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João Ricardo Faria, Peter McAdam

The Green Golden Rule:  
habit and anticipation of future  
consumption

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### **Abstract**

We derive the Green Golden Rule (GGR) in the Habit Formation (HF) and Anticipation of Future Consumption (AFC) frameworks. Since consumption is the key variable of GGR, time non-separabilities in preferences over consumption streams, given by the AFC and HF, may have important impacts on the environment and sustainability. We demonstrate that agents who smooth their consumption patterns, according to the HF hypothesis, are more likely to preserve the environment than those who anticipate future consumption or who do not so smooth consumption.

**Keywords:** Sustainability; Environment; Resources; Growth.

**JEL:** D90, Q56

## Non-Technical Summary

The seminal paper of Chichilnisky et al. (1995) introduced the concept of the Green Golden Rule (GGR). This gives the highest indefinitely maintainable level of instantaneous utility for a society concerned with the long-run values of consumption and environmental factors. In their sustainable-growth framework, environmental goods are valued in their own right (i.e., yield utility). Moreover, a state equation governs how the stock of the environmental good changes over time: the net change is given by the difference between its rate of renewal and the consumption rate of the stock. The resulting GGR is then given by the equality between the marginal rate of transformation and the marginal rate of substitution between consumption and the environmental good in the steady state.

The crucial variable of the GGR is thus consumption. Yet, surprisingly, no attention has been given to analyzing the robustness of the GGR to alternative (and otherwise popular) consumption preferences. Our contribution is to bridge that knowledge gap. Specifically, we re-assess the GGR in the light of habit formation in consumption (HF) and Anticipation of Future Consumption preferences (AFC). The AFC and HF models display important characteristics of consumption not captured by the standard Ramsey growth model. The AFC considers a stock of future consumption and the HF a stock of past consumption in the representative agent problem. Therefore, these frameworks allow us to investigate the long-term impact of consumption on the environment when there are time non-separabilities in preferences over consumption.

The AFC approach has psychological origins and was created to address anomalies in traditional discounted utility theory and explain some empirical puzzles. In the AFC framework, utility is a function of both current consumption and a reference consumption level based on expected future consumption.

The habit formation (HF) hypothesis was created to solve some empirical puzzles such as equity-premium puzzle. The basic assumption is that the utility function reflects present and past consumption, which is given by a stock of habits (a weighted average of past consumption). The HF explains the empirical regularity that consumption is excessively sensitive and smooth.

Our main result is that consumption smoothing is good for sustainability and helps environmental preservation. Contrasted with alternative consumption patterns (i.e., the neo-classical growth model and the AFC), the model with habit formation yields a greater stock of long-run renewable environmental good.

## 1 Introduction

The seminal paper of Chichilnisky et al. (1995) introduced the concept of the Green Golden Rule (GGR). This gives the highest indefinitely maintainable level of instantaneous utility for a society concerned with the long-run values of consumption and environmental factors.<sup>1</sup> In their sustainable-growth framework, environmental goods are valued in their own right (i.e., yield utility). Moreover, a state equation governs how the stock of the environmental good changes over time: the net change is given by the difference between its rate of renewal and the consumption rate of the stock. The resulting GGR is then given by the equality between the marginal rate of transformation and the marginal rate of substitution between consumption and the environmental good in the steady state.

The crucial variable of the GGR is thus consumption. Yet, surprisingly, no attention has been given to analyzing the robustness of the GGR to alternative (and otherwise popular) consumption preferences. Our contribution is to bridge that knowledge gap. Specifically, we re-assess the GGR in the light of habit formation in consumption (HF) and Anticipation of Future Consumption preferences (AFC). The AFC and HF models display important characteristics of consumption not captured by the standard Ramsey model. The AFC considers a stock of future consumption and the HF a stock of past consumption in the representative agent problem. Therefore, these frameworks allow us to investigate the long-term impact of consumption on the environment when there are time non-separabilities in preferences over consumption.

The AFC approach has psychological origins (e.g., Loewenstein, 1987), and was created to address anomalies in traditional discounted utility theory and explain some empirical puzzles. In the AFC framework, utility is a function of both current consumption and a reference consumption level based on expected future consumption (e.g., Monteiro and Turnovsky, 2016). Kuznitz et al. (2008) show that AFC preferences reduce the mean allocation to stocks, and that agents save more and invest less in risky assets.

The habit formation (HF) hypothesis was created to solve some empirical puzzles such as equity-premium puzzle (e.g. Constantinides, 1990). The basic assumption is that utility reflects present and past consumption, which is given by a stock of habits (a weighted average of past consumption). The HF explains the empirical regularity that consumption is excessively sensitive and smooth (Chetty and Szeidl, 2016).

Our main result is that consumption smoothing is good for sustainability and helps environmental preservation. Contrasted with alternative consumption patterns (i.e., the neo-classical growth model and the AFC), the model with habit formation yields a greater stock of long-run renewable environmental good.

## 2 The Green Golden Rule

The basic model adapts the Ramsey framework to include a proxy for environmental capital. It is derived from Dasgupta and Heal (1974) and Chichilnisky et al. (1995). The environmental

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<sup>1</sup> The conventional golden rule is the growth path that yields the maximum amount of per-capita consumption to members of each present and future generation.

capital is a renewable environmental good  $A$ , which yields utility. Utility is given by the strictly concave function  $U(C, A)$ , where  $C$  is the consumption of output. Output is produced according to the linear homogeneous production function  $F(K, A)$ , where  $K$  is the stock of capital. Capital accumulation is given by:

$$\dot{K} = F(K, A) - C \quad (1)$$

where  $\dot{K} = dK/dt$ . The dynamics of the renewable environmental good  $A$  is described by the difference between the rate of renewal,  $R(A)$ , satisfying  $R(0) = 0$ , and consumption:

$$\dot{A} = R(A) - C \quad (2)$$

We assume the net rate of renewal is given by a logistic reproduction function:<sup>2</sup>

$$R(A) = rA \left[ 1 - \frac{A}{A^S} \right] \quad (3)$$

which is bounded above, exhibits a threshold effect and is negative when  $A > A^S$  (where  $A^S$  is the ‘carrying capacity’ of the environment). Note that solving the derivative  $R_A(A) = 0$  yields the stock corresponding to the maximum sustainable yield (MSY):  $A = A^S/2$ .<sup>3</sup>

The utilitarian optimization problem for an isoelastic instantaneous utility function is:

$$Max_C \int_0^{\infty} \frac{(C^\alpha A^{1-\alpha})^{1-\sigma}}{1-\sigma} e^{-\delta t} dt \quad (4)$$

subject to equations (2) and (3) and ignoring capital accumulation, (1).<sup>4</sup> Regarding parameters,  $\alpha \in (0, 1)$ ,  $\delta > 0$  is the rate of time preference and  $\sigma > 0$  is the coefficient of relative risk aversion.

In the steady state,  $C = R(A)$ , and the maximum is characterized by the optimality condition:<sup>5</sup>

$$\frac{U_A}{U_C} = -R_A(A) \quad (5)$$

The GGR is defined by (5), namely the equality between the marginal rate of transformation and the marginal rate of substitution between consumption and the environmental good across steady states. In turn, the steady-state of (2) yields  $C = rA [1 - A/A^S]$ . This, combined with utility function (4) and the logistic function (3), yields the GGR of the Ramsey model (denoted  $G$ ):

$$A^G = \frac{A^S}{1 + \alpha} \quad (6)$$

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<sup>2</sup> The ‘1’ in (3) can be thought of as the normalized birth rate, and thus  $1 - \frac{A}{A^S}$  as the birth minus death rate. Later in section 6, we replace this ‘1’ with the generalized term  $\Psi \geq 1$  which represents a subsidy on the birth rate of new resources, for example like a subsidy to the planting of trees. A policy to directly boost the birth rate of new resources through subsidies, would be associated to  $\Psi > 1$ .

<sup>3</sup> Note that  $R_A(A) < 0$  for  $A > A^S/2$ .

<sup>4</sup> This does not change the solution to maximization of long-run utility. However, as Chichilnisky et al. (1995) note, it complicates the solution to the discounted utilitarian criterion.

<sup>5</sup> This is Proposition 1 in Chichilnisky et al. (1995).

Notice  $A^G > A^S/2$ . In other words one can have an optimal stock of the environmental resource larger than MSY due to the direct effect of the environmental good on utility.

### 3 The GGR with Anticipation of Future Consumption

Defining the stock of future per-capita consumption,  $F$ , as a function of future consumption, we have:

$$F = \rho e^{\rho t} \int_{\tau}^{\infty} e^{-\rho\tau} C(\tau) d\tau \quad (7)$$

where  $\rho > 0$  represents the speed of adjustment of the stock of future consumption (the relative weights of future consumption at different times). There is positive but exponentially declining weight to consumption in future periods. The larger is  $\rho$ , the less weight is given to future consumption in  $F$ .

Differentiating (7) with respect to time yields the dynamics of the stock of future consumption:

$$\dot{F} = \rho(F - C) \quad (8)$$

Faria and McAdam (2013) show that the isoelastic utility function is consistent with the stability conditions to introducing  $F$  in the utility function. Accordingly, we adapt the instantaneous utility function in (3) to include  $F$ :

$$Max_C \int_0^{\infty} \frac{((CF^\beta)^\alpha A^{1-\alpha})^{1-\sigma}}{1-\sigma} e^{-\delta t} dt \quad (9)$$

where parameter  $\beta \geq 0$  indexes the importance of the stock of future consumption in utility.

The utilitarian optimization problem with the AFC is to solve (9) subject to equations (2), (3) and (8). The GGR with anticipation of future consumption (denoted  $GF$ ) is found through (5), with the isoelastic instantaneous utility function in (9) and the logistic function in (3) – noting that in the steady state,  $F = C = R(A) \Rightarrow (CF^\beta)^\alpha = C^{\alpha(1+\beta)}$ :

$$A^{GF} = A^S \frac{1 + \alpha\beta}{1 + 2\alpha\beta + \alpha} \quad (10)$$

Naturally,  $A^{GF} \rightarrow A^G$  if  $\beta \rightarrow 0$ .

### 4 The GGR with Habit Formation

We assume the stock of habits is given by (see Ryder and Heal, 1973):

$$H = \rho e^{-\rho t} \int_{-\infty}^t C(\tau) e^{\rho\tau} d\tau \quad (11)$$

Thus,  $H$  is a weighted average of past consumption levels.<sup>6</sup> The larger is  $\rho$ , the less weight is given to past consumption in determining the  $H$ .<sup>7</sup> Differentiating (11) with respect to time yields the dynamics of habit formation:

$$\dot{H} = \rho(C - H) \quad (12)$$

For the  $HF$  we adapt the instantaneous isoelastic utility function proposed by Abel (1990):

$$Max_C \int_0^{\infty} \frac{((C/H^\gamma)^\alpha A^{1-\alpha})^{1-\sigma}}{1-\sigma} e^{-\delta t} dt \quad (13)$$

s.t. equations (2), (3) and (12), where  $\gamma \in [0, 1]$  indexes the importance of habits.<sup>8</sup> Given that in the steady state  $H = C = R(A) \Rightarrow (C/H^\gamma)^\alpha = C^{\alpha(1-\gamma)}$ , this implies,

$$A^{GH} = A^S \frac{1 - \alpha\gamma}{1 - 2\alpha\gamma + \alpha} \quad (14)$$

Analogously with our previous treatment,  $A^{GH} \rightarrow A^G$  if  $\gamma \rightarrow 0$ .

## 5 Model Comparisons

We now compare the models. First, note parameter  $\rho$  – which gives the weight to past consumption in determining the stock of habits  $H$ , and the weight of future consumption in determining the stock of future consumption  $F$  – plays no role in the GGR.<sup>9</sup> Only the utility parameters  $\beta$  and  $\gamma$  associated respectively with  $F$  and  $H$  matter. Second, also conspicuous by their absence in the determination of the GGR, are the time preference and relative risk aversion parameters ( $\delta; \sigma$ ).

It is easy to see that for parameter values such that:<sup>10</sup>

$$\alpha\gamma < \alpha \leq 2\alpha\gamma \leq 1 < 1 + \alpha$$

we have,

$$A^{GH} = A^S \frac{1 + \alpha\beta}{1 + 2\alpha\beta + \alpha} < A^G = A^S \frac{1}{1 + \alpha} < A^{GF} = A^S \frac{1 - \alpha\gamma}{1 - 2\alpha\gamma + \alpha} \quad (15)$$

<sup>6</sup> Note, we ignore the distinction between external and internal habit formation. The literature has shown that in the absence of endogenous labor supply, both specifications yield the same steady state. However, in an interesting contribution, Wendner (2011) shows that in the presence of exogenous technological change (or population growth), a consumption externality always affects the steady state equilibrium, even if labor supply is inelastic. The consumption externality affects the elasticity of marginal utility of consumption. Once there is technical change, the elasticity of marginal utility enters the Euler equation. As a consequence, the elasticity of marginal utility becomes a channel through which a consumption externality affects the steady state equilibrium - even in the absence of elastic labor supply and a consumption-labor tradeoff.

<sup>7</sup> For notational convenience we assume a common value for  $\rho$  in the HF and AFC models.

<sup>8</sup> If  $\gamma = 1, (= 0)$  consumption relative to habit stocks is very important (unimportant).

<sup>9</sup> Note, a possible extension of our exercises would be to analogously consider habits and anticipations of the environmental good directly.

<sup>10</sup> For instance,  $\alpha = 0.5, \gamma = 0.5$ .

The inequalities in (15) yield the important and interesting result that if agents form consumption habits (i.e., smooth their consumption patterns), they are more likely to treat the environment with greater care and to better preserve it relative to those with AFC preferences (or who do not smooth their consumption plans).<sup>11</sup>

## 6 Conclusions

We introduce and analyze the GGR in the AFC and HF frameworks. Since consumption is the key variable of GGR, time non-separabilities in preferences over consumption stream, given by these two frameworks, may have important impacts on the environment. We demonstrate that agents who smooth their consumption patterns, according to the HF hypothesis, are more likely to preserve the environment than agents who anticipate future consumption (or who do not smooth their consumption plans).

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<sup>11</sup> All agents given utility preferences smooth consumption but the stronger the degree of habit formation the lower the elasticity of intertemporal substitution, e.g., Jaccard (2014, section 5.2), and hence the higher the smoothing motive.



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### João Ricardo Faria

Florida Atlantic University, Boca Raton, United States; email: [jfaria@fau.edu](mailto:jfaria@fau.edu)

### Peter McAdam

European Central Bank, Frankfurt am Main, Germany; email: [peter.mcadam@ecb.europa.eu](mailto:peter.mcadam@ecb.europa.eu)

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Postal address 60640 Frankfurt am Main, Germany

Telephone +49 69 1344 0

Website [www.ecb.europa.eu](http://www.ecb.europa.eu)

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