (see, for example, Ravina (2007)) are substantially lower, with a range of 0.29-0.5.

⁶It should be noted that when the habits externality is sufficiently large this subsidy turns negative as households' overconsumption dominates the underproduction associated with the monopolistic competition distortion. The balancing of the two distortions is used as a device by Levine, Pearlman, and Piersie (2008) to achieve an efficient steady-state while avoiding the need to introduce a steady-state subsidy.

And finally, we have the following expressions defining habit-adjusted consumption \widehat{X}_t , CPI inflation $\widehat{\pi}_t$ (when habits are deep), and the real marginal cost \widehat{mc}_t :

$$\widehat{X}_t = \frac{1}{1-\theta} \left(\widehat{Y}_t - \theta \widehat{Y}_{t-1} \right) \quad (32)$$

$$\widehat{\pi}_t = \widehat{\pi}_t^m + \widehat{\mu}_t - \widehat{\mu}_{t-1} \quad (33)$$

$$\widehat{mc}_t = \sigma \widehat{X}_t + v \widehat{Y}_t - (1+v) \widehat{A}_t. \quad (34)$$

3 Optimal Policy

In this section we consider the nature of optimal monetary policy in response to technology shocks, under both cases of commitment and discretion by the monetary authority. The central bank's objective function is given by a second order approximation to the representative households' utility (see Appendix E for details),

$$\begin{aligned} \Gamma_0 &= -\frac{1}{2} \chi \bar{N}^{1+v} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left[\sigma \left(\frac{1-\theta}{1-\theta\beta} \right) \widehat{X}_t^2 + v \left(\widehat{Y}_t - \frac{1+v}{v} \widehat{A}_t \right)^2 \right] + \frac{\varepsilon}{\kappa} (\widehat{\pi}_t^m)^2 \right\} + tip + O[2]. \\ &= -\frac{1}{2} \chi \bar{N}^{1+v} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left[(\delta+v) \widehat{Y}_t^2 - 2\theta\delta \widehat{Y}_t \widehat{Y}_{t-1} + \theta^2 \delta \widehat{Y}_{t-1}^2 - 2(1+v) \widehat{Y}_t \widehat{A}_t \right] + \frac{\varepsilon}{\kappa} (\widehat{\pi}_t^m)^2 \right\} \\ &\quad + tip + O[2] \end{aligned}$$

where $\delta \equiv \frac{\sigma}{(1-\theta\beta)(1-\theta)}$. The last line was obtained by replacing \widehat{X}_t with its expression in terms of output. The weights given to the various elements in the objective function are derived from basic preference parameters, and the presence of the term in inflation reflects the costs of price dispersion due to nominal inertia. It should be noted that this objective function applies whether or not habits are deep or superficial.

While this welfare measure has the same basic components (output and inflation) as the benchmark New Keynesian model (without externalities due to consumption habits), this welfare measure looks different, in that it does not contain a single "output gap", defined as the difference between output and the flex-price level of output. However, the more complex term in the current set-up is conceptually similar. The output gap term in the standard analysis captures the extent to which output deviates from its efficient level (typically because of nominal inertia, rather than any other distortion). In a model with external habits, there is an additional externality which means that the flexible price equilibrium is unlikely to be efficient, such that it is not possible to rewrite output in gap form. Nevertheless, it is possible to show that minimisation of the terms in square brackets is equivalent to implementing the social planner's allocation. In other words, we are still trading off inflation control against minimising the deviation of the decentralised equilibrium to that which would be implemented by a benevolent social planner (see appendix D for the social planner's problem).

We measure the welfare cost of a particular policy as the fraction of permanent consumption that must be given up in order to equal welfare in the stochastic economy to that of the efficient steady state, $E \sum_{t=0}^{\infty} \beta^t u(X_t, N_t) = (1 - \beta)^{-1} u((1 - \theta)(1 - \xi)\bar{C}, \bar{N})$. Given the utility function adopted, the expression for ξ in percentage terms is

$$\xi = \left[1 - \frac{[(1 - \sigma)\Upsilon]^{\frac{1}{1-\sigma}}}{(1 - \theta)\bar{C}} \right] \times 100,$$

where $\Upsilon \equiv (1 - \beta)W + \chi \frac{\bar{N}^{1+v}}{1+v}$ and $W \equiv E \sum_{t=0}^{\infty} \beta^t u(X_t, N_t)$ represents the unconditional expectation of lifetime utility in the stochastic equilibrium.

3.1 Optimal Policy under Commitment

If the monetary authority can credibly commit to following its policy plans, it then chooses the policy that maximizes households' welfare subject to the private sector's optimal behaviour, as summarized in equations (28) - (34), and given the exogenous process for technology (see Appendix F for details of the policy problem under commitment). We analyze the implications of this policy in terms of impulse responses to exogenous technology shocks.

Optimal policy faces a trade-off between output and inflation stabilization in the face of technology shocks which would not be present with internal habits. With internal habits, policy would be loosened to ensure the flexible price equilibrium was recreated without generating any inflation (see Amato and Laubach (2004)).⁷ However, when habits are external, such that one household does not take account of the impact their increased consumption has on the utility of others, then with one policy instrument available, the monetary authority cannot simultaneously ensure output is at its efficient level and inflation is eliminated. Instead, while nominal inertia points to a relaxation of policy in the face of a positive technology shock to boost output, the consumption externality suggests that the higher consumption this entails need not be desirable.⁸ Figure 1 shows that the optimal response of the economy to a positive persistent technology shock is a positive output gap and an initial decline followed by an increase in inflation (pluses indicate the benchmark calibration with $\theta = 0.65$) when habits are superficial.

To achieve this outcome, the monetary authority reduces the nominal interest rate to boost demand to the socially optimal level. Because the policy is expansionary, we can implicitly say that the inefficiency due to price stickiness is dominating in this case. As the degree of importance of habits increases, inflation stabilisation remains the primary goal and the policy maker suffers a widening output gap due to the consumption

⁷Of course, this is also the case in the New Keynesian model without habits - in the face of technology shocks, the monetary policy maker can eliminate the output gap without generating inflation.

⁸Ljungqvist and Uhlig (2000) show how contractionary tax policy can be used for the purpose of aligning output with the efficient level in response to technology shocks.

externality.

We turn to the case of deep habits in Figure 2. Holding the monetary policy response constant, we would expect the slackening of monetary policy to increase the discounted profits of final goods firms, encouraging them to cut current markups, generating consumption habits in their goods and thereby widening the output gap further. As a result, the optimal monetary response does not slacken real interest rates by as much in order to discourage the reduction in mark-ups. In fact, once the degree of habits passes a certain level, real interest rates actually rise initially, as policy makers seek to dampen the initial rise in consumption which imposes an undesirably externality on households. This can be seen in Figure 2, where pluses depict impulse responses under the benchmark value of habits, $\theta = 0.65$, and solid lines the responses under an alternative higher value of $\theta = 0.75$.

3.2 Optimal Policy with Discretion

The previous sub-section examined policy under commitment. It is well known that not being able to commit to a time-inconsistent policy can give rise to a stabilisation bias in the New Keynesian economy whereby policy makers cannot obtain the most favourable trade-offs between output gap and inflation stabilisation.⁹ In our economy with a consumption externality, there may be additional sources of stabilisation bias which make it interesting to assess the importance of having access to a commitment technology. Appendix F defines the inputs to the iterative algorithm used to compute time-consistent policy in Soderlind (1999).

Figure 3 contrasts policy under discretion and commitment when habits are superficial. Aside from failing to exploit the expectational benefits of price level control, the discretionary policy also fails to achieve the initial relative tightening of policy which mitigates the generation of undesirable habits effects. The desirability of undertaking the commitment policy emerges in the significantly different paths for inflation across commitment and discretion. The welfare cost of not being able to commit to future policies amounts to 0.016% of steady state consumption and it is 0.77% higher than under commitment, in the benchmark case of $\theta = 0.65$.

When we undertake the same comparison in the case of deep habits (see Figure 4), the time inconsistency problem is even more significant than in the case of superficial habits. This is because under deep habits there is a stronger desire to tighten policy initially in order to prevent an undesirable increase in consumption habits, exacerbated by the profit-maximising cuts in mark-ups by final goods producing firms. In fact, under commitment, interest rates actually rise initially if the extent of habits formation exceeds $\theta = 0.73$. Since such a policy is designed to improve policy trade-offs in the future, it is not possible to engineer such a monetary tightening under time consistent

⁹It should be noted that the stabilisation bias can exist in economies which do not contain steady-state distortions, and are therefore not subject to the familiar inflation bias.

policy. The costs of not having access to a commitment technology are correspondingly higher under deep habits, where the welfare costs under discretion are 1.98% higher than commitment, under the benchmark of $\theta = 0.65$.

4 Simple Rules

Having derived the optimal policy under commitment and discretion for our new Keynesian economy with either superficial or deep habits, we now turn to consider the following simple monetary policy rule,

$$\widehat{R}_t = \phi_\pi \widehat{\pi}_t^m + \phi_y \widehat{Y}_t^{gap} + \phi_R \widehat{R}_{t-1}.$$

This is similar to that considered in Schmitt-Grohe and Uribe (2007), but with some differences noted below. \widehat{R}_t is the nominal interest rate, $\widehat{\pi}_t^m$ is the rate of inflation in the intermediate goods sector and \widehat{Y}_t^{gap} is the log-deviation of output from the level that would be chosen by the social planner. The choice of inflation rate does not matter in the case of superficial habits since consumer and producer price inflation are identical in this case. However, it does make a difference in the case of deep habits since endogenous variations in mark-ups in the final goods sector induce a time-varying wedge between the two inflation measures. In this case, it is natural to adopt producer price inflation as the inflation measure in the rule, since this captures the costs of price dispersion in social welfare (see Appendix E and the discussion of this point in Kirsanova, Leith, and Wren-Lewis (2006)).¹⁰ We also include a term in the output gap rather than output itself. Schmitt-Grohe and Uribe (2007) demonstrate that a term in output plays no role in an optimally parameterised simple rule, and we wish to assess, in contrast to this finding, whether there is a role for policy responding to a measure of the inefficiency in the level of output in an economy with a potentially large consumption externality.

In this section we begin by considering the determinacy properties of our simple rule, under both forms of habits formation. We then turn to consider the welfare maximising parameterisation of the rule, and assessing to what extent this can mimic the optimal policy under commitment described above.

Figure 5 details the determinacy properties of this rule when habits are of the superficial form. Each sub-plot details the combinations of ϕ_π and ϕ_y which ensure determinacy (light grey dots), indeterminacy (blanks) and instability (dark grey stars).¹¹

¹⁰Furthermore, adopting a rule specified in terms of CPI inflation when habits are of the deep kind can induce cyclical responses to shocks which are not conducive to mimicking the optimal response to shocks. These results are available from the authors upon request.

¹¹Following Blanchard and Kahn (1980), we write the dynamic model in matrix form as $AE_t x_{t+1} = Bx_t$ where x_t is an $n \times 1$ vector of the model's endogenous and exogenous variables. A and B are square matrices of size $n \times n$. Let us define $J = A^{-1}B$, m as the number of non-predetermined variables in x , $n - m$ the number of predetermined variables in x , and q the number of eigenvalues of J that are greater than one in absolute value, i.e. explosive eigenvalues. If $q = m$, the system is determinate (determinacy). In other words the solution to $AE_t x_{t+1} = Bx_t$ is unique and converges to the steady

Moving from left to right across subplots increases the degree of interest rate inertia in the rule, ϕ_R , while moving down the page increases the extent of habits formation, θ . Consider the first sub-plot in the top left hand corner with $\phi_R = 0$ and $\theta = 0$, which re-states the stability properties of the original Taylor rule. Here the importance of the Taylor principle is revealed as $\phi_\pi > 1$ in combination with a non-negative response to the output gap is a sufficient condition for determinacy. Within this region there is limited scope for compensating for failing to fulfil the Taylor principle through increasing the positive response of the interest rate to the output gap, and there is slightly greater scope for using a more aggressive monetary policy response to compensate for a mildly negative interest rate response to the output gap. It is also interesting to note that a second region of determinacy exists where the interest rate rule fails to satisfy the Taylor principle, such that $\phi_\pi < 1$, and the response to the output gap is strongly negative. This region is not often discussed in the literature, but is mentioned in Rotemberg and Woodford (1999). Typically, when monetary policy fails to satisfy the Taylor principle, inflation can be driven by self-fulfilling expectations which are validated by monetary policy. However, when the output gap response is sufficiently negative there is an additional destabilising element in policy, which overturns the excessive stability generated by a passive monetary policy, implying a unique saddlepath where any deviation from that saddlepath will imply an explosive path for inflation.

As we move down the sub-plots in the first column of Figure 5 we increase the degree of superficial habits. This means that the output response to both policy and shocks is more muted as current consumption is increasingly tied to past levels of consumption. This has two effects on the determinacy properties of the basic Taylor rule. Firstly, a rule which satisfies the Taylor principle will do so even if the response to the output gap is increasingly negative. Secondly, the additional instability caused by adopting a negative interest rate response to the output gap becomes insufficient to move a passive interest rate rule to a position of determinacy. Accordingly, the importance of the Taylor principle is enhanced when consumption is subject to superficial habits effects.

As we move across the page from left to right we increase the extent of interest rate inertia in the rule. In this case, as Woodford (2001) shows, the Taylor principle needs to be rewritten in terms of the long-run interest rate response to excess inflation, $\frac{\phi_\pi}{1-\phi_R} > 1$. As a result, the determinacy region in the positive quadrant spreads further into the adjacent quadrants since a given level of instantaneous policy response to inflation ϕ_π has a far greater long-run effect.

Finally, when we combine superficial habits effects with interest rate inertia, it becomes possible to induce instability in our economy when the rule is passive, $\frac{\phi_\pi}{1-\phi_R} < 1$, and the interest rate response to the output gap is negative, $\phi_y < 0$. Essentially, the

state. If $q < m$ there are an infinite number of solutions to $AE_t x_{t+1} = Bx_t$, the system is therefore indeterminate (indeterminacy). Ultimately, if $q > m$ there is no solution to $AE_t x_{t+1} = Bx_t$ and the system is unstable (instability).

slow evolution of consumption under habits combined with interest rate inertia and a perverse policy response to output gaps and inflation serves to induce a cumulative instability in the model.

Figure 6 constructs a similar set of sub-plots when habits are of the deep, rather than superficial, kind. If the extent of habits formation is relatively low, the determinacy properties of the model are similar to those observed under superficial habits. However, when the degree of habits formation exceeds $\theta > 0.77$, then there are some significant differences. Firstly, the usual determinacy region in the positive quadrant disappears and becomes indeterminate. This indeterminacy is linked to the additional dynamics displayed in the final goods sectors, where the markup is time-varying under deep habits formation. Suppose economic agents expect an increase in inflation. Given an active interest rate rule, $\phi_\pi > 1$, this will give rise to a tightening of monetary policy. Typically, such a policy would lead to a contraction in aggregate demand, invalidating the inflation expectations. However, in the presence of deep habits, the higher real interest rates will encourage final goods firms to raise current mark-ups as they discount the lost future sales such price increases would imply more heavily. If the size of habits effects is sufficiently large, then this increase in mark-ups can validate the initial increase in inflationary expectations, leading to self-fulfilling inflationary episodes and indeterminacy.

Furthermore, the region of instability identified under superficial habits, becomes determinate when combined with the excessive stability implied by endogenous mark-up behaviour, such that the only determinate rule in the presence of a large deep habits effect is where the rule is passive, $\frac{\phi_\pi}{1-\phi_R} < 1$, and the policy response to the output gap is sufficiently strongly negative.

Optimal Simple Rules Having explored the determinacy properties of the simple rule described above when embedded in our economy featuring either superficial or deep habits, we now turn to consider the optimal parameterisation of the rule in each case.¹² In the case of superficial habits and under the benchmark value of $\theta = 0.65$, the optimal simple rule implies a lot of interest rate smoothing, with a strong positive response to inflation but a negative response to the output gap ($\phi_\pi = 18.47$, $\phi_y = -0.09$, and $\phi_R = 0.93$). With this type of optimal monetary policy rule, the economy's response to a technology shock essentially replicates the responses obtained under a full commitment policy, as shown in Figure 7. To explore the intuition underpinning this result, Figure 8 explores how the optimal policy rule parameters vary with the degree of habits formation, θ .

In the absence of habits effects, in a New Keynesian economy, a positive technology shock leads to a decrease in inflation and, due to the nominal inertia, an insufficiently

¹²We search across the rule parameter space using the Simplex method employed by the Fminsearch algorithm in Matlab (see, Lagarias, Reeds, Wright, and Wright (1998)) in order to minimise the unconditional welfare losses associated with the rule.

large increase in output. Optimally, a decrease in the nominal interest rate stimulates demand by reducing the real interest rate. This can be achieved by having a very large coefficient on inflation relative to all other parameters, which essentially allows the simple policy rule to achieve the flex-price equilibrium with zero inflation and a zero output gap. As we introduce superficial habits effects, in the face of the same shock households overconsume and the output gap becomes positive suggesting that policy be tightened rather than relaxed. This trade-off, which is not present in the model without external habits, affects the optimal parameterisation of the simple policy rule. Specifically, as we increase the degree of habits formation, the optimal parameter on inflation in the simple rule falls and the extent of interest rate inertia increases. Furthermore, the negative coefficient on the output gap also falls, eventually turning positive.

A key feature of optimal policy under commitment is price level control where the optimal policy achieves expectational benefits in seeking to ensure that price level returns to base following any shock. As the degree of superficial habits formation is increased, this price level control can be achieved most effectively through a combination of interest rate inertia and output gap response. Consider the impact of the positive technology shock depicted in Figure 7. Essentially, the rule is able to maintain a cut in real interest rates, even when inflation is slightly positive (to undo the price level effects of the initial fall in inflation), by responding negatively to the persistent positive output gap and maintaining that stance for longer by increasing the amount of interest rate inertia. When the degree of habits formation becomes sufficiently large, the coefficient on the output gap becomes positive in order to reduce the initial relaxation of policy, and the degree of interest rate inertia is increased to ensure price level control.

Figure 9 plots the optimal parameters of the simple rule, when the economy features an increasing level of deep habits formation.¹³ When habits are deep there is less of a desire to cut interest rates initially, to prevent final goods firms' cutting their mark-ups and generating even greater consumption externalities. For relatively low levels of habits, this implies a more muted response to inflation and output gaps. However, there is a surprising 'blip' in the optimal parameters for intermediate levels of the deep habits effect. At intermediate levels of deep habits this desire to tighten policy is finely balanced against the need to avoid falls in intermediate goods inflation which induces undesirable increases in price dispersion. Despite the large rule parameters this implies, the rule still successfully mimics optimal policy under commitment, and the simple rule in all cases comes close to achieving the welfare levels observed under commitment - see Figure 10 for an illustration of impulse responses to a technology shock in the case of $\theta = 0.5$.¹⁴

¹³For very high degrees of habits (exceeding 0.75), we can obtain a determinate rule, lying in the determinate region identified in Figure 6. However, welfare is deteriorating relative to commitment policy.

¹⁴More reasonable parameters in the simple rule do not significantly affect results in this region of habits formation. The very large optimal parameters help achieve small numerical improvements in welfare.

5 Conclusion

In this paper we considered the optimal policy response to technology shocks in a New Keynesian economy subject to habits effects in consumption. These effects were assumed to be external, such that one household fails to take account of the impact their consumption behaviour has on other households as each household seeks to ‘catch up with the Joneses’. This consumption externality needs to be traded-off against the monetary policy maker’s usual desire to stabilise inflation (a trade-off which would not exist if habits were internal) and generates a new form of stabilisation bias as time consistent policy is unable to mimic the initial policy response under commitment. This framework is further enriched by allowing the habits effects to be either superficial (at the level of the household’s total consumption) or deep (at the level of individual consumption goods). Under deep habits, firms face dynamic demand curves which imply an intertemporal dimension to price setting and endogenous mark-up behaviour. This, in turn, further modifies the optimal policy response to technology shocks when habits are deep.

In addition to considering optimal policy, we also consider the stabilising properties of simple rules. We investigate the determinacy properties of such rules and find that superficial habits effects tend to increase the range of parameters consistent with determinacy, provided the Taylor principle is satisfied. However, for sufficiently large measures of deep habits (which fall within the range of econometric estimates) the Taylor principle ceases to be either a necessary or sufficient condition for determinacy. We demonstrate that optimally parameterised determinate simple rules can typically come close to achieving the welfare levels observed under optimal commitment policy. Overall our work suggests that the choice of internal or external habits effects will have non trivial implications for optimal policy, even if the implied dynamics of the model when policy is described by an ad hoc rule could be similar (Kozicki and Tinsley (2002)).

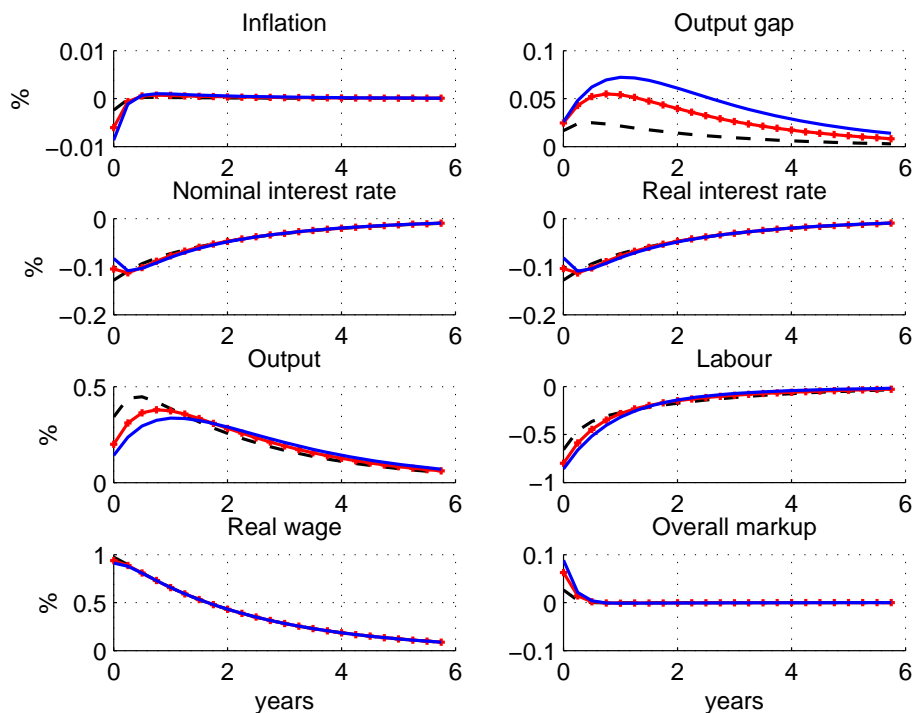


Figure 1: Impulse responses to a 1% positive technology shock under *optimal commitment policy*, in the case of *superficial habits*: $\theta = 0.4$ (dash lines), $\theta = 0.65$ (benchmark value, pluses), $\theta = 0.75$ (solid lines).

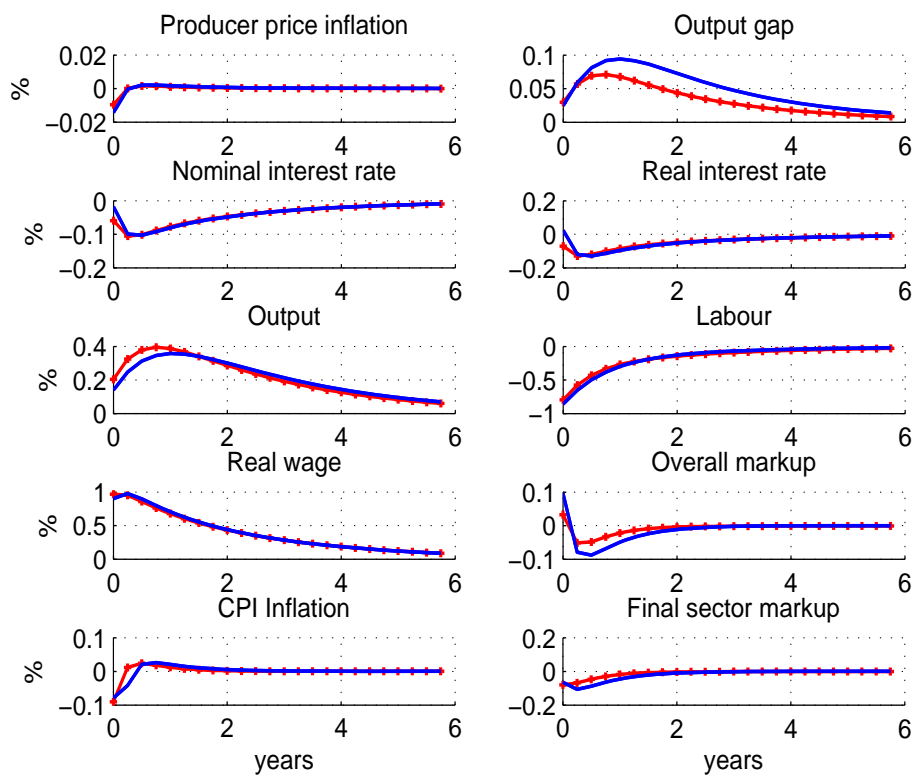


Figure 2: Impulse responses to a 1% positive technology shock under *optimal commitment policy*, in the case of *deep habits*, for $\theta = 0.65$ (benchmark value, pluses) and $\theta = 0.75$ (solid lines).

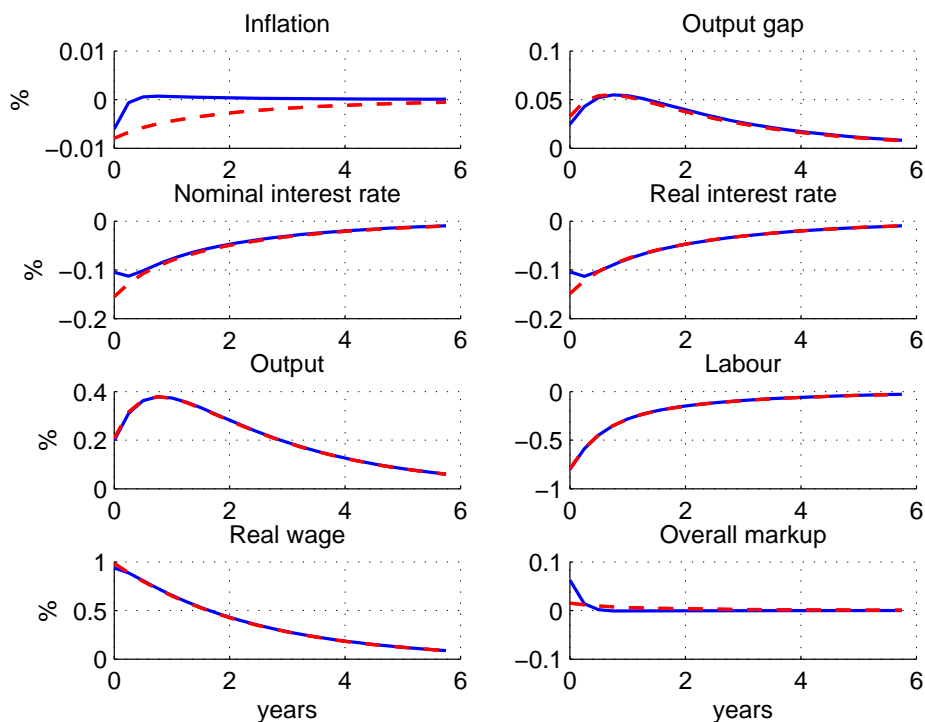


Figure 3: Impulse responses to a 1% positive technology shock in the case of *superficial habits* under optimal policy with *commitment* (solid lines) and with *discretion* (dash lines).

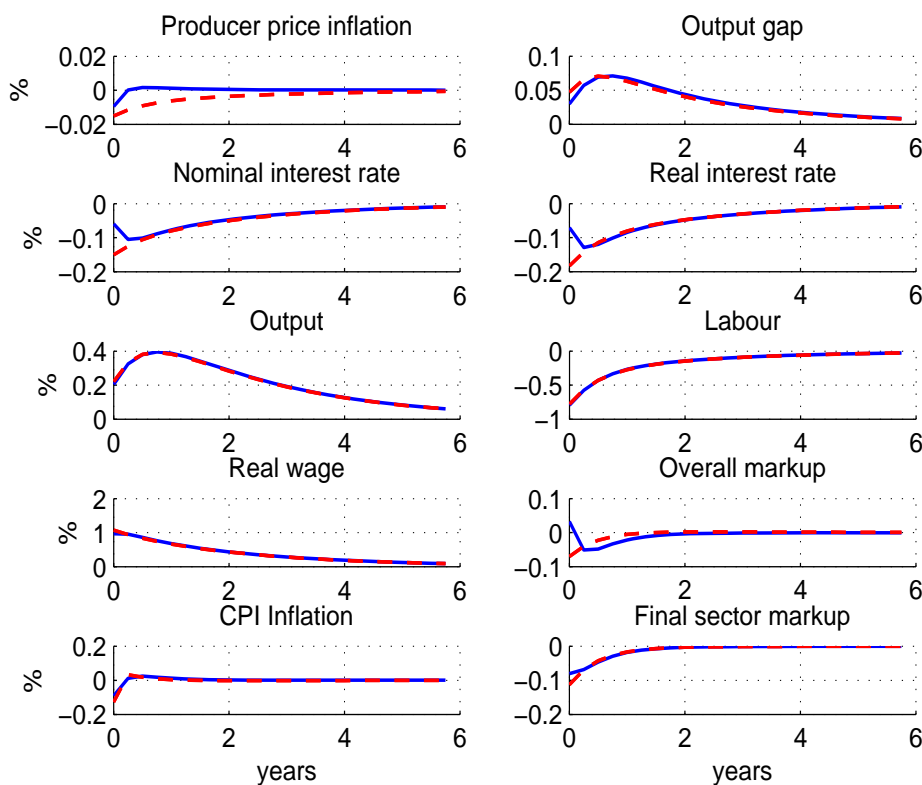


Figure 4: Impulse responses to a 1% positive technology shock in the case of *deep habits* under optimal policy with *commitment* (solid lines) and with *discretion* (dash lines).

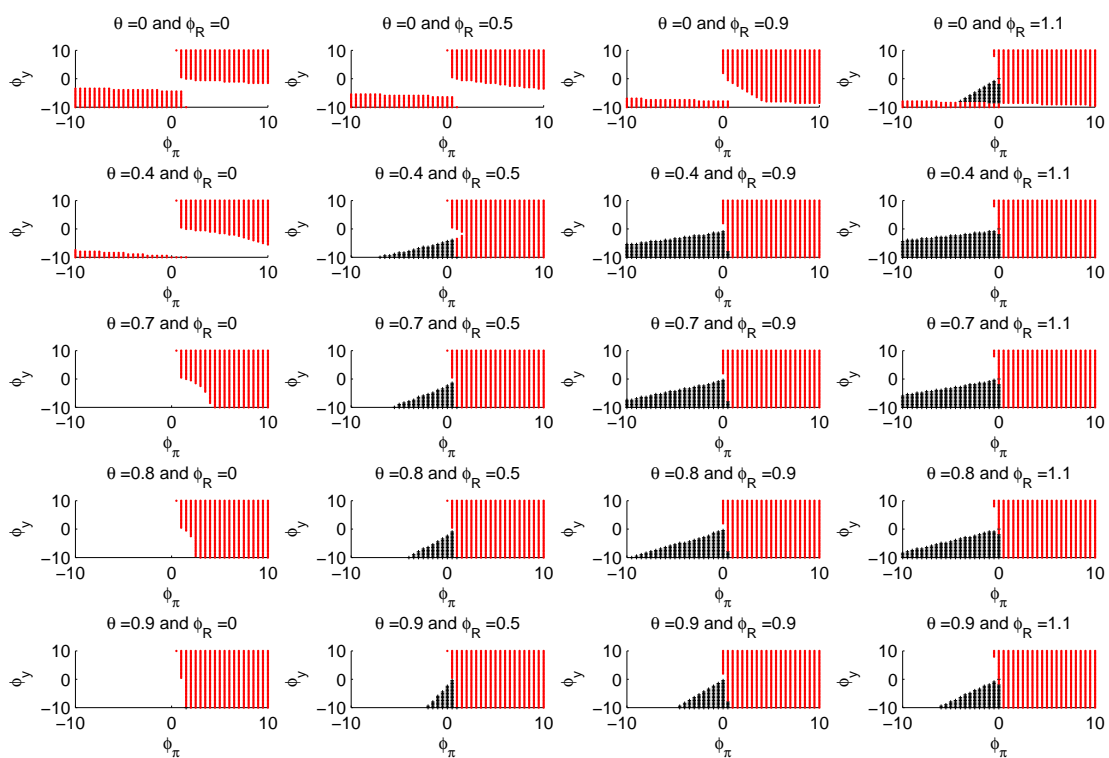


Figure 5: Determinacy properties of the model with *superficial habits*, when monetary policy follows the rule $\hat{R}_t = \phi_\pi \hat{\pi}_t^m + \phi_y \hat{Y}_t^{gap} + \phi_R \hat{R}_{t-1}$: determinacy (light grey dots), indeterminacy (blanks), and instability (dark grey stars).

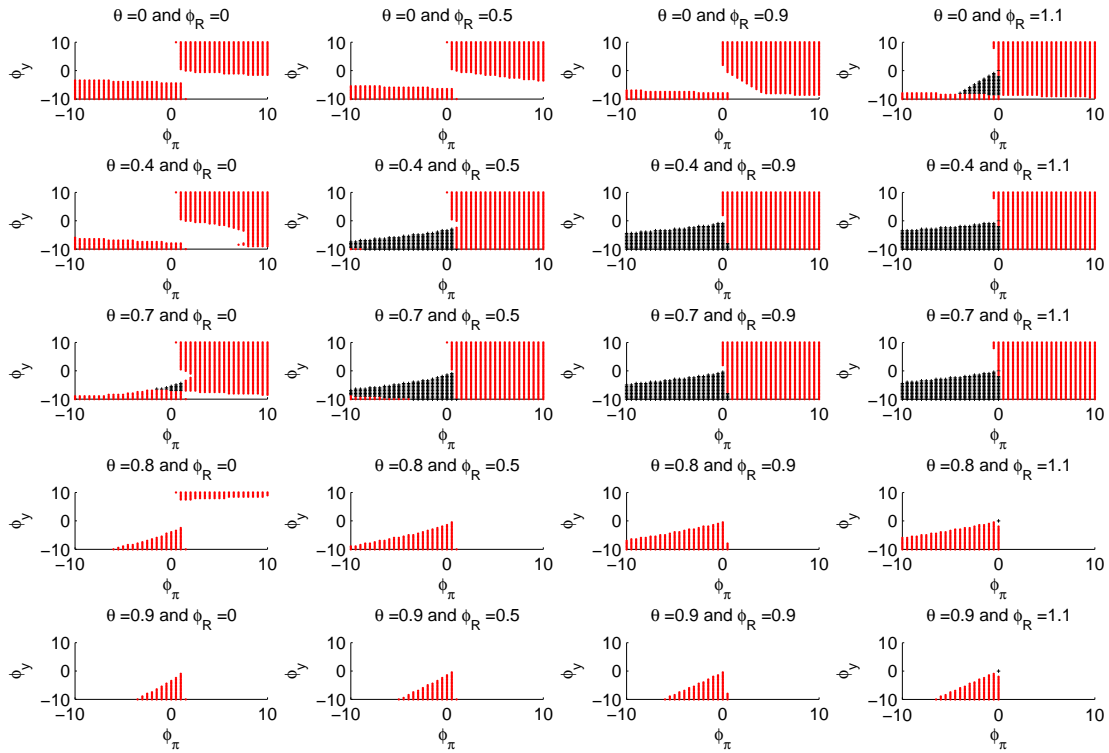


Figure 6: Determinacy properties of the model with *deep habits*, when monetary policy follows the rule $\widehat{R}_t = \phi_\pi \widehat{\pi}_t^m + \phi_y \widehat{Y}_t^{gap} + \phi_R \widehat{R}_{t-1}$: determinacy (light grey dots), indeterminacy (blanks), and instability (dark grey stars).

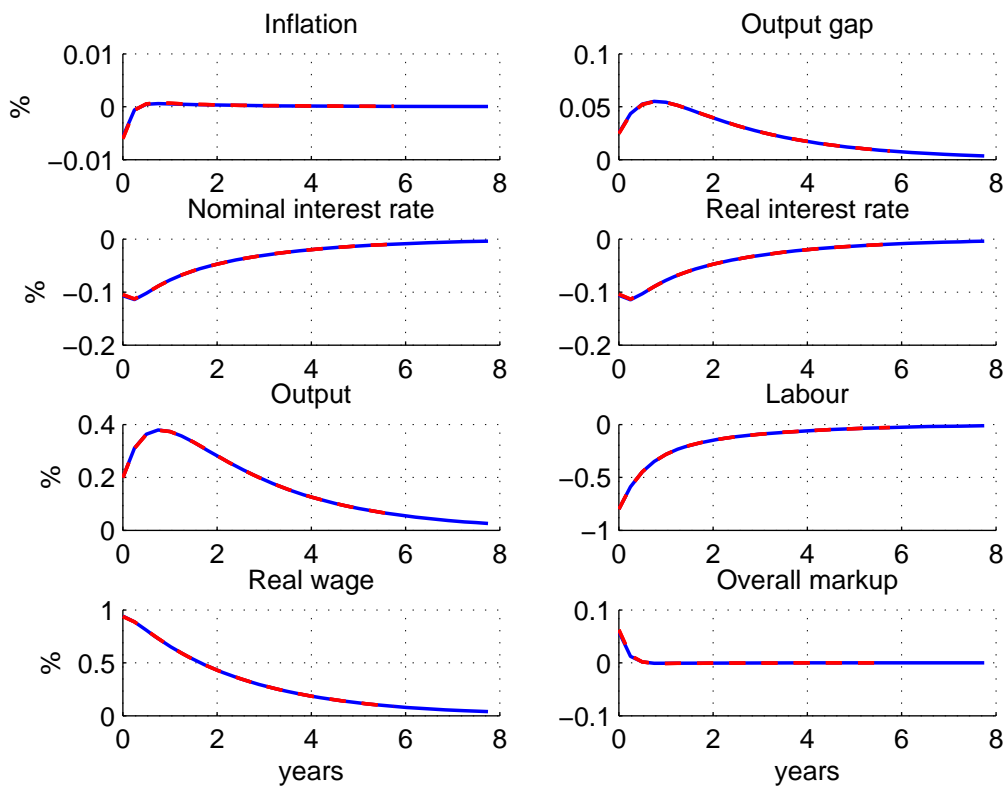


Figure 7: Impulse responses to a 1% positive technology shock in the model with *superficial habits* with $\theta = 0.65$, under the optimal Taylor rule (solid lines) and optimal commitment policy (dash lines).

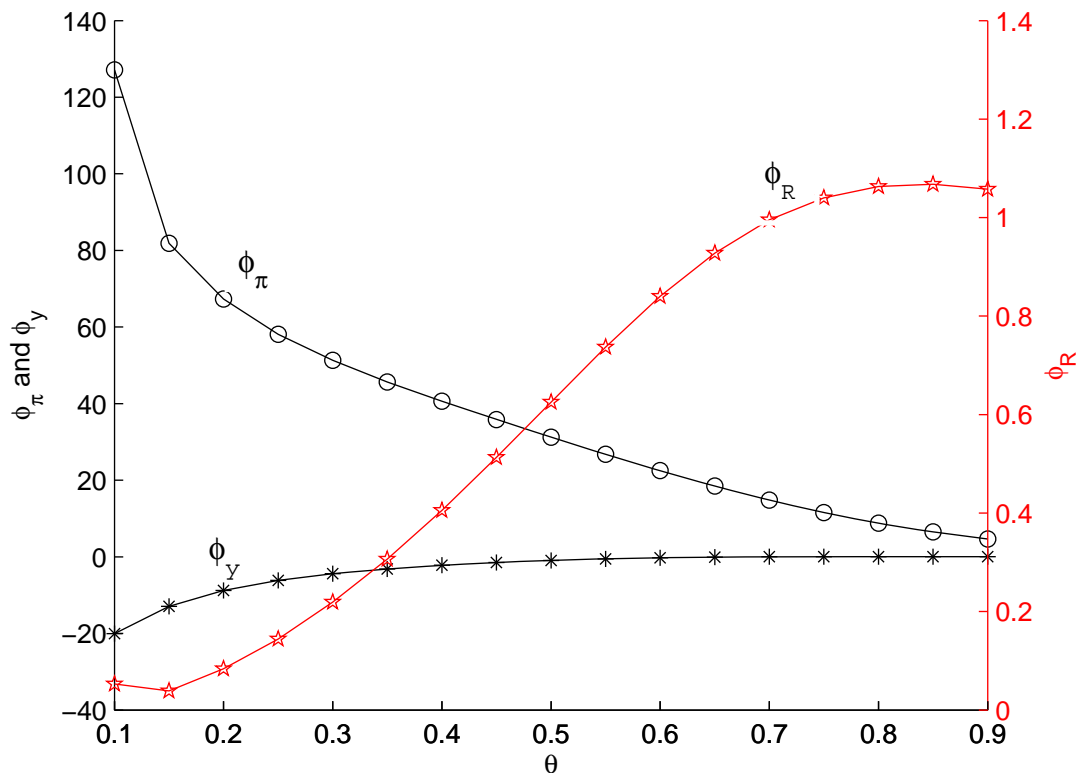


Figure 8: Optimal policy rule parameters for varying degrees of *superficial habits*.

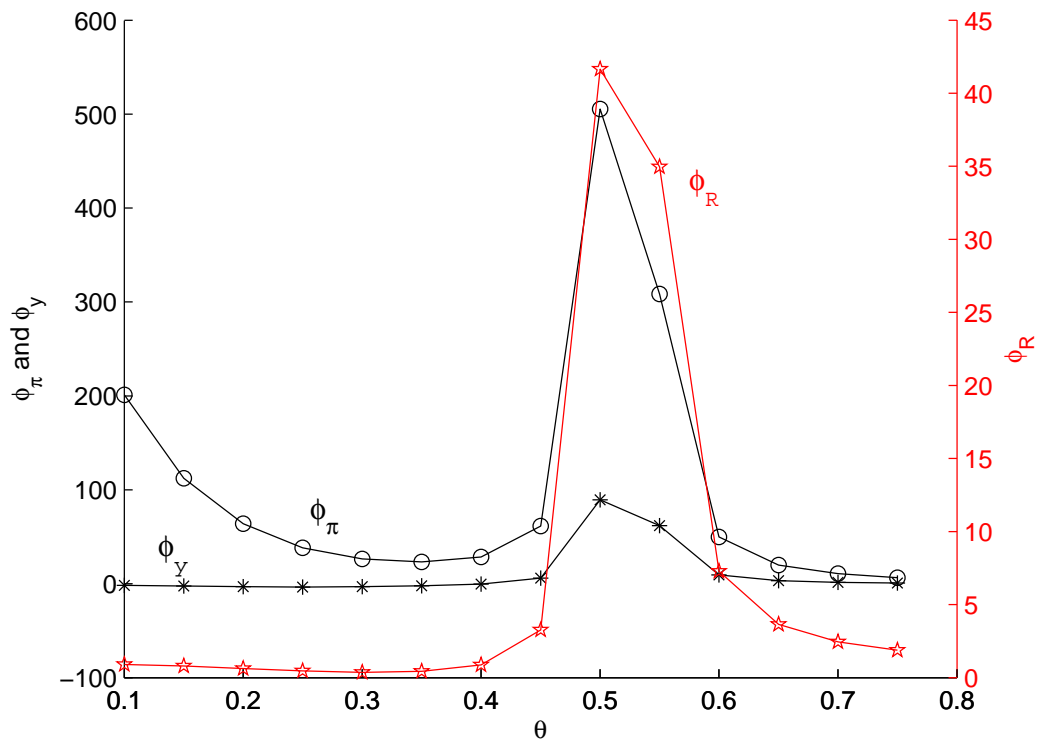


Figure 9: Optimal policy rule parameters for varying degrees of *deep habits*.

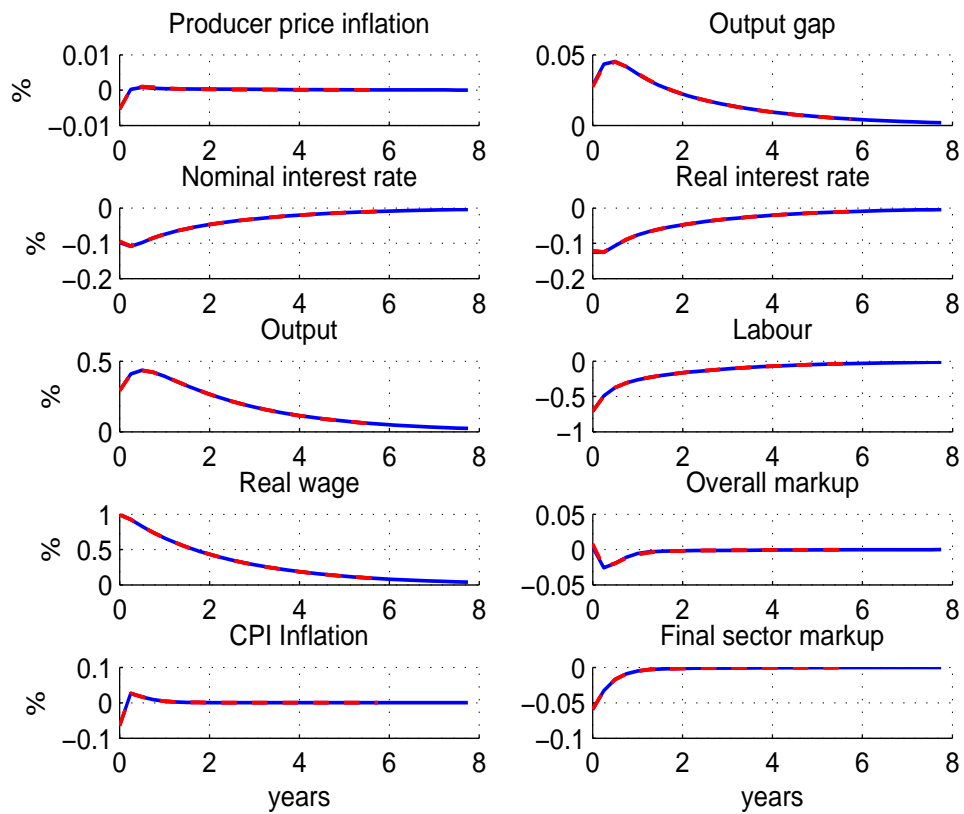


Figure 10: Impulse responses to a 1% positive technology shock in the model with *deep habits* with $\theta = 0.5$, under the optimal Taylor rule (solid lines) and optimal commitment policy (dash lines).

A Analytical Details

A.1 Households

Cost Minimization Households decide the composition of the consumption basket to minimize expenditures

$$\begin{aligned} & \min_{\{C_{it}^k\}_i} \int_0^1 P_{it} C_{it}^k di \\ \text{s.t.} & \left(\int_0^1 \left(C_{it}^k - \theta C_{it-1} \right)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}} \geq \bar{X}_t^k \end{aligned}$$

The demand for individual goods i is

$$C_{it}^k = \left(\frac{P_{it}}{P_t} \right)^{-\eta} X_t^k + \theta C_{it-1},$$

where P_t is the overall price level, expressed as an aggregate of the good i prices, $P_t = \left(\int_0^1 P_{it}^{1-\eta} di \right)^{\frac{1}{1-\eta}}$.

Utility Maximization The solution to the utility maximization problem is obtained by solving the Lagrangian function,

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left[u \left(X_t^k, N_t^k \right) - \lambda_t^k \left(P_t X_t^k + P_t \vartheta_t + Q_{t,t+1} D_{t+1}^k - W_t N_t^k - D_t^k - \Phi_t + T_t \right) \right].$$

In the budget constraint, we have re-expressed the total spending on the consumption basket, $\int_0^1 P_{it} C_{it}^k di$, in terms of quantities that affect the household's utility,

$$\int_0^1 P_{it} C_{it}^k di = P_t X_t^k + P_t \vartheta_t,$$

where under deep habits ϑ_t is given as $\vartheta_t \equiv \theta \int_0^1 \left(\frac{P_{it}}{P_t} \right) C_{it-1} di$, while under superficial habits it takes the simpler form, $\vartheta_t \equiv \theta C_{t-1}$. Households take ϑ_t as given when maximising utility.

The first order conditions are then,

$$(X_t^k) : \quad u_X(t) = \lambda_t^k P_t$$

$$(N_t^k) : \quad -u_N(t) = u_X(t) \frac{W_t}{P_t}$$

$$(D_t^k) : \quad 1 = \beta E_t \left[\frac{u_X(t+1)}{u_X(t)} \frac{P_t}{P_{t+1}} \right] R_t$$

where $R_t = \frac{1}{E_t[Q_{t,t+1}]}$ is the one-period gross return on nominal riskless bonds.

With utility given by $u(X, N) = \frac{X^{1-\sigma}}{1-\sigma} - \chi \frac{N^{1+v}}{1+v}$, the first derivatives are

$$u_X(\cdot) = X^{-\sigma} \quad \text{and} \quad u_N(\cdot) = -\chi N^v.$$

A.2 Final Goods Producers

Final goods producers choose the amount of intermediate inputs to minimize the cost of production subject to the available technology

$$\begin{aligned} & \min_{\{Y_{jit}\}_j} \int_0^1 P_{jit} Y_{jit} dj \\ \text{s.t.} \quad & \left(\int_0^1 (Y_{jit})^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \geq \bar{Y}_{it} \end{aligned}$$

The resulting demand functions are:

$$Y_{jit} = \left(\frac{P_{jit}}{P_{it}^m} \right)^{-\varepsilon} Y_{it}, \quad \forall j \quad (35)$$

where $P_{it}^m = \left(\int_0^1 P_{jit}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$ is an aggregate of intermediate goods prices. Profits are defined as: $\Phi_{it} \equiv P_{it} Y_{it} - \int_0^1 P_{jit} Y_{jit} dj = (P_{it} - P_{it}^m) Y_{it}$.

Due to the dynamic nature of the demand they face, final goods producers choose processes for P_{it} and Y_{it} to maximize the present discounted value of profits, under the restriction that all demand be satisfied at the chosen price ($C_{it} = Y_{it}$):

$$\begin{aligned} \max_{\{P_{it}, Y_{it}\}} E_t \sum_{s=0}^{\infty} Q_{t,t+s} \Phi_{it+s} &= E_t \sum_{s=0}^{\infty} Q_{t,t+s} (P_{it+s} - P_{it+s}^m) Y_{it+s} \\ \text{s.t.} Y_{it+s} &= \left(\frac{P_{it+s}}{P_{t+s}} \right)^{-\eta} X_{t+s} + \theta Y_{it+s-1} \\ Q_{t,t+s} &= \beta^s \left(\frac{X_{t+s}}{X_t} \right)^{-\sigma} \frac{P_t}{P_{t+s}} \end{aligned}$$

The first order conditions are:

$$v_{it} = (P_{it} - P_{it}^m) + \theta E_t [Q_{t,t+1} v_{it+1}]$$

and

$$Y_{it} = v_{it} \left[\eta \left(\frac{P_{it}}{P_t} \right)^{-\eta-1} \frac{X_t}{P_t} \right]$$

where v_{it} is the Lagrange multiplier on the dynamic demand constraint and represents the shadow price of producing good i .

A.3 Intermediate Goods Producers

The cost minimization of intermediate goods producers involves the choice of labour input N_{jit} subject to the available production technology

$$\begin{aligned} \min_{N_{jit}} (1 - \varkappa) W_t N_{jit} \\ \text{s.t. } A_t N_{jit} = Y_{jit} \end{aligned}$$

Costs are subsidized at rate \varkappa , which is determined to ensure that the long-run equilibrium of the economy is efficient. The minimization problem implies a labour demand, $N_{jit} = \frac{Y_{jit}}{A_t}$, and a nominal marginal cost which is the same across all brand producing firms, $MC_t = (1 - \varkappa) \frac{W_t}{A_t}$. Profits are defined as:

$$\begin{aligned} \Phi_{jit} &\equiv P_{jit} Y_{jit} - (1 - \varkappa) W_t N_{jit} = P_{jit} Y_{jit} - (1 - \varkappa) W_t \frac{Y_{jit}}{A_t} \\ &= (P_{jit} - MC_t) Y_{jit} \end{aligned}$$

The profit maximization is subject to the Calvo-style of price setting behavior where, with fixed probability $(1 - \alpha)$ each period, a firm can set its price and with probability α the firm keeps the price from the previous period. When a firm can set the price it does so in order to maximize the present discounted value of profits, subject to the demand for its own goods. Profits are discounted by the stochastic discount factor, adjusted for the probability of not being able to set prices in future periods:

$$\begin{aligned} \max_{P_{jit}^*} E_t \sum_{s=0}^{\infty} \alpha^s Q_{t,t+s} \Phi_{jit+s} &= E_t \sum_{s=0}^{\infty} \alpha^s Q_{t,t+s} [(P_{jit}^* - MC_{t+s}) Y_{jit+s}] \\ \text{s.t. } Y_{jit+s} &= \left(\frac{P_{jit}^*}{P_{it+s}^m} \right)^{-\varepsilon} Y_{it+s} \\ Q_{t,t+s} &= \beta^s \left(\frac{X_{t+s}}{X_t} \right)^{-\sigma} \frac{P_t}{P_{t+s}} \end{aligned}$$

Optimally, the relative price is set at

$$\frac{P_{jit}^*}{P_t} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{E_t \sum_{s=0}^{\infty} (\alpha\beta)^s (X_{t+s})^{-\sigma} mc_{t+s} (P_{it+s}^m)^\varepsilon Y_{it+s}}{E_t \sum_{s=0}^{\infty} (\alpha\beta)^s (X_{t+s})^{-\sigma} \left(\frac{P_{t+s}}{P_t} \right)^{-1} (P_{it+s}^m)^\varepsilon Y_{it+s}}$$

where $mc_t = \frac{MC_t}{P_t}$ is the real marginal cost. The relative price can also be expressed as

$$\frac{P_{jit}^*}{P_t} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{K_{1t}}{K_{2t}}$$

where K_{1t} and K_{2t} have a recursive representation:

$$\begin{aligned} K_{1t} &\equiv E_t \sum_{s=0}^{\infty} (\alpha\beta)^s (X_{t+s})^{-\sigma} mc_{t+s} (P_{it+s}^m)^\varepsilon Y_{it+s} \\ &= X_t^{-\sigma} mc_t \mu_t^{-\varepsilon} Y_t + \alpha\beta E_t [K_{1t+1} \pi_{t+1}^\varepsilon] \end{aligned}$$

and

$$\begin{aligned} K_{2t} &\equiv E_t \sum_{s=0}^{\infty} (\alpha\beta)^s (X_{t+s})^{-\sigma} \left(\frac{P_{t+s}}{P_t}\right)^{-1} (P_{it+s}^m)^\varepsilon Y_{it+s} \\ &= X_t^{-\sigma} \mu_t^{-\varepsilon} Y_t + \alpha\beta E_t [K_{2t+1} \pi_{t+1}^{\varepsilon-1}] \end{aligned}$$

B Equilibrium Conditions

B.1 Aggregation and Symmetry

Aggregate Output: The market clearing condition at the level of intermediate goods is

$$\left(\frac{P_{jit}}{P_{it}^m}\right)^{-\varepsilon} Y_{it} = A_t N_{jit}, \quad \forall j, \forall i$$

which, upon aggregation across the j firms, becomes:

$$Y_{it} \Delta_{it} = A_t N_{it}, \quad \forall i$$

where $\Delta_{it} \equiv \int_0^1 \left(\frac{P_{jit}}{P_{it}^m}\right)^{-\varepsilon} dj$ represents intermediate goods price dispersion in sector i . With final goods producing sectors being symmetric, we can drop the i subscript and write the aggregate production function as

$$Y_t = A_t \frac{N_t}{\Delta_t}.$$

Aggregate Profits: The economy wide profits are given by the aggregate profits from final goods producers and intermediate goods producers:

$$\begin{aligned} \Phi_t &= \int_0^1 \Phi_{it} di + \int_0^1 \int_0^1 \Phi_{jit} dj di \\ &= \int_0^1 P_{it} Y_{it} di - (1 - \varkappa) W_t N_t \\ &= P_t Y_t - (1 - \varkappa) W_t N_t \end{aligned}$$

where we have used the assumption of symmetric final goods sectors to obtain the final result.

B.2 System of Non-linear Equations

$$X_t = C_t - \theta C_{t-1} \quad (36)$$

$$\chi N_t^v X_t^\sigma = \frac{W_t}{P_t} \equiv w_t \quad (37)$$

$$X_t^{-\sigma} = \beta E_t [X_{t+1}^{-\sigma} R_t \pi_{t+1}^{-1}] \quad (38)$$

$$Y_t = \eta \omega_t X_t \quad (39)$$

$$\omega_t = \left(1 - \frac{1}{\mu_t}\right) + \theta \beta E_t \left[\left(\frac{X_{t+1}}{X_t}\right)^{-\sigma} \omega_{t+1} \right] \quad (40)$$

$$Y_t = \frac{A_t}{\Delta_t} N_t \quad (41)$$

$$Y_t = C_t \quad (42)$$

$$\Delta_t \equiv \int_0^1 \left(\frac{P_{jt}}{P_t^m}\right)^{-\varepsilon} dj = (1 - \alpha) \left(\frac{P_{jt}^*}{P_t^m}\right)^{-\varepsilon} + \alpha (\pi_t^m)^\varepsilon \Delta_{t-1} \quad (43)$$

$$(P_t^m)^{1-\varepsilon} = \alpha (P_{t-1}^m)^{1-\varepsilon} + (1 - \alpha) (P_{jt}^*)^{1-\varepsilon} \quad (44)$$

$$\frac{P_{jt}^*}{P_t} = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{K_{1t}}{K_{2t}} \quad (45)$$

$$\text{where} : K_{1t} = u_X(t) m c_t \mu_t^{-\varepsilon} Y_t + \alpha \beta E_t [K_{1t+1} \pi_{t+1}^\varepsilon] \quad (46)$$

$$: K_{2t} = u_X(t) \mu_t^{-\varepsilon} Y_t + \alpha \beta E_t [K_{2t+1} \pi_{t+1}^{\varepsilon-1}] \quad (47)$$

$$m c_t = (1 - \varkappa) \frac{W_t/P_t}{A_t} \quad (48)$$

$$\pi_t^m = \pi_t \frac{\mu_{t-1}}{\mu_t} \quad (49)$$

$$\ln A_t = \rho \ln A_{t-1} + \epsilon_t \quad (50)$$

Definitions:

$$\pi_t \equiv \frac{P_t}{P_{t-1}}$$

$$\pi_t^m \equiv \frac{P_t^m}{P_{t-1}^m}$$

$$\mu_t \equiv \frac{P_t}{P_t^m}$$

B.3 The Deterministic Steady State

The non-stochastic long-run equilibrium is characterized by constant real variables and nominal variables growing at a constant rate. The equilibrium conditions (36) - (50) reduce to:

$$X = (1 - \theta) C \quad (51)$$

$$\chi N^\nu X^\sigma = w \quad (52)$$

$$1 = \beta (R\pi^{-1}) = \beta r \quad (53)$$

$$\omega = [(1 - \theta)\eta]^{-1} \quad (54)$$

$$\mu = [1 - (1 - \theta\beta)\omega]^{-1} \quad (55)$$

$$Y = A \frac{N}{\Delta} \quad (56)$$

$$Y = C \quad (57)$$

$$\Delta = \frac{1 - \alpha}{1 - \alpha(\pi^m)^\varepsilon} \left(\frac{P^*}{P} \right)^{-\varepsilon} \mu^{-\varepsilon} \quad (58)$$

$$1 = \alpha(\pi^m)^{\varepsilon-1} + (1 - \alpha) \left(\frac{P^*}{P} \right)^{1-\varepsilon} \mu^{1-\varepsilon} \quad (59)$$

$$\frac{P^*}{P} = \frac{\varepsilon}{\varepsilon - 1} \frac{K_1}{K_2} = \left[\frac{\varepsilon}{\varepsilon - 1} \frac{1 - \alpha\beta\pi^{\varepsilon-1}}{1 - \alpha\beta\pi^\varepsilon} \right] mc \quad (60)$$

$$K_1 = \frac{u_X mc \mu^{-\varepsilon} Y}{1 - \alpha\beta\pi^\varepsilon} \quad (61)$$

$$K_2 = \frac{u_X \mu^{-\varepsilon} Y}{1 - \alpha\beta\pi^{\varepsilon-1}} \quad (62)$$

$$mc = (1 - \varkappa) \frac{w}{A} \quad (63)$$

$$\pi^m = \pi$$

$$A = 1$$

Table 1 contains the imposed calibration restrictions. We assume values for the real interest rate, the Frisch labour supply elasticity, the steady state inflation, and the following parameters, σ , η , ε , α , θ , and χ . The discount factor β matches the assumed real rate of interest, $\beta = r^{-1}$, while, given the specification of the utility function, ν is the inverse of the Frisch labour supply elasticity, $\varepsilon_{Nw} = \frac{1}{\nu}$.

The steady state values of the shadow price ω and the markup μ are given by equations (54) and (55). The relative price $\frac{P^*}{P}$ can then be obtained from equation (59)

as,

$$\frac{P^*}{P} = \left[\frac{1}{1-\alpha} \left(1 - \alpha (\pi^m)^{\varepsilon-1} \right) \right]^{\frac{1}{1-\varepsilon}} \mu^{-1},$$

while equation (58) gives the steady state value of price dispersion Δ and, from equation (60), the marginal cost is

$$mc = \left(\frac{P^*}{P} \right) \left[\frac{\varepsilon}{\varepsilon-1} \frac{1 - \alpha\beta\pi^{\varepsilon-1}}{1 - \alpha\beta\pi^\varepsilon} \right]^{-1}.$$

Under the assumption of zero steady state inflation ($\pi = \pi^m = 1$), the following long-run equilibrium expressions simplify to:

$$\begin{aligned} \frac{P^*}{P} &= \frac{1}{\mu} \\ \Delta &= 1 \\ mc &= \frac{1}{\mu} \left(\frac{\varepsilon-1}{\varepsilon} \right). \end{aligned}$$

To determine the steady state value of labour, we substitute for X in terms of Y in (52) and then, using the aggregate production function, we obtain the following expression,

$$\chi N^{\sigma+\nu} [(1-\theta)A]^\sigma = w, \quad (64)$$

which can be solved for N . Note that this expression depends on the real wage w , which can be obtained from equation (63). However, in order to make the long-run equilibrium efficient, we impose the condition

$$w = 1 - \theta\beta.$$

This condition is equivalent to setting the cost subsidy \varkappa so as to ensure that the allocation under the decentralized equilibrium (64) matches the social planner's allocation (75), i.e. $\varkappa = 1 - \frac{1}{1-\theta\beta} \left(\frac{1}{\mu} \frac{\varepsilon-1}{\varepsilon} \right)$. See Appendix D for the social planner's problem.

Finally, equations (56), (57), and (51) can be solved for aggregate output Y , consumption C and habit-adjusted consumption X .

B.4 System of Log-linear Equations

Log-linearizing the equilibrium conditions (36) - (50) around the efficient deterministic steady state with zero inflation gives the following set of equations:

$$\begin{aligned}\widehat{X}_t &= (1 - \theta)^{-1} \left(\widehat{C}_t - \theta \widehat{C}_{t-1} \right) \\ \sigma \widehat{X}_t + v \widehat{N}_t &= \widehat{w}_t\end{aligned}\quad (65)$$

$$\begin{aligned}\widehat{X}_t &= E_t \widehat{X}_{t+1} - \frac{1}{\sigma} \left(\widehat{R}_t - E_t \widehat{\pi}_{t+1} \right) \\ \widehat{Y}_t &= \widehat{w}_t + \widehat{X}_t \\ \widehat{w}_t &= \frac{1}{\mu\omega} \widehat{\mu}_t + \theta\beta E_t \widehat{w}_{t+1} + \theta\beta\sigma \left(\widehat{X}_t - E_t \widehat{X}_{t+1} \right) \\ \widehat{Y}_t &= \widehat{A}_t + \widehat{N}_t - \widehat{\Delta}_t\end{aligned}\quad (66)$$

$$\begin{aligned}\widehat{Y}_t &= \widehat{C}_t \\ \widehat{\Delta}_t &= \alpha \widehat{\Delta}_{t-1} \\ \widehat{P}_t^m &= \alpha \widehat{P}_{t-1}^m + (1 - \alpha) \widehat{P}_t^*\end{aligned}\quad (67)$$

$$\widehat{P}_t^* - \widehat{P}_t = \widehat{K}_{1t} - \widehat{K}_{2t}$$

$$\begin{aligned}\text{where} \quad : \quad \widehat{K}_{1t} &= (1 - \alpha\beta) \left(-\sigma \widehat{X}_t + \widehat{m}c_t - \varepsilon \widehat{\mu}_t + \widehat{Y}_t \right) + \alpha\beta E_t \left[\widehat{K}_{1t+1} + \varepsilon \widehat{\pi}_{t+1} \right] \\ : \quad \widehat{K}_{2t} &= (1 - \alpha\beta) \left(-\sigma \widehat{X}_t - \varepsilon \widehat{\mu}_t + \widehat{Y}_t \right) + \alpha\beta E_t \left[\widehat{K}_{2t+1} + (\varepsilon - 1) \widehat{\pi}_{t+1} \right]\end{aligned}$$

Upon substitution of the expressions in \widehat{K}_{1t} and \widehat{K}_{2t} , we obtain the following expression

$$\widehat{P}_t^* - \widehat{P}_t = \alpha\beta E_t \left(\widehat{P}_{t+1}^* - \widehat{P}_{t+1} \right) + \alpha\beta E_t \widehat{\pi}_{t+1} + (1 - \alpha\beta) \widehat{m}c_t \quad (68)$$

$$\begin{aligned}\widehat{m}c_t &= \widehat{w}_t - \widehat{A}_t \\ \widehat{\pi}_t &= \widehat{\pi}_t^m + \widehat{\mu}_t - \widehat{\mu}_{t-1} \\ \widehat{A}_t &= \rho_A \widehat{A}_{t-1} + \varepsilon_t^A\end{aligned}\quad (69)$$

and, finally the following definitions

$$\begin{aligned}\widehat{\pi}_t &= \widehat{P}_t - \widehat{P}_{t-1} \\ \widehat{\pi}_t^m &= \widehat{P}_t^m - \widehat{P}_{t-1}^m \\ \widehat{\mu}_t &= \widehat{P}_t - \widehat{P}_t^m.\end{aligned}$$

Combine equations (68), (67), (69), together with the definitions of producer price inflation and markup to obtain a New Keynesian Phillips Curve in terms of producer inflation

$$\widehat{\pi}_t^m = \beta E_t \widehat{\pi}_{t+1}^m + \left[\frac{(1 - \alpha\beta)(1 - \alpha)}{\alpha} \right] (\widehat{mc}_t + \widehat{\mu}_t),$$

while the real marginal cost can be re-written as

$$\widehat{mc}_t = \sigma \widehat{X}_t + v \widehat{Y}_t - (1 + v) \widehat{A}_t,$$

where we have used the first order conditions (65) to substitute for the real wage \widehat{w}_t , and the production function (66) and the fact that price dispersion in the linear model is deterministic to write $\widehat{N}_t = \widehat{Y}_t - \widehat{A}_t$.

C Model with Superficial Habits

C.1 Households

Habits are “superficial” when they are formed at the level of the aggregate consumption good. Households derive utility from the habit-adjusted composite good X_t^k ,

$$X_t^k = C_t^k - \theta C_{t-1},$$

where household k 's consumption, C_t^k , is an aggregate of a continuum of final goods, indexed by $i \in [0, 1]$,

$$C_t^k = \left(\int_0^1 (C_{it}^k)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}},$$

with $\eta > 1$ the elasticity of substitution between them and $C_{t-1} \equiv \int_0^1 C_{t-1}^k dk$ the cross-sectional average of consumption.

Cost Minimization Households decide the composition of the consumption basket to minimize expenditures

$$\begin{aligned} & \min_{\{C_{it}^k\}_i} \int_0^1 P_{it} C_{it}^k di \\ \text{s.t.} & \left(\int_0^1 (C_{it}^k)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}} \geq \bar{C}_t^k. \end{aligned}$$

The demand for individual goods i is

$$C_{it}^k = \left(\frac{P_{it}}{P_t} \right)^{-\eta} C_t^k,$$

where $P_t \equiv \left(\int_0^1 P_{it}^{1-\eta} di \right)^{\frac{1}{1-\eta}}$ is the consumer price index. The overall demand for good i is obtained by aggregating across all households

$$C_{it} = \int_0^1 C_{it}^k dk = \left(\frac{P_{it}}{P_t} \right)^{-\eta} C_t. \quad (70)$$

Unlike in the case of deep habits, this demand is *not* dynamic.

C.2 Final Goods Producers

Final goods producers' cost minimization problem is unchanged. However, the profit maximization is the typical static problem whereby firms choose the price to maximize current profits, $\Phi_{it} = (P_{it} - P_{it}^m) Y_{it}$, subject to the demand for their good (70) and under the restriction that all demand be satisfied at the chosen price ($C_{it} = Y_{it}$). The

optimal price is set at a *constant* markup, $\mu = \frac{\eta}{\eta-1}$, over the marginal cost,

$$P_{it} = \mu P_{it}^m.$$

C.3 Equilibrium

With a constant markup in the final goods sectors, the system of equilibrium conditions (36) - (50) in appendix B changes along the following dimensions: in a symmetric equilibrium, producer price inflation equals CPI inflation, $\pi_t = \pi_t^m$. We impose a constant markup, $\mu_t = \mu$, in the pricing equation of intermediate goods firms and we exclude equations (39) and (40), which are no longer valid under constant markup at the level of final goods producers.

In this setup, we obtain the familiar looking New Keynesian Phillips Curve,

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \widehat{mc}_t, \quad \kappa \equiv \frac{(1 - \alpha\beta)(1 - \alpha)}{\alpha}, \quad (71)$$

to which we add the IS curve,

$$\hat{X}_t = E_t \hat{X}_{t+1} - \frac{1}{\sigma} \hat{R}_t + \frac{1}{\sigma} E_t \hat{\pi}_{t+1}, \quad (72)$$

and two equations defining the habit-adjusted consumption and the real marginal cost,

$$\hat{X}_t = \frac{1}{1 - \theta} (\hat{Y}_t - \theta \hat{Y}_{t-1}) \quad (73)$$

$$\widehat{mc}_t = \sigma \hat{X}_t + v \hat{Y}_t - (1 + v) \hat{A}_t. \quad (74)$$

D The Social Planner's Problem

The subsidy level that ensures an efficient long-run equilibrium is obtained by comparing the steady state solution of the social planner's problem with the steady state obtained in the decentralized equilibrium. The social planner ignores the nominal inertia and all other inefficiencies and chooses real allocations that maximize the representative consumer's utility subject to the aggregate resource constraint, the aggregate production function, and the law of motion for habit-adjusted consumption:

$$\begin{aligned} & \max_{\{X_t^*, C_t^*, N_t^*\}} E_0 \sum_{t=0}^{\infty} \beta^t u(X_t^*, N_t^*) \\ \text{s.t. } Y_t^* &= C_t^* \\ Y_t^* &= A_t N_t^* \\ X_t^* &= C_t^* - \theta C_{t-1}^* \end{aligned}$$

The optimal choice implies the following relationship between the marginal rate of substitution between labour and habit-adjusted consumption and the intertemporal marginal rate of substitution in habit-adjusted consumption

$$\frac{\chi (N_t^*)^v}{(X_t^*)^{-\sigma}} = A_t \left[1 - \theta \beta E_t \left(\frac{X_{t+1}^*}{X_t^*} \right)^{-\sigma} \right].$$

The steady state equivalent of this expression can be written as,

$$\chi (N^*)^{v+\sigma} [(1-\theta)A]^\sigma = A(1-\theta\beta). \quad (75)$$

The dynamics of this model are driven by technology shocks to the system of equilibrium conditions composed of the Euler equation, the resource constraint, and the evolution of habit-adjusted consumption. In log-linear form, these are:

$$\widehat{X}_t^* = \theta\beta E_t \widehat{X}_{t+1}^* + \frac{1-\theta\beta}{\sigma} (-v\widehat{N}_t^* + \widehat{A}_t)$$

$$\widehat{Y}_t^* = \widehat{A}_t + \widehat{N}_t^*$$

$$\widehat{X}_t^* = \frac{1}{1-\theta} (\widehat{Y}_t^* - \theta\widehat{Y}_{t-1}^*),$$

which combined yield the following dynamic equation

$$\widehat{Y}_t^* = \theta\beta\zeta E_t \widehat{Y}_{t+1}^* + \theta\zeta \widehat{Y}_{t-1}^* + \left(\frac{1+v}{\delta} \right) \zeta \widehat{A}_t$$

where $\zeta \equiv (1 + \theta^2\beta + \frac{v}{\delta})^{-1}$ and $\delta \equiv \frac{\sigma}{(1-\theta\beta)(1-\theta)}$. In the absence of deep habits, $\theta = 0$, the model reduces to the basic New Keynesian model where $\widehat{Y}_t^* = \left(\frac{1+v}{\sigma+v} \right) \widehat{A}_t$.

E Derivation of Welfare

Individual utility in period t is

$$\frac{X_t^{1-\sigma}}{1-\sigma} - \chi \frac{N_t^{1+v}}{1+v}$$

where $X_t = C_t - \theta C_{t-1}$ is the habit-adjusted aggregate consumption. Before considering the elements of the utility function, we need to note the following general result relating to second order approximations

$$\frac{Y_t - Y}{Y_t} = \widehat{Y}_t + \frac{1}{2} \widehat{Y}_t^2 + O[2]$$

where $\widehat{Y}_t = \ln\left(\frac{Y_t}{\bar{Y}}\right)$ and $O[2]$ represents terms that are of order higher than 2 in the bound on the amplitude of the relevant shocks. This will be used in various places in the derivation of welfare. Now consider the second order approximation to the first term,

$$\frac{X_t^{1-\sigma}}{1-\sigma} = \bar{X}^{1-\sigma} \left(\frac{X_t - \bar{X}}{\bar{X}} \right) - \frac{\sigma}{2} \bar{X}^{1-\sigma} \left(\frac{X_t - \bar{X}}{\bar{X}} \right)^2 + tip + O[2]$$

where *tip* represents ‘terms independent of policy’. Using the results above this can be rewritten in terms of hatted variables

$$\frac{X_t^{1-\sigma}}{1-\sigma} = \bar{X}^{1-\sigma} \left\{ \widehat{X}_t + \frac{1}{2} (1-\sigma) \widehat{X}_t^2 \right\} + tip + O[2].$$

In pure consumption terms, the value of X_t can be approximated to second order by:

$$\widehat{X}_t = \frac{1}{1-\theta} \left(\widehat{C}_t + \frac{1}{2} \widehat{C}_t^2 \right) - \frac{\theta}{1-\theta} \left(\widehat{C}_{t-1} + \frac{1}{2} \widehat{C}_{t-1}^2 \right) - \frac{1}{2} \widehat{X}_t^2 + O[2]$$

To a first order,

$$\widehat{X}_t = \frac{1}{1-\theta} \widehat{C}_t - \frac{\theta}{1-\theta} \widehat{C}_{t-1} + O[1]$$

which implies

$$\widehat{X}_t^2 = \frac{1}{(1-\theta)^2} \left(\widehat{C}_t - \theta \widehat{C}_{t-1} \right)^2 + O[2]$$

Therefore,

$$\frac{X_t^{1-\sigma}}{1-\sigma} = \bar{X}^{1-\sigma} \left\{ \frac{1}{1-\theta} \left(\widehat{C}_t + \frac{1}{2} \widehat{C}_t^2 \right) - \frac{\theta}{1-\theta} \left(\widehat{C}_{t-1} + \frac{1}{2} \widehat{C}_{t-1}^2 \right) + \frac{1}{2} (-\sigma) \widehat{X}_t^2 \right\} + tip + O[2]$$

Summing over the future,

$$\sum_{t=0}^{\infty} \beta^t \frac{X_t^{1-\sigma}}{1-\sigma} = \bar{X}^{1-\sigma} \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1-\theta\beta}{1-\theta} \left(\widehat{C}_t + \frac{1}{2} \widehat{C}_t^2 \right) - \frac{1}{2} \sigma \widehat{X}_t^2 \right\} + tip + O[2].$$

The term in labour supply can be written as

$$\frac{\chi N_t^{1+v}}{1+v} = \chi \bar{N}^{1+v} \left\{ \hat{N}_t + \frac{1}{2} (1+v) \hat{N}_t^2 \right\} + tip + O[2]$$

Now we need to relate the labour input to output and a measure of price dispersion. Aggregating the individual firms' demand for labour yields

$$N_t = \int_0^1 \left(\frac{Y_{it}}{A_t} \right) \int_0^1 \left(\frac{P_{jit}}{P_{it}^m} \right)^{-\varepsilon} dj di.$$

Note that in the absence of any sector specific shocks or heterogeneity, $Y_{it} = Y_t$. It can be shown (see Woodford (2003), Chapter 6) that

$$\begin{aligned} \hat{N}_t &= \hat{Y}_t - \hat{A}_t + \ln \left[\int_0^1 \left(\frac{P_{jit}}{P_{it}^m} \right)^{-\varepsilon} di \right] \\ &= \hat{Y}_t - \hat{A}_t + \frac{\varepsilon}{2} var_j \{ p_{jit} \} + O[2] \end{aligned}$$

so we can write

$$\frac{\chi N_t^{1+v}}{1+v} = \chi \bar{N}^{1+v} \left\{ \hat{Y}_t + \frac{1}{2} (1+v) \hat{Y}_t^2 - (1+v) \hat{Y}_t \hat{A}_t + \frac{\varepsilon}{2} var_j \{ p_{jit} \} \right\} + tip + O[2]$$

Welfare is then given by

$$\begin{aligned} \Gamma_0 &= \bar{X}^{1-\sigma} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1-\theta\beta}{1-\theta} \left(\hat{C}_t + \frac{1}{2} \hat{C}_t^2 \right) - \frac{1}{2} \sigma \hat{X}_t^2 \right\} \\ &\quad - \chi \bar{N}^{1+v} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \hat{Y}_t + \frac{1}{2} (1+v) \hat{Y}_t^2 - (1+v) \hat{Y}_t \hat{A}_t + \frac{\varepsilon}{2} var_j \{ p_{jit} \} \right\} \\ &\quad + tip + O[2] \end{aligned}$$

From the social planner's problem we know, $\bar{X}^{-\sigma} (1-\theta\beta) = \chi \bar{N}^v$ such that $\bar{X}^{1-\sigma} (1-\theta\beta) = (1-\theta) \chi \bar{N}^{1+v}$. If we use the appropriate subsidy to render the steady-state efficient and the second order approximation to the national accounting identity,

$$\hat{C}_t = \hat{Y}_t - \frac{1}{2} \hat{C}_t^2 + \frac{1}{2} \hat{Y}_t^2 + O[2],$$

we can eliminate the level terms and rewrite the loss function as

$$\begin{aligned}\Gamma_0 &= \chi \bar{N}^{1+v} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ -\frac{1}{2} \sigma \left(\frac{1-\theta}{1-\theta\beta} \right) \widehat{X}_t^2 \right\} \\ &\quad - \chi \bar{N}^{1+v} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2} v \widehat{Y}_t^2 - (1+v) \widehat{Y}_t \widehat{A}_t + \frac{\varepsilon}{2} \text{var}_j \{p_{jit}\} \right\} \\ &\quad + tip + O[2]\end{aligned}$$

Using the result from Woodford (2003) that

$$\sum_{t=0}^{\infty} \beta^t \text{var}_j \{p_{jit}\} = \frac{\alpha}{(1-\alpha)(1-\alpha\beta)} \sum_{t=0}^{\infty} \beta^t (\widehat{\pi}_t^m)^2 + tip + O[2]$$

we can write the discounted sum of utility as,

$$\begin{aligned}\Gamma_0 &= -\frac{1}{2} \chi \bar{N}^{1+v} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \sigma \left(\frac{1-\theta}{1-\theta\beta} \right) \widehat{X}_t^2 + v \left(\widehat{Y}_t - \frac{1+v}{v} \widehat{A}_t \right)^2 + \frac{\alpha\varepsilon}{(1-\alpha)(1-\alpha\beta)} (\widehat{\pi}_t^m)^2 \right\} \\ &\quad + tip + O[2]\end{aligned}$$

Note that due to the dynamic nature of the social planner's problem it is not as straightforward to rewrite the welfare function in the usual "gap" form.

E.1 Welfare Measure

We measure welfare as the unconditional expectation of lifetime utility in the stochastic equilibrium,

$$\begin{aligned}W &= E \sum_{t=0}^{\infty} \beta^t u(X_t, N_t) \\ &= \frac{1}{1-\beta} \left\{ \bar{u} - \frac{1}{2} \chi \bar{N}^{1+v} \left[(\delta + v + \theta^2 \delta) \text{var}(\widehat{Y}_t) - 2\theta\delta \text{cov}(\widehat{Y}_t, \widehat{Y}_{t-1}) - 2(1+v) \text{cov}(\widehat{Y}_t, \widehat{A}_t) \right. \right. \\ &\quad \left. \left. + (1+v) \text{var}(\widehat{A}_t) + \frac{\varepsilon}{\kappa} \text{var}(\widehat{\pi}_t^m) \right] \right\}\end{aligned}$$

with $\kappa \equiv \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}$, $\delta \equiv \frac{\sigma}{(1-\theta\beta)(1-\theta)}$, and \bar{u} the steady-state level of the momentary utility. To obtain the above expression we have used the second order approximation of utility derived above (inclusive of *tip* terms) and also imposed the condition that $\text{var}(\widehat{Y}_t) = \text{var}(\widehat{Y}_{t-1})$.

F Optimal Policy: Commitment

Upon substitution of the habit-adjusted consumption term, the central bank's objective function becomes

$$\frac{1}{2}\Omega E_0 \sum_{t=0}^{\infty} \beta^t \left[(\delta + \nu) \widehat{Y}_t^2 - 2\theta\delta\widehat{Y}_t\widehat{Y}_{t-1} + \theta^2\delta\widehat{Y}_{t-1}^2 - 2(1 + \nu) \widehat{Y}_t\widehat{A}_t + \frac{\varepsilon}{\kappa}(\widehat{\pi}_t^m)^2 \right]$$

where $\Omega \equiv \chi\bar{N}^{1+\nu}$ and $\delta \equiv \frac{\sigma}{(1-\theta\beta)(1-\theta)}$ and we re-write the constraints as,

$$\left(\frac{1+\theta}{1-\theta}\right)\widehat{Y}_t = \frac{1}{1-\theta}E_t\widehat{Y}_{t+1} + \frac{1}{\sigma}E_t(\widehat{\pi}_{t+1}^m + \widehat{\mu}_{t+1}) + \frac{\theta}{1-\theta}\widehat{Y}_{t-1} - \frac{1}{\sigma}(\widehat{R}_t + \widehat{\mu}_t)$$

$$\widehat{\pi}_t^m = \beta E_t\widehat{\pi}_{t+1}^m - \kappa_1\widehat{Y}_t - \kappa_2\widehat{Y}_{t-1} - \kappa_3\widehat{A}_t + \kappa\widehat{\mu}_t$$

$$\widehat{\mu}_t = \gamma_1\beta E_t\widehat{Y}_{t+1} - \gamma_2\widehat{Y}_t - \gamma_3\widehat{Y}_{t-1}$$

where

$$\left\{ \begin{array}{l} \kappa \equiv \frac{(1-\alpha\beta)(1-\alpha)}{\alpha} \\ \kappa_1 \equiv -\kappa \left(\frac{\sigma}{1-\theta} + \nu \right) \\ \kappa_2 \equiv \kappa \frac{\sigma\theta}{1-\theta} \\ \kappa_3 \equiv \kappa(1 + \nu) \end{array} \right. \quad \text{and} \quad \left\{ \begin{array}{l} \gamma \equiv \mu\omega \frac{\theta}{1-\theta} \\ \gamma_1 \equiv \gamma(\theta + \sigma) \\ \gamma_2 \equiv \gamma[1 + \theta\beta + \beta\sigma(1 + \theta)] \\ \gamma_3 \equiv -\gamma(1 + \theta\beta\sigma) \end{array} \right.$$

We can then form the Lagrangian function defining the policy problem under commitment as:

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \frac{1}{2}\Omega \left[-(\delta + \nu)\widehat{Y}_t^2 + 2\theta\delta\widehat{Y}_t\widehat{Y}_{t-1} - \theta^2\delta\widehat{Y}_{t-1}^2 + 2(1 + \nu)\widehat{Y}_t\widehat{A}_t - \frac{\varepsilon}{\kappa}(\widehat{\pi}_t^m)^2 \right] \\ -\varsigma_t \left[\left(\frac{1+\theta}{1-\theta}\right)\widehat{Y}_t - \frac{1}{1-\theta}\widehat{Y}_{t+1} - \frac{1}{\sigma}(\widehat{\pi}_{t+1}^m + \widehat{\mu}_{t+1}) - \frac{\theta}{1-\theta}\widehat{Y}_{t-1} + \frac{1}{\sigma}(\widehat{R}_t + \widehat{\mu}_t) \right] \\ -\varphi_t \left[\widehat{\pi}_t^m - \beta\widehat{\pi}_{t+1}^m + \kappa_1\widehat{Y}_t + \kappa_2\widehat{Y}_{t-1} + \kappa_3\widehat{A}_t - \kappa\widehat{\mu}_t \right] \\ -\nu_t \left[\widehat{\mu}_t - \gamma_1\beta\widehat{Y}_{t+1} + \gamma_2\widehat{Y}_t + \gamma_3\widehat{Y}_{t-1} \right] \end{array} \right.$$

The government chooses paths for \widehat{R}_t , \widehat{Y}_t , $\widehat{\pi}_t^m$, and $\widehat{\mu}_t$. The first order condition with respect to the nominal interest rate gives:

$$-\sigma^{-1}E_0\beta^t\varsigma_t = 0, \quad \forall t \geq 0 \quad (76)$$

which implies that the IS curve is not binding and it can therefore be excluded from the optimization problem. Once the optimal rules for the other variables have been obtained, we use the IS curve to determine the path of the nominal interest rate. So,

the central bank now chooses $\{\widehat{Y}_t, \widehat{\pi}_t^m, \widehat{\mu}_t\}$ and the Lagrangian takes the form:

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \frac{1}{2} \Omega \left[-(\delta + v) \widehat{Y}_t^2 + 2\theta\delta \widehat{Y}_t \widehat{Y}_{t-1} - \theta^2 \delta \widehat{Y}_{t-1}^2 + 2(1+v) \widehat{Y}_t \widehat{A}_t - \frac{\varepsilon}{\kappa} (\widehat{\pi}_t^m)^2 \right] \\ -\varphi_t \left[\widehat{\pi}_t^m - \beta \widehat{\pi}_{t+1}^m + \kappa_1 \widehat{Y}_t + \kappa_2 \widehat{Y}_{t-1} + \kappa_3 \widehat{A}_t - \kappa \widehat{\mu}_t \right] \\ -\nu_t \left[\widehat{\mu}_t - \gamma_1 \beta \widehat{Y}_{t+1} + \gamma_2 \widehat{Y}_t + \gamma_3 \widehat{Y}_{t-1} \right] \end{array} \right\}.$$

The first order condition for the markup, $\widehat{\mu}_t$, gives the relationship between the two Lagrange multipliers,

$$\nu_t = \kappa \varphi_t,$$

while for inflation we have the rather usual expression,

$$\widehat{\pi}_t^m = -\frac{\kappa}{\varepsilon \Omega} (\varphi_t - \varphi_{t-1}).$$

The first order condition for output is

$$-\Omega \delta \zeta \widehat{Y}_t + \Omega \delta \theta \widehat{Y}_{t-1} + \Omega (1+v) \widehat{A}_t - \kappa_1 \varphi_t - \gamma_2 \nu_t + \gamma_1 \nu_{t-1} + \beta E_t \left[\Omega \delta \theta \widehat{Y}_{t+1} - \kappa_2 \varphi_{t+1} - \gamma_3 \nu_{t+1} \right] = 0$$

which, after eliminating ν_t based on the above expression relating Lagrange multipliers and collecting terms, becomes

$$\zeta^{-1} \widehat{Y}_t + \left(\frac{\kappa_1 + \kappa \gamma_2}{\delta \Omega} \right) \varphi_t = \theta \beta E_t \widehat{Y}_{t+1} - \left(\frac{\kappa_2 + \kappa \gamma_3}{\delta \Omega} \right) \beta E_t \varphi_{t+1} + \theta \widehat{Y}_{t-1} + \frac{\kappa \gamma_1}{\delta \Omega} \varphi_{t-1} + \left(\frac{1+v}{\delta} \right) \widehat{A}_t \quad (77)$$

where $\zeta \equiv (1 + \theta^2 \beta + \frac{v}{\delta})^{-1}$.

Under full commitment, the central bank ignores past commitments in the first period by setting pre-existing conditions to zero, $\varphi_{-1} = 0$. To find the solution, we solve the system of equations composed of the first order conditions, the three constraints, and the technology shock process.

F.1 Optimal Policy: Discretion

In order to solve the time-consistent policy problem we employ the iterative algorithm of Soderlind (1999), which follows Currie and Levine (1993) in solving the Bellman equation. The per-period objective function can be written in matrix form as $Z_t' Q Z_t$, where $Z_{t+1} = \left[\widehat{A}_{t+1} \quad \widehat{Y}_t \quad E_t \widehat{Y}_{t+1} \quad E_t \widehat{\mu}_{t+1} \quad E_t \widehat{\pi}_{t+1}^m \right]'$ and

$$Q = \frac{1}{2} \Omega \begin{bmatrix} 0 & 0 & -(1+v) & 0 & 0 \\ 0 & \theta^2 \delta & -\theta \delta & 0 & 0 \\ -(1+v) & -\theta \delta & (\delta + v) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\varepsilon}{\kappa} \end{bmatrix}.$$

and the structural description of the economy is given by,

$$Z_{t+1} = AZ_t + Bu_t + \xi_{t+1},$$

where $u_t = [\widehat{R}_t]$, $\xi_{t+1} = [\varepsilon_{t+1}^A \ 0 \ 0 \ 0 \ 0]'$, $A \equiv A_0^{-1}A_1$, $B \equiv A_0^{-1}B_0$,

$$A_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \beta\gamma_1 & 0 & 0 \\ 0 & 0 & \frac{1}{1-\theta} & \sigma^{-1} & \sigma^{-1} \\ 0 & 0 & 0 & 0 & \beta \end{bmatrix}, \quad A_1 = \begin{bmatrix} \rho & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & \gamma_3 & \gamma_2 & 1 & 0 \\ 0 & -\frac{\theta}{1-\theta} & \frac{1+\theta}{1-\theta} & \sigma^{-1} & 0 \\ \kappa_3 & \kappa_2 & \kappa_1 & -\kappa & 1 \end{bmatrix}$$

and

$$B_0 = [0 \ 0 \ 0 \ \sigma^{-1} \ 0]'$$

This completes the description of the required inputs for Soderlind (1999)'s Matlab code which computes optimal discretionary policy.

Parameter	Value	Description
r	$(1.04)^{1/4}$	Real interest rate
σ	2	Inverse of intertemporal elasticity of substitution
η	11.0	Elasticity of substitution across final goods
ε	11.0	Elasticity of substitution across intermediate goods
α	0.75	Degree of price stickiness
θ	0.65	Degree of habit formation
ϵ_{Nw}	4.0	Frisch labour supply elasticity
χ	3.0	Relative weight on labour in the utility function
π	1	Gross CPI inflation rate in steady-state
ρ	0.9	Persistence of exogenous shocks
σ_A	0.009	Standard deviation of exogenous shocks

Table 1: Parameter values used in simulations

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