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Attenuating the forward guidance puzzle: implications for optimal monetary policy

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Abstract

We examine the implications of less powerful forward guidance for optimal policy using a sticky-price model with an effective lower bound (ELB) on nominal interest rates as well as a discounted Euler equation and Phillips curve. When the private-sector agents discount future economic conditions more in making their decisions today, an announced cut in future interest rates becomes less effective in stimulating current economic activity. While the implication of such discounting for optimal policy depends on its degree, we find that, under a wide range of plausible degrees of discounting, it is optimal for the central bank to compensate for the reduced effect of a future rate cut by keeping the policy rate at the ELB for longer.

JEL: E52, E58, E61

Keywords: Forward Guidance, Optimal Policy, Discounted Euler Equation, Discounted Phillips Curve, Effective Lower Bound.

Non-technical summary

When current policy rates are at or close to their effective lower bounds, central banks often turn to communication about the future path of policy rates—known as forward guidance—as an alternative means to stimulate economic activity. According to the standard sticky-price model often used in academia and central banks to analyze monetary policy, forward guidance is a powerful substitute for a change in the current policy rate and should be used by central banks to improve welfare when the current policy rate is constrained by the lower bound. In particular, in the standard model, the central bank finds it optimal to announce that it will keep the policy rate at the lower bound for longer than would be warranted by future output and inflation stabilization considerations alone.

An intriguing feature of this standard model is that the economic effects of forward guidance can be implausibly large. This feature—often referred to as the forward guidance puzzle—has generated concern among researchers that the standard model is of limited use for the analysis of forward guidance policies and, as a result, has also generated an interest in modifying the standard model to mitigate the implausibly large effects of forward guidance. A number of recent papers have shown that various economically sensible departures from the standard framework go a long way in attenuating the forward guidance puzzle, but they have done so under the assumption that the interest rate policy is characterized by a simple feedback rule.

In this paper, we examine the implications of attenuating the forward guidance puzzle for the optimal design of forward guidance policy. We do so by introducing private-sector discounting—discounting of the expected future income in the Euler equation and discounting of the expected future marginal costs of production in the Phillips curve—into an otherwise standard sticky-price model and characterizing how the degree of discounting affects optimal commitment policy. When private-sector agents discount future economic conditions more in making their decisions today, an announced cut in future interest rates becomes less effective in stimulating current economic activity. While the implication of such discounting for optimal policy depends on its degree, we find that, under a wide range of plausible degrees of discounting, it is optimal for the central bank to compensate for the reduced effect of a future rate cut by keeping the policy rate at the effective lower bound for longer than in the standard model without private-sector discounting.

1 Introduction

When current policy rates are at or close to their effective lower bounds (ELB), central banks often turn to communication about the future path of policy rates—known as forward guidance—as an alternative means to stimulate economic activity. According to the standard sticky-price model often used in academia and central banks to analyze monetary policy, forward guidance is a powerful substitute for a change in the current policy rate and should be used by central banks to improve welfare when the current policy rate is constrained by the ELB (see Eggertsson and Woodford (2003); and Jung, Teranishi, and Watanabe (2005)). In particular, in this model, the central bank finds it optimal to announce that it will keep the policy rate at the ELB for longer than would be warranted by future output and inflation stabilization considerations alone.

An intriguing feature of this standard model is that the economic effects of forward guidance can be implausibly large (Carlstrom, Fuerst, and Paustian (2015); Del Negro, Giannoni, and Patterson (2015); and Kiley (2016)). This feature—often referred to as the forward guidance puzzle—has generated concern among researchers that the standard model is of limited use for the analysis of forward guidance policies and, as a result, has also generated an interest in modifying the standard model to mitigate the implausibly large effects of forward guidance. As we will review shortly, a number of recent papers have shown that various economically sensible departures from the standard framework go a long way in attenuating the forward guidance puzzle, but they have done so under the assumption that the interest rate policy is characterized by a simple feedback rule.

In this paper, we examine the implications of attenuating the forward guidance puzzle for the optimal design of forward guidance policy. We do so by introducing private-sector discounting—discounting of the expected future income in the Euler equation and discounting of the expected future marginal costs of production in the Phillips curve—into an otherwise standard sticky-price model and characterizing how the degree of discounting affects optimal commitment policy. We assume that the central bank is concerned about inflation and output stabilization. Our setup is motivated by McKay, Nakamura, and Steinsson (2016b) and Gabaix (2016), among others, who relate their models of less powerful forward guidance to the discounted Euler equation and/or the discounted Phillips curve. We begin our analysis by examining the effect of discounting on optimal policy in a three-period model in which the cost and the benefit of adjusting future policy rates can be transparently described. We then move on to examining the effect of discounting in a series of infinite-horizon models and study how the degree of discounting affects optimal policy.

A priori, the implication of less powerful forward guidance for optimal policy is not clear. When forward guidance is less powerful, the central bank may want to promise to keep the policy rate at the ELB for longer, as the economy would not be sufficiently stimulated otherwise. However, it is also plausible that the central bank wants to keep the policy rate at the ELB for a shorter period when forward guidance is less powerful; in an extreme case in which forward guidance has no effect on current economic activities, it would be pointless for the central bank to promise that it will keep the policy rate at the ELB after the natural rate of interest becomes positive.

The main insight from the three-period model in the first part of the paper is as follows. The cost of a commitment by the central bank during a crisis involving the binding ELB constraint to keep the policy rate at the ELB for longer is that the economy experiences a temporary overheating in the aftermath of the crisis. The benefit of such a commitment is that, because of the overheating, the declines in output and inflation are mitigated during the crisis. Because the private-sector discounting makes the economy less sensitive to a future rate change, the discounting reduces both the cost and the benefit of a commitment to keep the policy rate at the ELB for longer. Since the cost and the benefit are both lower, whether the discounting makes it optimal for the central bank to promise to keep future policy rates low for longer or shorter than in the standard model is a quantitative question and depends on the parameters of the model.

Turning to the infinite-horizon models, when there is discounting only in the Euler equation, we find that optimal monetary policy in a liquidity trap entails committing to keeping the policy rate at the ELB for longer than in the standard model without discounting. Even though the policy rate is kept at the ELB for longer with higher discounting, the declines in output and inflation at the outset of the liquidity trap are larger, as the extension of the ELB duration is not large enough to compensate for the reduced power of forward guidance. Likewise, we find that the central bank will find it optimal to keep the policy rate at the ELB for longer in the model with discounting only in the Phillips curve than in the standard model without any discounting.

Even when we allow for discounting in both the Euler equation and the Phillips curve, the optimal duration of keeping the policy rate at the ELB continues to be longer with discounting than without discounting, unless the degree of discounting is large in both equations. When households and price-setters heavily discount their expected future income and marginal costs, respectively, in making their decisions, forward guidance becomes so ineffective that the central bank finds it optimal to keep the policy rate at the ELB for a shorter period than in the standard model.

Given our finding that the implication of less powerful forward guidance importantly depends on the degree of discounting, we will end our analysis by reviewing the evidence regarding the degree of discounting in the Euler equation and the Phillips curve. We find that, through the lens of existing micro-founded models of less powerful forward guidance, minor deviations of the standard Euler equation and the standard Phillips curve are empirically plausible and are enough to meaningfully attenuate the forward guidance puzzle. According to our analysis of the infinite-horizon models, a small degree of discounting makes it optimal for the central bank to keep the policy rate at the ELB for longer. The increase in the optimal ELB duration due to discounting can be quantitatively large. Under some specifications considered in the literature, we find that the discounting increases the expected ELB duration associated with optimal policy by about one year.

An important caveat to our analysis is that it is semi-structural. We are silent—and purposefully so—about the model's primitives that justify the discounting in the log-linearized equilibrium conditions characterizing the households and price-setters. Also, we assume that the central bank's objective function is given by the standard quadratic function of inflation and the output gap. Although this objective function can be justified as the second-order approximation to household welfare in the standard model, it may not be necessarily consistent with household welfare in more micro-founded models of Euler equation and/or Phillips curve discounting.

While it would be ideal to characterize the optimal commitment policy in each of the growing number of micro-founded models of less powerful forward guidance, we think that our semi-structural approach has some merits. First, as we discussed, many different micro-founded models will end up with something that looks like the discounted Euler equation and Phillips curve. As a result, the insights from our analysis can be seen as providing a useful starting point for understanding optimal policy in various micro-founded models of less powerful forward guidance. Second, the minimal departure from the standard model entailed by our approach allows us to sharply characterize the cost and benefit of keeping interest rates at the ELB for an extended period and the way they are affected when the forward guidance becomes less powerful, and our approach facilitates the interpretation of the results on optimal policy. Finally, from a more pragmatic perspective, solving the optimal commitment policy problem in some of the micro-founded models discussed above without abstracting from the ELB constraint is computationally very hard or infeasible.[1]

Our paper builds on recent papers that either examine the forward guidance puzzle in a standard sticky-price model, or propose ways to attenuate the puzzle, or both. Carlstrom, Fuerst, and Paustian (2015) and Kiley (2016) show that the effects of a future rate cut are much smaller in sticky-information models than in sticky-price models. Del Negro, Giannoni, and Patterson (2015) attenuate the forward guidance puzzle by introducing an overlapping-generations structure into an otherwise standard New Keynesian model, while Angeletos and Lian (2016); Gabaix (2016), Haberis, Harrison, and Waldron (2017); and Wiederholt (2015) attenuate the puzzle by departing from the assumption of common knowledge, rational expectations, perfect credibility, and perfect information, respectively. McKay, Nakamura, and Steinsson (2016a) and Kaplan, Moll, and Violante (2016) show that the effect of forward guidance is much smaller in a model with heterogeneous households and incomplete markets than in the standard New Keynesian model with a representative household and complete markets. All of these papers restrict the scope of their analysis to Taylor-type interest rate

[1] In Appendix G, we characterize optimal commitment policy in a structural model of less powerful forward guidance—an overlapping-generation New Keynesian model with perpetual youth—and find that the results from the structural analysis are consistent with those from our semi-structural analysis.

rules in analyzing the dynamics of the economy at the ELB.[2] Our contribution is to depart from the assumption of a suboptimal monetary policy and analyze the implications of attenuated forward guidance for optimal commitment policy when the policy rate is constrained by the ELB.

The two papers closest to ours are Bilbiie (2017) and Andrade, Gaballo, Mengus, and Mojon (2017). Bilbiie (2017) uses a novel framework developed by Bilbiie (2016) to analyze the implications of discounting in the Euler equation for the optimal ELB duration. Andrade, Gaballo, Mengus, and Mojon (2017) build a model in which the presence of "pessimists" who do not believe in the central bank's commitment to keeping the policy rate at the ELB for an extended period attenuates the forward guidance puzzle, and they examine their model's implication for the optimal ELB duration. Our work differs methodologically from theirs, as we characterize the optimal commitment policy whereas they compute the optimal ELB duration assuming that, after liftoff, the policy rate either returns to its steady state immediately (Bilbiie (2017)) or is determined by a simple feedback rule (Andrade, Gaballo, Mengus, and Mojon (2017)). There are also interesting substantive differences between—and similarities across—these two papers on one hand and our paper on the other, which will be discussed in detail in the main body of the paper.

Finally, our work is related to the literature on optimal monetary policy under commitment in the New Keynesian model with the ELB constraint (Eggertsson and Woodford (2003); Jung, Teranishi, and Watanabe (2005); Adam and Billi (2006); and Nakov (2008)). These papers established the desirability of keeping the policy rate at the ELB for an extended period in the New Keynesian model with the standard Euler equation and the standard Phillips curve. Our paper extends their analysis to models with a discounted Euler equation and a discounted Phillips curve, showing that the desirability of "low-for-long" policy survives even in models with discounting and that the central bank often finds it optimal to keep the policy rate low for longer with discounting than without discounting.

The remainder of the paper is organized as follows. Section 2 presents our stylized analysis based on a three-period model. Section 3 extends the analysis to an infinite-horizon model. Section 4 reviews the evidence on the degree of discounting and Section 5 briefly summarizes the result of an optimal policy analysis in a micro-founded model of less powerful forward guidance (discussed in detail in Appendix G). Section 6 concludes.

2 A three-period model

In this section, we examine the implication of private-sector discounting for optimal policy in a deterministic three-period model. For simplicity, we only allow discounting in the Euler equation. The three-period structure allows us to transparently describe how the Euler

[2] Gabaix (2016) studies optimal policy in his model but abstracts from the ELB constraint when characterizing the optimal interest rate policy under commitment.

equation discounting affects the marginal costs and benefits of forward guidance and thus optimal policy.

2.1 Private sector

Time is discrete and indexed by t. The economy starts at $t = 1$ and ends in $t = 3$.[3] There is no uncertainty. In periods 1 and 2, the private-sector equilibrium conditions are given by the following two equations:

$$y_t = (1-\alpha)y_{t+1} - \sigma(i_t - \pi_{t+1} - r_t^n), \quad (1)$$

$$\pi_t = \kappa y_t + \beta \pi_{t+1}, \quad (2)$$

where y_t is the output gap, π_t denotes the inflation rate between periods $t-1$ and t, i_t is the nominal interest rate between periods t and $t+1$ on a risk-free bond, and r_t^n is a demand shock that captures exogenous fluctuations in the natural real rate of interest. Equation (1) is the Euler equation, and equation (2) is the Phillips curve. α is a discounting parameter on the expected output next period. If $\alpha = 0$, these two equations represent the textbook New Keynesian model studied in detail in Woodford (2003) and Gali (2015), among others. In the final period 3, the private-sector equilibrium conditions are given by

$$y_3 = -\sigma(i_3 - r_3^n), \quad (3)$$

$$\pi_3 = \kappa y_3. \quad (4)$$

The demand shock is assumed to be negative in period 1 and stays constant at a positive value in periods 2 and 3. That is, $r_1^n = r_L^n < 0$, and $r_2^n = r_3^n = r^n > 0$.

2.2 Monetary policy

The central bank has the following objective function:

$$u(y, \pi) = -\frac{1}{2}\left(\pi^2 + \lambda y^2\right). \quad (5)$$

At the beginning of the initial period $t = 1$, the central bank chooses sequences of the output gap, inflation, and the nominal interest rate in order to maximize the discounted sum of current and future utility flows. The central bank's optimization problem is given by

$$\max_{\{y_t,\pi_t,i_t\}_{t=1}^3} u(y_1,\pi_1) + \beta u(y_2,\pi_2) + \beta^2 u(y_3,\pi_3),$$

[3] More precisely, we are working with an infinite-horizon model in which the economy is assumed to be at the steady state from period 4 onwards (in a pure three-period model, there would not be any demand for government bonds at period 3). We simply refer to this setup as "the three-period model" for the sake of brevity.

subject to the private-sector equilibrium conditions described earlier and the ELB constraint on the policy rate:

$$i_t \geq i_{ELB},$$

for all $t \in \{1, 2, 3\}$. As a reference, we will also consider how the economy behaves when the nominal interest rate is determined according to the following simple interest rate feedback rule:

$$i_t = \max[r_t^n + \phi_\pi \pi_t, i_{ELB}], \tag{6}$$

where $\phi_\pi > 1$.

2.3 Results

While our simple three-period model can be solved in closed form in principle, the solution turns out to depend on the discounting parameter α in a complicated, nonlinear way. We therefore use a numerical example to describe how discounting affects optimal policy.[4] Table 1 shows the parameter values used for the numerical analysis. For this three-period model, they are chosen not based on empirical realism, but to make the key takeaways from the analysis transparent.

Table 1: Parameter Values: Three-Period Model

Parameter	Description	Values
β	Discount factor	0.9925
σ	Intertemporal elasticity of substitution	1
κ	Slope of the Phillips Curve	0.2
θ	Relative price elasticity of demand	10
λ	Relative weight on output gap volatility (κ/θ)	0.02
α	Discounting parameter in the Euler equation	$[0, 0.5]$
i_{ELB}	The effective lower bound	0
r^n	Long-run natural real rate ($1/\beta - 1$)	0.0075
r_L^n	Natural real rate in period 1	-0.03825

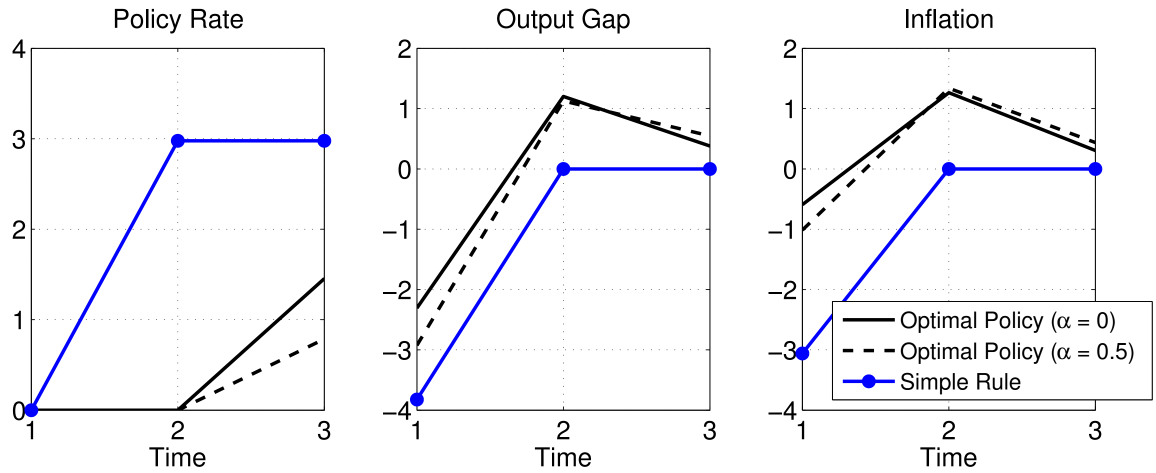
Figure 1 shows the paths of the output gap, inflation and the policy rate in the model with discounting ($\alpha = 0.5$) and in the model without discounting ($\alpha = 0$).[5] Under the simple interest-rate rule, output and inflation are fully stabilized at their steady state values from period 2—when the crisis shock disappears—regardless of the value of α. Because the allocation in period 2 does not depend on α, and because the policy rate in period 1 is constrained at the ELB regardless of α, output and inflation in period 1 do not depend on α.[6] Output declines by about 4 percent and inflation drops by 3 percentage points in period 1.

[4]In Appendix A, we provide some analytical results in a version of the three-period model with a static Phillips curve.

[5]The parameterization $\alpha = 0.5$ in the model with discounting is chosen for illustrative purposes.

[6]In the three-period model of this section and the infinite-horizon model of the next section, the allocation under the simple interest rate rule is identical to the allocation under the optimal discretionary policy.

Figure 1: Optimal Policy in a Three-Period Model
with and without Discounting



Note: Units are annualized percent, annualized percentage points, and percent deviation for policy rate, inflation, and output gap, respectively.

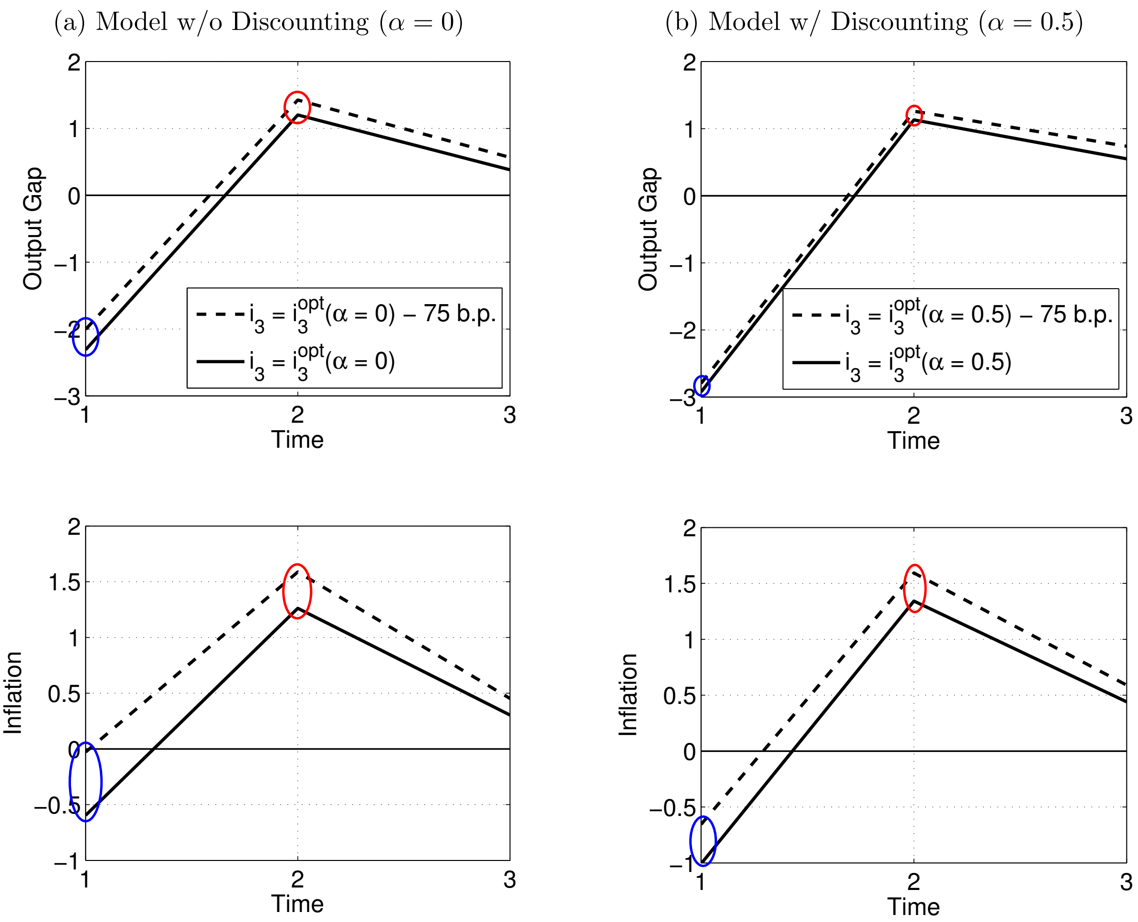
Under the optimal policy, the central bank keeps the policy rate at the ELB in period 2 and chooses the policy rate below the natural rate of interest in period 3 ($i_3 < r_3^n$). This low-for-longer policy results in overshooting of output and inflation in periods 2 and 3, which mitigates the declines in output and inflation in period 1 through expectations. The policy rate in period 3 depends on the degree of discounting, with the central bank in the economy with discounting choosing a lower policy rate than the central bank in the economy without discounting. Even though the policy rate in period 3 is lower under discounting, the initial declines in output and inflation are larger under discounting, reflecting the reduced power of forward guidance under discounting. However, regardless of the degree of discounting, the declines in output and inflation in period 1 are smaller under optimal policy than under the interest rate feedback rule.

To understand how the degree of discounting affects optimal policy, Figure 2 shows how differently an announcement of a future rate cut—here a reduction in i_3 by 75 basis points below the optimal level—affects the paths of output and inflation in the models with and without discounting. In both models, a promise to lower the policy rate further in period 3 from its optimal level mitigates the declines in output and inflation in period 1, but amplifies the overshooting of output and inflation in period 2, as the comparison of dashed and solid lines in each panel of Figure 2 demonstrates. Hence, the rate cut in period 3 bears the benefit of smaller output and inflation deviations from their targets in period 1 and the cost of larger target deviations in period 2 in both models.[7]

Because a rate cut in period 3 has a smaller effect on inflation and output in earlier

[7]The optimal commitment policy accounts for this tradeoff when choosing the level of the policy rate for period 3 so the central bank has no incentive in period 1 to announce a period-3 policy rate that deviates from the one that solves the optimization problem under commitment.

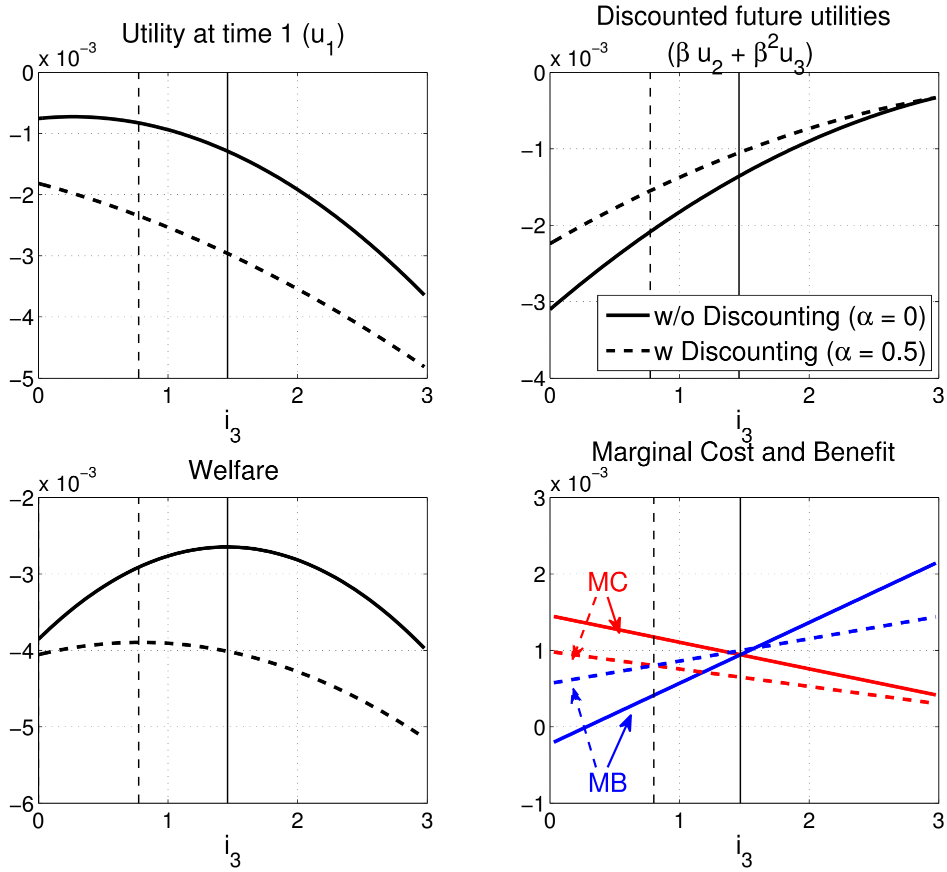
Figure 2: Effects of a Reduction in the Future Policy Rate on the Macroeconomy with and without Discounting



Note: The thin solid line represents the steady states where u_t is at maximum. Units are percent deviation and annualized percentage points for output gap and inflation, respectively.

periods, it reduces and increases the target deviation in periods 1 and 2, respectively, by less in the model with discounting than in the standard model. In our three-period model, for any given level of the third-period policy rate, the degree of discounting does not matter for allocations in period 3, and thus the level of overshooting in period 2 is smaller with discounting than without discounting for any $i_3 < r^n$. As a result, *for any given level of the third-period policy rate*, a smaller increase in the target deviation in period 2 induced by a third-period rate cut means a smaller increase in the welfare cost of overshooting in period 2 in the model with discounting than in the standard model. However, the smaller reduction in the target deviation in period 1 induced by a smaller increase in the target overshooting in period 2 does not necessarily mean that the welfare benefit of a less severe recession is smaller in the model with discounting; for any given level of the third-period policy rate, the recession is more severe to begin with in the model with discounting than in the standard model, and the objective function is quadratic. Thus, a smaller increase in inflation and output in period one can lead to a larger increase in welfare in the model with discounting than in the standard model.

Figure 3: Tradeoff of Adjusting the Future Policy Rate
with and without Discounting



Note: The solid (dashed) vertical line denotes maximum welfare for $\alpha = 0$ ($\alpha = 0.5$) in the bottom two panels. MB and MC stand for marginal welfare benefit and cost, respectively.

Figure 3 shows how the discounting affects the cost and benefit of a future rate cut in detail. In each panel, solid and dashed lines are for the standard model and the model with discounting, respectively. For different values of i_3, the upper left panel plots the central bank's utility flow in period 1, while the upper right panel plots the discounted sum of utility flows in periods 2 and 3. Consistent with the discussion in the previous paragraph, in both the models with and without discounting, the utility flow of the initial period is decreasing in i_3, whereas the discounted sum of utility flows of the second and third periods is increasing in i_3 (as long as $i_3 < r^n$). The bottom left panel shows welfare, the sum of the top two panels. The optimal i_3 that maximizes welfare equates the marginal benefit and cost of reducing the third-period policy rate, which are shown in the bottom right panel. Consistent with the discussion in the previous paragraph, the marginal cost of reducing the third-period policy rate—in the form of a larger overshooting—is smaller in the model with discounting than in the standard model for any level of i_3. However, the marginal benefit of reducing the third-period policy rate—in the form of a less severe recession—may be smaller or larger in the model with discounting than in the standard model. In our numerical example, the marginal benefit is higher in the model with discounting if i_3 is sufficiently small, but is lower otherwise.

Appendix B reinforces the idea that the effect of discounting for optimal policy is ambiguous by providing an alternative parameterization of the three-period model in which the optimal third-period policy rate is higher with discounting than without discounting. Given our finding that the effect of discounting for optimal policy depends importantly on the specifics of the model, we now turn our attention to an infinite-horizon model that is stylized, but is parameterized in a way that can speak to the implications of discounting in a more empirically relevant setup.

3 An infinite-horizon model

In this section, we examine the effect of the private-sector discounting on optimal policy in infinite-horizon models. In addition to discounting in the Euler equation, we allow for discounting in the Phillips curve. The economy is buffeted by two exogenous disturbances, a demand shock (r_t^n) and a cost-push shock (e_t).

3.1 The private sector and monetary policy

The aggregate private-sector behavior is summarized by the following two loglinear equilibrium conditions:

$$y_t = (1-\alpha_1)\mathbb{E}_t y_{t+1} - \sigma(i_t - \mathbb{E}_t\pi_{t+1} - r_t^n), \quad (7)$$

$$\pi_t = \kappa y_t + (1-\alpha_2)\beta\mathbb{E}_t\pi_{t+1} + e_t, \quad (8)$$

where α_1 and α_2 are the discount parameters in the Euler equation and the Phillips curve, respectively.[8] The two exogenous shocks, (r_t^n, e_t), are perfectly correlated, as in Boneva, Braun, and Waki (2016) and Hills and Nakata (2018), and follow a two-state Markov process. In the first period, the economy is in the crisis state. It moves to the normal state—which is assumed to be an absorbing state—with a positive probability $(1-\mu)$ each period.[9] In the crisis state, $r_t^n = r_L^n$ and $e_t = e_L$. In the normal state, $r_t^n = r^n = 1/\beta - 1$ and $e_t = 0$.

At the beginning of the first period, the central bank chooses a state-contingent sequence of (y_t, π_t, i_t) in order to maximize the expected discounted sum of future utility flows given

[8] Some papers, including McKay, Nakamura, and Steinsson (2016b), have considered an alternative specification of the discounted Euler equation in which there is an additional discounting parameter for the expected real rate. The key results of our analysis are robust to this alternative specification of the discounted Euler equation. These results are available upon request.

[9] The assumption of an absorbing state is common in the literature and is without loss of generality for the purpose of this paper. However, the departure from this assumption does have interesting implications, as explored by Nakata (2018) and Nakata and Schmidt (2014).

by

$$\max_{\{y_t,\pi_t,i_t\}_{t=1}^{\infty}} \mathbb{E}_1 \sum_{t=1}^{\infty} \beta^{t-1} u(y_t, \pi_t)$$

subject to equations ((7) - (8)) and the ELB constraint on the policy rate ($i_t \geq i_{ELB}$). The utility flow, $u(\cdot)$, is given by the standard quadratic objective function (see equation (5)). Further details on the optimization problem and the associated optimality conditions are provided in Appendix D.

3.2 Parameterization

Table 2 shows the baseline parameter values. Our choice of $\beta = 0.9925$ implies a real interest rate in the normal state of 3 percent (annualized). $\sigma = 1$ is consistent with the log-utility specification of the household preference for consumption in the standard intertemporal optimization problem of the household. $\theta = 8$ is within the range of values found in the literature (see Broda and Weinstein (2006); Denes, Eggertsson, and Gilbukh (2013)). $\kappa = 0.007$ is consistent with a Calvo parameter of 0.84 in the Calvo model of price-setting. Our choice of $\mu = 5/6$ is within the range of values used in the literature and implies an expected duration of crisis of one and a half year. The value of λ is set to be consistent with the value implied by the second-order approximation of the household's welfare.[10] The main exercise of this section is to examine how α_1 and α_2 affect optimal policy. We will consider the range from 0 to 1 for both parameters.

Table 2: Parameter Values: Infinite-Horizon Model

Parameter	Description	Values
β	Discount factor	0.9925
σ	Intertemporal elasticity of substitution	1
κ	Slope of the Phillips Curve	0.007
θ	Relative price elasticity of demand	8
λ	Relative weight on output gap volatility (κ/θ)	0.875×10^{-3}
i_{ELB}	The effective lower bound on the policy rate	0
α_1	Discounting parameter in the Euler equation	$[0, 1]$
α_2	Discounting parameter in the Phillips curve	$[0, 1]$
μ	Persistence probability of the crisis state	$5/6$
(r_t^n, e_t)	(demand shock,cost-push shock)	*Chosen so that $(y_1, \pi_1) = (-0.07, -0.01/4)$ under the simple rule

Conditional on these parameter values, we choose the size of the two shocks, r_L^n and e_L, such that output and inflation fall by 7 percent and 1 percentage point, respectively, if the central bank follows the simple interest rate rule described earlier; see equation (6).[11]

[10] In Appendix E, we conduct an analysis in which the value of λ is much higher and is consistent with placing equal weights on inflation and output stabilization objectives, a common practice in central banks.

[11] In the infinite-horizon model with an absorbing state, the allocation under policy rule (6) is identical to the allocation under the optimal discretionary policy.

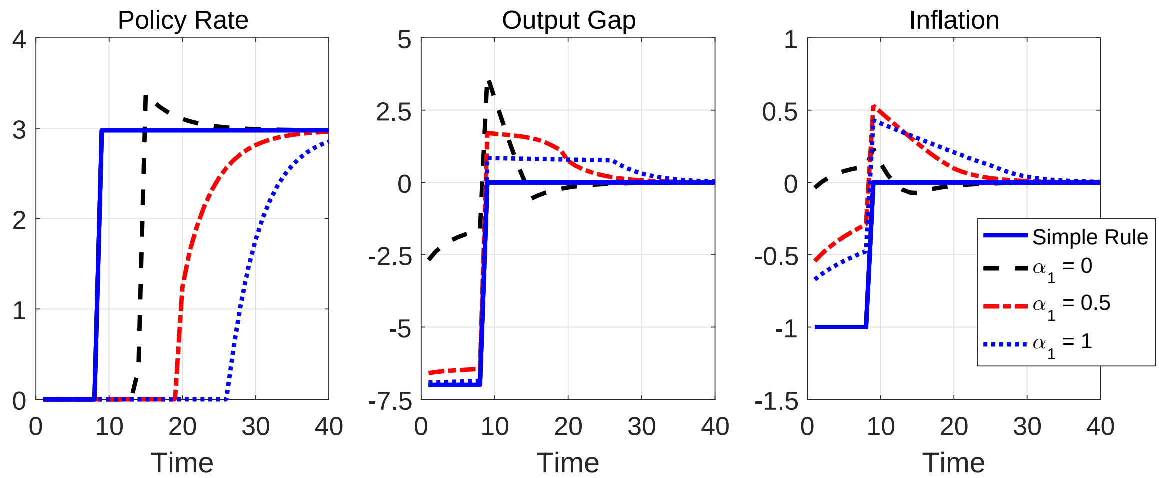
This way of choosing the shock size means that the values of r_L^n and e_L are different when we choose different values of α_1 and α_2. The approach of keeping the severity of the crisis constant as one varies the model's parameter values is adopted by Boneva, Braun, and Waki (2016) and Hills and Nakata (2018), who adjust the size of shocks in their models to keep the magnitudes of the declines in output and inflation unchanged when conducting sensitivity analyses.[12,13]

3.3 Results

We will first examine the effect of α_1 on optimal policy while holding $\alpha_2 = 0$ (i.e., the model with discounting only in the Euler equation). We then examine the effect of α_2 on optimal policy while holding $\alpha_1 = 0$ (i.e., the model with discounting only in the Phillips curve). Finally, we will explore the implication of varying both α_1 and α_2.

Figure 4 plots the impulse response functions of the policy rate, inflation and output under the simple interest rate rule and under the optimal monetary policy for various degrees of Euler equation discounting, when the crisis state persists for eight quarters.

Figure 4: Optimal Policy w/ Discounting in the Euler Equation



Note: In all models, $\alpha_2 = 0$. Units are annualized percent, annualized percentage points, and percent deviation for the policy rate, inflation, and the output gap, respectively.

By construction, the allocation under the simple rule does not depend on α_1, because, as we vary α_1, we vary the size of the shocks so that the declines in output and inflation in the crisis state are unchanged under the simple rule. A key feature of the economy under the simple rule is that the economy returns to its steady state once the crisis shock disappears.

[12] Unlike in the three-period model, the discounting factors in the private-sector behavioral constraints will influence the level of output and inflation in the crisis state when monetary policy follows the simple rule. This dependency reflects the fact that, conditional on being in the crisis state, the crisis shock will persist in the subsequent period with positive probability μ, affecting private-sector expectations about future output and inflation.

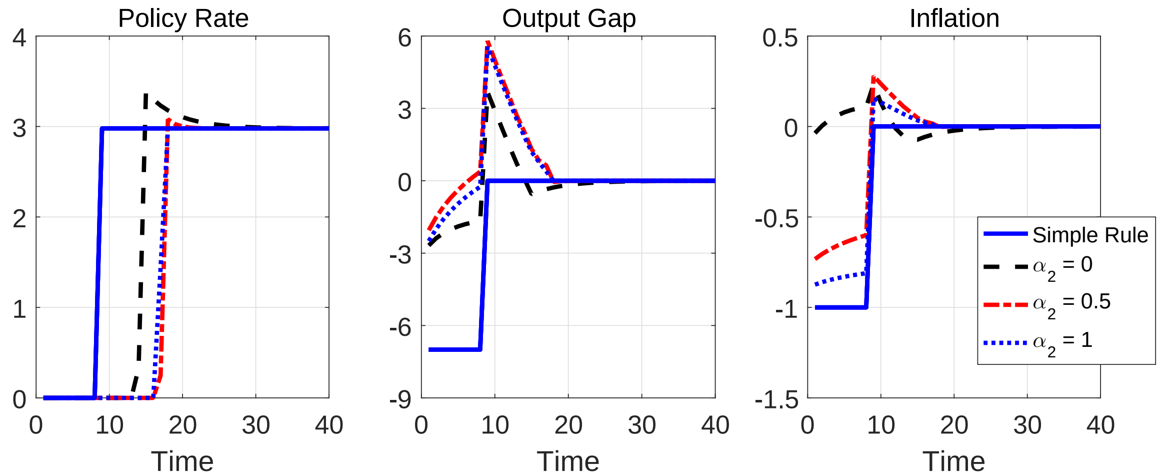
[13] In Appendix F, we conduct an alternative analysis in which the size of shocks are kept constant as we vary discounting parameters.

Under the optimal policy with $\alpha_1 = 0$—which corresponds to the standard Euler equation—the central bank keeps the policy rate at the ELB beyond the point in time where the shocks jump back to the normal state. The policy of keeping the interest rate at the ELB for an extended period stimulates the economy by raising expectations about future output and inflation. The optimality of the low-for-long policy in this standard version of the model is well known (Jung, Teranishi, and Watanabe (2005); Eggertsson and Woodford (2003)).

Comparing the optimal policy for alternative degrees of discounting, we find that the more households discount expected future real interest rates, the longer the central bank keeps the policy rate at zero. When $\alpha_1 = 0$, the policy rate is kept at the ELB for 13 quarters. As α_1 increases, the ELB duration increases monotonically, reaching 26 quarters when $\alpha_1 = 1$. Nevertheless, as α_1 increases, the initial declines in output and inflation are larger, reflecting the diminished effectiveness of a cut in future interest rates. The initial decline in output is more sensitive to the degree of discounting than the initial decline of inflation, reflecting the fact that the discounting is in the Euler equation.

One way to summarize the effect of discounting on optimal policy is to inspect the expected ELB duration under different degrees of discounting. As shown in the left panel of Figure 6, when $\alpha_1 = 0$, the expected ELB duration is about 10 quarters. As α_1 increases, the expected ELB duration increases monotonically and in a quantitatively significant way. When $\alpha_1 = 1$, the expected ELB duration is about 18 quarters, two years longer than when $\alpha_1 = 0$.

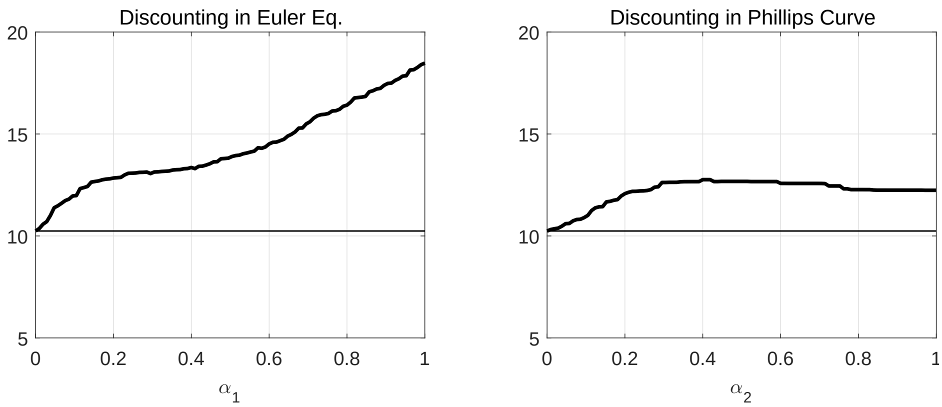
Figure 5: Optimal Policy w/ Discounting in the Phillips Curve



Note: In all models, $\alpha_1 = 0$. Units are annualized percent, annualized percentage points, and percent deviation for the policy rate, inflation, and the output gap, respectively.

We now turn to the analysis of optimal policy in the model with discounting only in the Phillips curve. Figure 5 plots the same set of impulse response functions with alternative degrees of α_2 when the crisis shock lasts for eight quarters. As in the previous model with discounting only in the Euler equation, by construction, the allocations under the simple rule do not depend on the degree of discounting. Under optimal policy, the central bank keeps the policy rate at the ELB for longer in the models with discounting than in the model without discounting. However, unlike in the model with discounting only in the Euler equation, the effect of discounting on the optimal ELB duration is not monotonic, as can be seen in the right panel of Figure 6 which shows the effect of α_2 on the expected ELB duration. As for the allocation, the initial decline in inflation is larger when α_2 is higher, as can be seen in the right panel of Figure 5. Interestingly, the initial decline in output depends on the degree of discounting in a non-monotonic way. As α_2 increases from 0, the initial output decline becomes smaller at first, but increases eventually as α_2 approaches 1, as can be seen in the middle panel of Figure 5.

Figure 6: Expected ELB Durations

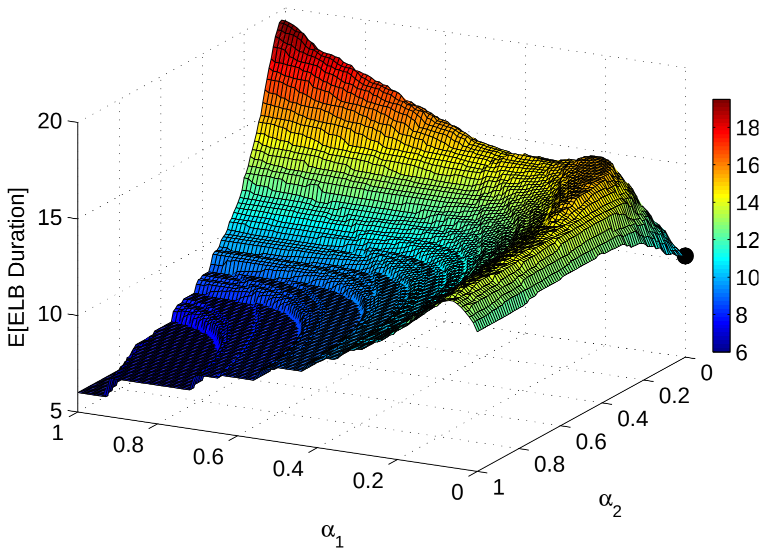


Note: The thin dashed lines represent the expected duration of the crisis shock. Thin dash-dotted lines represent expected ELB duration in the absence of discounting. Units are in quarters.

Finally, we examine the effect of discounting in the model that allows for discounting in both the Euler equation and in the Phillips curve. Figure 7(a) plots the expected ELB duration for various combinations of α_1 and α_2, while Figure 7(b) demarcates the parameter space into the part in which the expected ELB duration is shorter or longer than the expected ELB duration in the standard model (that is, the model with $\alpha_1 = \alpha_2 = 0$). According to these figures, as long as either α_1 or α_2 is sufficiently low, the expected ELB duration is higher with discounting than without discounting, in line with our previous results. The forward-looking element in households' and firms' decision making is too small to allow for an effective use of forward guidance as a means to steer private sector behavior and the expected ELB duration is lower with discounting than without discounting, only when α_1 and α_2 are both high. The effect of discounting on the ELB duration can be large. According to Figure 7(c), when both α_1 and α_2 are around 0.15, the expected ELB duration is about five quarters longer than when they are 0. When both α_1 and α_2 are 1, the expected ELB duration is about four quarters shorter than when they are 0.

Figure 7: Optimal Policy w/ Discounting in the Euler Equation and the Phillips Curve

(7.a) E[ELB Duration] with $\alpha_1 \in [0,1]$ and $\alpha_2 \in [0,1]$

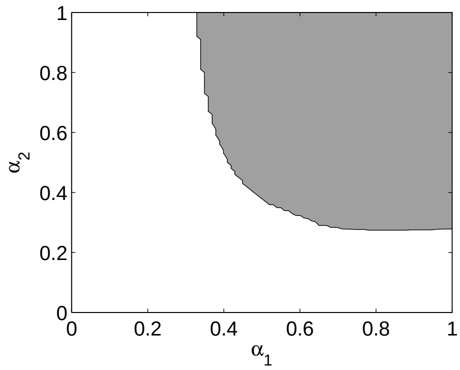


3.4 Relation to Bilbiie (2017) and Andrade, Gaballo, Mengus, and Mojon (2017)

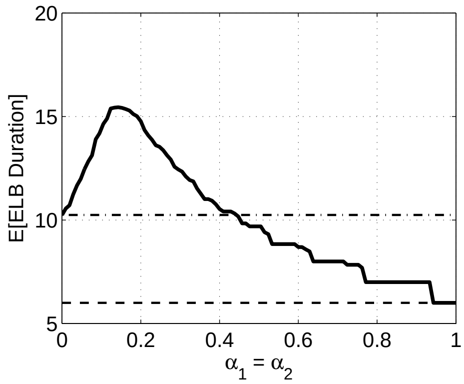
Bilbiie (2017) studies the implication of discounting in the Euler equation in a model with a static Phillips curve for the optimal ELB duration using a novel framework developed in Bilbiie (2016) in which the central bank optimally chooses—before the crisis shock hits the economy—the probability of keeping the policy rate at the ELB each period after the crisis shock disappears. He finds that the optimal expected ELB duration is lower when the discounting in the Euler equation is higher. A corresponding analysis in our paper is to vary α_1 while keeping $\alpha_2 = 1$. According to Figure 7(a), the effect of increasing α_1 is non-monotonic when $\alpha_2 = 1$. One key factor accounting for the difference in the result is that, as discussed in Section 3.2, we keep the severity of the shock constant as we vary discounting parameters, whereas Bilbiie (2017) keeps the size of the exogenous shock constant. Accordingly, as the discounting parameter increases in Bilbiie (2017), the crisis becomes less severe, adding a force to reduce the optimal expected ELB duration.

Andrade, Gaballo, Mengus, and Mojon (2017) study the implication for the optimal ELB duration of less powerful forward guidance associated with the presence of pessimists who do not believe in the central bank's commitment to keep the policy rate at the ELB after the natural rate of interest becomes positive. They assume that the policy rate will be set according to a feedback rule after liftoff and let the central bank to choose the duration of keeping the policy rate at the ELB. They do find a nonlinear effect of less powerful forward guidance on the optimal ELB duration, which is reminiscent of Figure 7(c) in which we vary discounting parameters in both Euler equation and Phillips curve at the same time. The

(7.b) Pairs of (α_1, α_2) under which E[ELB Duration] is shorter than in the standard model



(7.c) E[ELB Duration] with $\alpha_1 = \alpha_2 \in [0, 1]$



qualitative similarity makes sense, as the fraction of pessimists affect both the Euler equation and the Phillips curve in their model. Like Bilbiie (2017), they keep the size of the exogenous shock constant as they vary the fraction of pessimists. Interestingly, their nonlinearity is much more pronounced than ours. Also, the optimal ELB duration becomes the same as the ELB duration under optimal discretionary policy when a fraction of pessimists reaches a certain threshold in their model, while we find that the expected ELB duration under a fully optimal policy gradually approaches the ELB duration under optimal discretionary policy, as seen in Figure 7(c).

4 Discussion

A key finding from the previous two sections is that the implication of private-sector discounting for optimal policy depends importantly on its degree. Given this finding, a natural question is, "what are the plausible values of discounting parameters?" In this section, we investigate this question by reviewing the values of the discounting parameters in the Euler equation and the Phillips curve consistent with plausible parameterizations of different micro-founded models of less powerful forward guidance in the literature.[14]

Del Negro, Giannoni, and Patterson (2015) show that the New Keynesian model combined with an overlapping-generations structure a la Blanchard (1985) and Yaari (1965) goes a long way in attenuating the forward guidance puzzle. The log-linearized private-sector equilibrium conditions of their overlapping-generations model feature a discounted Euler equation and a discounted Phillips curve akin to those considered in our paper. The discounting parameters in their Euler equation and the Phillips curve are related to the probability of death, which can be interpreted more broadly to capture different forms of wealth resetting, such as a

[14]There are vast empirical literatures estimating the consumption Euler equation and Phillips curve. A comprehensive review of these literatures is beyond the scope of this section.

default. They consider two parameterizations; one implies $\alpha_1 = 0.013$ and $\alpha_2 = 0.191$ and the other implies $\alpha_1 = 0.04$ and $\alpha_2 = 0.209$.

McKay, Nakamura, and Steinsson (2016a) show that the forward guidance puzzle is substantially muted in an incomplete-markets model in which households face uninsurable income risks, and McKay, Nakamura, and Steinsson (2016b) show that the dynamics of the model of McKay, Nakamura, and Steinsson (2016a) is well approximated by the sticky-price model with the discounted Euler equation and the standard Phillips curve. They show that $\alpha_1 = 0.03$ and $\alpha_1 = 0.06$ are consistent with the dynamics of their incomplete-markets model with low-risk and high-risk calibrations, respectively.

Gabaix (2016) proposes a version of the New Keynesian model in which agents are non-rational, demonstrating that forward guidance is less powerful in his model and that the dynamics of his model are characterized by a discounted Euler equation and a discounted Phillips curve. While his analysis is mainly analytical, he presents a numerical example in which he sets $\alpha_1 = \alpha_2 = 0.15$, informed by the estimates of a hybrid Phillips curve from Galí and Gertler (1999). Angeletos and Lian (2016) show the attenuation of the forward guidance puzzle in a sticky-price model with imperfect common knowledge and derive the discounted Euler equation and the discounted Phillips curve. In their numerical example, the values assigned to two parameters governing the degree of departure from common knowledge imply $\alpha_1 = 0.003$ and $\alpha_2 = 0.037$.

Carlstrom, Fuerst, and Paustian (2015) and Kiley (2016) show that the forward guidance puzzle is attenuated in sticky-information models. In sticky-information models, inflation today depends on the infinite sum of the expected current marginal costs that were forecast in the past. As a result, there is no straightforward one-to-one mapping between our discounted Phillips curve and the sticky-information model. However, to the extent that the expected future inflation does not show up at all in the sticky-information model, the corresponding α_2 can be thought of as 1. Both papers use the standard Euler equation (that is, $\alpha_1 = 0$).

Finally, Chung (2015) shows that the effect of a future interest rate change is much more muted in FRB/US—a macroeconomic model used by the staff at the Board of Governors of the Federal Reserve System for policy analysis—than in other more standard sticky-price models, such as Smets and Wouters (2007). FRB/US developers in the 1990s introduced a discounting coefficient on the expected future income in the consumption Euler equation on the grounds that (i) risk-averse households would respond less to changes in the future income when they face uninsurable income risks than when they do not and (ii) households are finitely-lived.[15]. The discounting parameter in the Euler equation in FRB/US is 0.06. While there is various built-in inertia in the FRB/US Phillips curve, there is no explicit discounting of the expected inflation, so $\alpha_2 = 0$.[16]

[15]Interestingly, Reifschneider (1996)—a technical document that describes the consumption sector of FRB/US—motivates the introduction of the discounting parameter on the expected future income to the consumption Euler equation by referring to the early literature on uninsurable income risk and precautionary saving, such as Carroll (1997), as well as the models of perpetual youth by Blanchard (1985) and Yaari (1965).

[16]Kiley and Roberts (2017) find that inflation and output are better stabilized under an inertial interest

All told, according to the existing models of less powerful forward guidance, minor deviations from the standard Euler equation and the standard Phillips curve are empirically plausible and are enough to meaningfully attenuate the forward guidance puzzle. According to our analysis in the previous section, when the degree of discounting is small, the central bank wants to keep the policy rate at the ELB for longer than in the standard model. Under some choices of α_1 and α_2 considered in the literature (Del Negro, Giannoni, and Patterson (2015); McKay, Nakamura, and Steinsson (2016b); Angeletos and Lian (2016); and Chung (2015)), the optimal ELB duration is only slightly longer than in the standard model. However, under other choices of α_1 and α_2 (Gabaix (2016); Carlstrom, Fuerst, and Paustian (2015); and Kiley (2016)), the optimal ELB duration is extended in a quantitatively significant way. With $\alpha_1 = \alpha_2 = 0.15$ considered in Gabaix (2016), the expected ELB duration is more than one year longer than in the standard model, as can be seen in Figure 7(c). With $\alpha_1 = 0$ and $\alpha_2 = 1$ implied by the sticky-information model, the expected ELB duration is about half a year longer than in the standard model, as can be seen in the right panel of Figure 6.

5 Analysis based on a micro-founded model of less powerful forward guidance

So far, we have focused on analyzing the implication of less powerful forward guidance in New Keynesian models with a discounted Euler equation and/or Phillips curve without taking a stance on the structural model that generates discounting in these two equations. In Appendix G, we characterize an optimal commitment policy in a structural model of less powerful forward guidance. The structural model we examine is a New Keynesian model with overlapping generations (OLG) developed by Piergallini (2006), Castelnuovo and Nisticó (2010), and Nisticó (2012). Recently, Del Negro, Giannoni, and Patterson (2015) show that the log-linearization of the OLG New Keynesian model leads to discounting in both the consumption Euler equation and the Phillips curve and that the effects of anticipated monetary policy shock on today's economic activities is smaller in this model than in the standard New Keynesian model. In particular, the higher the probability of dying, the smaller the effects of a future interest rate adjustment are on today's economic activities.

In Appendix G, we find that the result from the OLG New Keynesian model is consistent with the main result from the model with a discounted Euler equation and Phillips curve. That is, the higher the death probability is—the less powerful forward guidance is—the more future accommodation the central bank would choose to promise.

rate rule that implements low-for-long policy than under other more standard rules in the FRB/US model, whereas Brayton, Laubach, and Reifschneider (2014) show that low-for-long policy is a key feature of optimal commitment policy in FRB/US model. These results are consistent with our result that low-for-long policy remains desirable in the model with discounting, unless the discounting parameter is close to one in both the Euler equation and Phillips curve.

It would be useful to characterize optimal commitment policy in other micro-founded models of less powerful forward guidance. We leave that effort to future research. Our hope is that our analysis based on the discounted Euler equation and Phillips curve can serve as a useful reference for other researchers when they analyze the optimal conduct of forward guidance policy in other micro-founded models of less powerful forward guidance.

6 Conclusion

Standard sticky-price models imply implausibly large responses of output and inflation to announcements about interest rate changes far in the future. As a result, there is widespread concern that they may be of limited use as a framework for analyzing forward guidance policies. Researchers have recently proposed various micro-founded modifications of the standard modeling framework that attenuate the effect of forward guidance on economic activities. The dynamics of these modified models are often consistent with those of an otherwise standard New Keynesian model with a discounted Euler equation and/or Phillips curve.

In this paper, we have examined the implication of less powerful forward guidance for the optimal design of forward guidance policy by analyzing how discounting on the part of the private sector affects the optimal commitment policy of the central bank caught in a liquidity trap. We have shown that forward guidance not only continues to be part of the optimal policy toolkit, but should also be used more heavily by the central bank than in the standard model under a wide range of the degree of discounting. In particular, it is optimal for the central bank to keep the policy rate at the ELB for a longer period of time in the model with discounting than in the standard model. This low-for-longer strategy allows the central bank to partially compensate for the reduced power of forward guidance due to discounting on current economic activities.

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Technical Appendix

The technical appendix is organized as follows. Appendix A provides necessary and sufficient conditions under which the optimal time-three policy rate declines as the discounting parameter increases in the three-period model with a static Phillips curve, while Appendix B shows a numerical example from the three-period model in which an increase in the discounting parameter leads to a higher time-three policy rate. Appendix C describes the details of optimal commitment policy in the three-period model, while Appendix D does so in the infinite-horizon model, including the solution method and its accuracy.

Appendix E analyzes the implication of discounting for optimal policy in an infinite-horizon model with equal weights on inflation and output stabilization terms in the central bank's objective function, while Appendix F analyzes the implication of discounting for optimal policy in an infinite-horizon model in a setup where we keep the shock size constant as we change discounting coefficients. Appendix G analyzes the implication of less powerful forward guidance for optimal policy in a New Keynesian model with overlapping generations and perpetual youth.

A Some analytical results in the three-period model

This section derives a necessary and sufficient condition for the marginal effect of an increase in the degree of discounting for the optimal policy rate and output in period 3 to be positive and negative, respectively. The condition is derived under the assumption that the ELB is binding in periods 1 and 2 and not binding in period 3.

The perfect-foresight New Keynesian model with a static Phillips curve and a Consumption Euler equation with discounting consists of the following two private sector behavioral constraints:

$$\pi_t = \kappa y_t \tag{A.1}$$

$$y_t = (1-\alpha)y_{t+1} - \sigma(i_t - \pi_{t+1} - r_t^n) \tag{A.2}$$

We can substitute (A.1) into (A.2), and obtain

$$y_t = (1-\alpha+\kappa\sigma)y_{t+1} - \sigma(i_t - r_t^n) \tag{A.3}$$

Assuming that the central bank faces the objective function introduced in the main text, the optimization problem of a central bank acting under commitment is

$$\max_{y_t, i_t} \sum_{t=0}^{\infty} \beta^t \left[-\frac{1}{2}(\kappa^2+\lambda)y_t^2 + \phi_t^{EE}\left(y_t - (1-\alpha+\kappa\sigma)y_{t+1} + \sigma(i_t - r_t^n)\right) + \phi_t^{ELB} i_t \right], \tag{A.4}$$

where we have used (A.1) to substitute out the inflation rate in the central bank objective function.

The first order necessary conditions can be combined to

$$\sigma(\kappa^2+\lambda)y_t = -\phi_t^{ELB} + \frac{1-\alpha+\kappa\sigma}{\beta}\phi_{t-1}^{ELB}, \tag{A.5}$$

where $\phi^{ELB} \geq 0$.

As in the main text, we assume $r_1^n = r_L^n < 0$ and $r_2^n = r_3^n = r^n > 0$.

Assumption 1 (for local analysis): For a given parameterization of the model, r_L^n and r^n are such that under the optimal commitment policy $\phi_1^{ELB}, \phi_2^{ELB} > 0$ and $\phi_3^{ELB} = 0$.

Define

$$\Omega(\alpha) \equiv 1 - \alpha + \kappa\sigma. \tag{A.6}$$

The system of equilibrium conditions for the three-period model conditional on Assumption 1 being satisfied then reads

$$\sigma(\kappa^2 + \lambda)y_1 + \phi_1^{ELB} = 0, \tag{A.7}$$
$$y_1 - \Omega y_2 = \sigma r_L^n, \tag{A.8}$$
$$\sigma(\kappa^2 + \lambda)y_2 + \phi_2^{ELB} - \frac{\Omega}{\beta}\phi_1^{ELB} = 0, \tag{A.9}$$
$$y_2 - \Omega y_3 = \sigma r^n, \tag{A.10}$$
$$\sigma(\kappa^2 + \lambda)y_3 - \frac{\Omega}{\beta}\phi_2^{ELB} = 0, \tag{A.11}$$
$$y_3 + \sigma i_3 = \sigma r^n, \tag{A.12}$$

where I use Ω as a shortcut for $\Omega(\alpha)$.
Substituting (A.9) into (A.11)

$$y_3 = \frac{\Omega^2}{\beta^2\sigma(\kappa^2 + \lambda)}\phi_1^{ELB} - \frac{\Omega}{\beta}y_2 \tag{A.13}$$

Substituting out ϕ_1^{ELB} using (A.7)

$$y_3 = -\frac{\Omega^2}{\beta^2}y_1 - \frac{\Omega}{\beta}y_2 \tag{A.14}$$

Substituting out y_1 and y_2 using (A.8) and (A.10), to get the policy function for y_3

$$\left(1 + \frac{\Omega^2}{\beta} + \frac{\Omega^4}{\beta^2}\right) y_3 = -\sigma\frac{\Omega^2}{\beta^2}r_L^n - \sigma\left(\frac{\Omega}{\beta} + \frac{\Omega^3}{\beta^2}\right) r^n \tag{A.15}$$

One can then show that $\frac{\partial y_3}{\partial \alpha} > 0$ if and only if

$$\left(\frac{\Omega^5}{\beta^4} - \frac{\Omega}{\beta^2}\right) 2\sigma(-r_L^n) + \left(\frac{1}{\beta} + 2\frac{\Omega^2}{\beta^2} - 2\frac{\Omega^4}{\beta^3} - \frac{\Omega^6}{\beta^4}\right) \sigma r^n > 0. \tag{A.16}$$

We can further simplify this expression. Note that from (A.11), $y_3 > 0$. Hence, from (A.15), we have $\frac{\Omega^2}{\beta^2}(-r_L^n) - \left(\frac{\Omega}{\beta} + \frac{\Omega^3}{\beta^2}\right) r^n > 0$. Let us rewrite (A.16):

$$2\frac{\Omega^3}{\beta^2}\left[\frac{\Omega^2}{\beta^2}(-r_L^n) - \left(\frac{\Omega}{\beta} + \frac{\Omega^3}{\beta^2}\right) r^n\right] + \left(\frac{1}{\beta} + 2\frac{\Omega^2}{\beta^2} + \frac{\Omega^6}{\beta^4}\right) r^n - 2\frac{\Omega}{\beta^2}(-r_L^n), \tag{A.17}$$

where we know that the term in square brackets has to be positive. Further reorganizing terms, we get

$$2\frac{1}{\Omega}\left(\frac{\Omega^4}{\beta^2}-1\right)\left[\frac{\Omega^2}{\beta^2}(-r_L^n)-\left(\frac{\Omega}{\beta}+\frac{\Omega^3}{\beta^2}\right)r^n\right]+\frac{1}{\beta}\left(\frac{\Omega^6}{\beta^3}-1\right)r^n. \tag{A.18}$$

Again, we know that the term in square brackets is positive if Assumption 1 is satisfied. Also, we know that $\Omega(\alpha) > 0$.

Thus, $\frac{\partial y_3}{\partial \alpha} > 0$ if and only if $\Omega(\alpha) > \sqrt{\beta}$, or, equivalently, $1-\sqrt{\beta}+\kappa\sigma > \alpha$. An interesting implication of this condition is that, if Assumption 1 is satisfied, a marginal increase in α at $\alpha = 0$ (no Euler equation discounting) will always lead to an increase in y_3, and a reduction in i_3.

Numerical example

We use the parameterization from Section 2 in the main text for the three-period model. The only difference (besides having a static Phillips curve) is that we choose a slightly lower value of -0.03 (instead of -0.03825) for r_L^n. This ensures that Assumption 1 is satisfied for $\alpha \in [0, 0.9]$.

Figure A.1 shows $\{y_1, y_2, y_3, \phi_1^{ELB}, \phi_2^{ELB}, i_3\}$ for $\alpha \in [0, 0.9]$.

Figure A.1: Allocations in the three-period model with a static Phillips curve

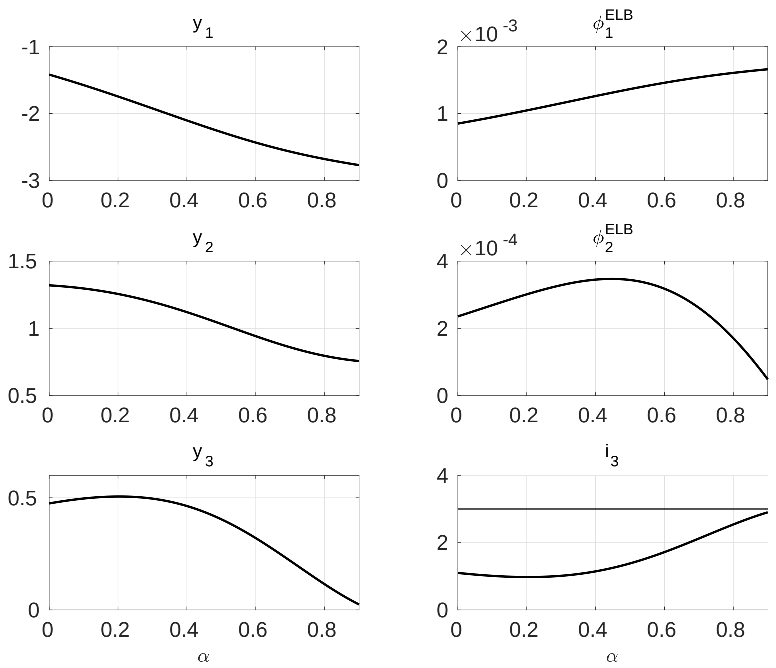
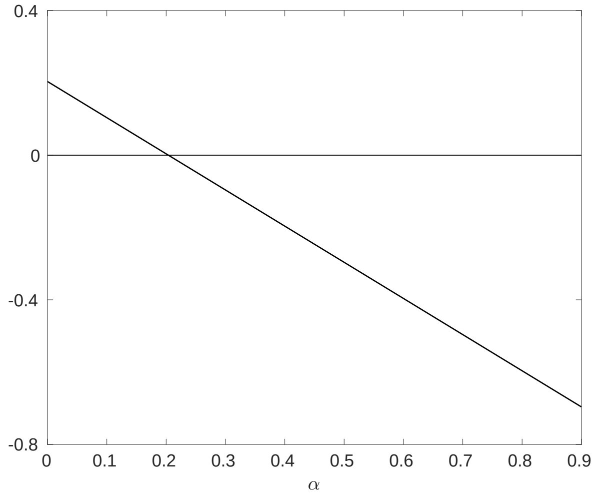


Figure A.2 plots $1-\sqrt{\beta}+\kappa\sigma-\alpha$ (solid black line) as a function of α. Hence, for those values of α for which the solid black line is above the horizontal black zero line, the necessary and sufficient condition for $\frac{\partial y_3}{\partial \alpha} > 0$ is satisfied.

Figure A.2: Indicators for $\frac{\partial y_3}{\partial \alpha} > 0$ and $\frac{\partial i_3}{\partial \alpha} < 0$



Note: Solid black line: $1 - \sqrt{\beta} + \kappa\sigma - \alpha$. For those values of α for which the solid black line is above the horizontal black zero line, the necessary and sufficient condition for $\frac{\partial y_3}{\partial \alpha} > 0$ and $\frac{\partial i_3}{\partial \alpha} < 0$ is satisfied.

B Additional results for the three-period model

In Section 2 of the main text, we analyze the implications of discounting in the Euler equation for optimal policy in a three-period model. One takeaway from that section is that whether the discounting makes optimal policy more or less accommodative depends on parameter values. In this section, we present a case in which the discounting make optimal policy less accommodative. The parameter value for $\kappa = 0.02$ in this example is excerpted from Eggertsson and Woodford (2003). All other parameter values are the same as in Table 1. The setup is the same as in Section 2, i.e. the contractionary shock lasts only for one period. As before, we set the discounting parameters equal to 0.5 in the model with discounting.

Figure B.1: Optimal Policy in a Three-Period Model
(An Alternative Parameterization)

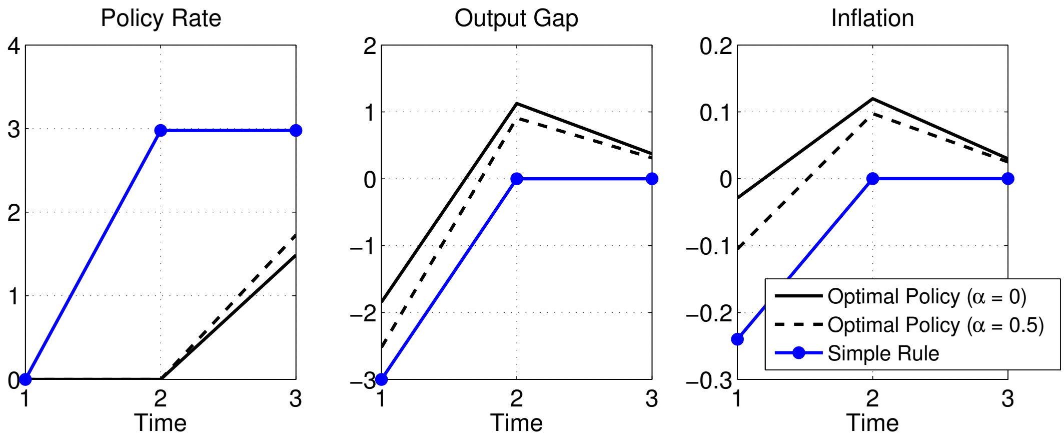


Figure B.1 shows the paths of the policy rate, inflation and the output gap under the alternative parameterization. Under the optimal policy, the central bank keeps the policy rate at zero in period 2 both in the model with $\alpha = 0$ and in the model with $\alpha = 0.5$. The optimal level of the policy rate in period 3 is higher in the model with discounting than in the model without discounting.

C Solving the three-period model under optimal commitment policy

The system of equilibrium conditions under optimal commitment policy are given by:

$$
\begin{aligned}
y_1 &= (1-\alpha)y_2 - \sigma(i_1 - \pi_2) + \sigma r_1^n \\
y_2 &= (1-\alpha)y_3 - \sigma(i_2 - \pi_3) + \sigma r^n \\
y_3 &= -\sigma i_3 \sigma r^n \\
\pi_1 &= \kappa y_1 + \beta \pi_2 \\
\pi_2 &= \kappa y_2 + \beta \pi_3 \\
\pi_3 &= \kappa y_3 \\
0 &= \lambda y_1 + \kappa \phi_1^{PC} + \frac{1}{\sigma}\phi_1^{ELB} \\
0 &= \lambda y_2 + \kappa \phi_2^{PC} + \frac{1}{\sigma}\phi_2^{ELB} - \frac{1-\alpha}{\beta\sigma}\phi_1^{ELB} \\
0 &= \lambda y_3 + \kappa \phi_3^{PC} + \frac{1}{\sigma}\phi_3^{ELB} - \frac{1-\alpha}{\beta\sigma}\phi_3^{ELB} \\
0 &= \pi_1 - \phi_1^{PC} \\
0 &= \pi_2 - \phi_2^{PC} + \phi_1^{PC} - \frac{1}{\beta}\phi_1^{ELB} \\
0 &= \pi_3 - \phi_3^{PC} + \phi_2^{PC} - \frac{1}{\beta}\phi_2^{ELB}
\end{aligned}
$$

and

$$
\begin{aligned}
&(i_1 - i_{ELB})\phi_1^{ELB} = 0, \quad (i_2 - i_{ELB})\phi_2^{ELB} = 0, \quad (i_3 - i_{ELB})\phi_3^{ELB} = 0 \\
&i_1 \geq i_{ELB}, \quad i_2 \geq i_{ELB}, \quad i_3 \geq i_{ELB} \\
&\phi_1^{ELB} \geq 0, \quad \phi_2^{ELB} \geq 0, \quad \phi_3^{ELB} \geq 0
\end{aligned}
$$

where ϕ_t^{PC} and ϕ_t^{ELB} denote the Lagrange multipliers associated with the Phillips curve and the ELB constraint.

The system of nonlinear equations can be solved as follows. First, assume that the ELB constraint is never binding. Set $\phi_t^{ELB} = 0$ for $t = 1, 2, 3$ and solve the resulting system of linear equations. If $i_t \geq 0$ for all t, the model is solved. If $i_1 < i_{ELB}$, set $i_1 = i_{ELB}$ and solve the system of linear equations with $\phi_1^{ELB} > 0$. If $i_2, i_3 \geq i_{ELB}$, the model is solved. If $i_2 < i_{ELB}$, set $i_2 = i_{ELB}$ and solve the system of linear equations with $\phi_1^{ELB}, \phi_2^{ELB} > 0$. If $i_3 \geq i_{ELB}$, the model is solved. If $i_3 < i_{ELB}$, set $i_3 = i_{ELB}$ and solve the system of linear equations with $\phi_1^{ELB}, \phi_2^{ELB}, \phi_3^{ELB} > 0$.

D Details of the optimal commitment policy

D.1 The recursive characterization

Following the common practice in the literature, we characterize the optimal commitment policy recursively based on the saddle-point functional equation. The saddle-point functional equation corresponding to the infinite-horizon problem of the Ramsey planner is given by,

$$
\begin{aligned}
&W_t(r_t^n, e_t, \omega_{1,t-1}, \omega_{2,t-1}) \\
=&\min_{\{\gamma_{1,t},\gamma_{2,t}\}}\max_{\{y_t,\pi_t,i_t\}} h(y_t, \pi_t, i_t, \omega_{1,t-1}, \omega_{2,t-1}, \gamma_{1,t}, \gamma_{2,t}) + \beta \mathbb{E}_t W_{t+1}(r_{t+1}^n, e_{t+1}, \omega_{1,t}, \omega_{2,t})
\end{aligned}
$$

where

$$
\begin{aligned}
&h(y_t, \pi_t, i_t, \omega_{1,t-1}, \omega_{2,t-1}, \gamma_{1,t}, \gamma_{2,t}) \\
&\qquad = \left[-\frac{1}{2}\left(\pi_t^2 + \lambda y_t^2\right)\right] - \omega_{1,t-1}((1-\alpha_1)y_t + \sigma\pi_t) - \omega_{2,t-1}(1-\alpha_2)\beta\pi_t \\
&\qquad\qquad + \gamma_{1,t}(y_t + \sigma(i_t - r_t^n)) + \gamma_{2,t}(\pi_t - \kappa y_t - e_t)
\end{aligned}
$$

and the optimization is subject to the the ELB constraint, and the following law of motions for $\omega_{1,t}$ and $\omega_{2,t}$,

$$
\begin{aligned}
\omega_{1,t} &= \gamma_{1,t} \\
\omega_{2,t} &= \gamma_{2,t}
\end{aligned}
$$

Note that $\omega_{1,t}$ (the Lagrange multiplier on the Euler equation) and $\omega_{2,t}$ (the Lagrange multiplier on the Phillips curve) are the only relevant state variables because the expectation terms show up only in these two equations.[17] Define the state $\mathbb{S}_t := [r_t^n, e_t, \omega_{1,t-1}, \omega_{2,t-1}]$. A Ramsey equilibrium can be defined as the set of time-invariant value and policy functions $\{W(\mathbb{S}_t),\ y(\mathbb{S}_t),\ \pi(\mathbb{S}_t),\ i(\mathbb{S}_t),\ \omega_1(\mathbb{S}_t),\ \omega_2(\mathbb{S}_t),\ \omega_3(\mathbb{S}_t)\}$ that solve the saddle-point functional equation equation above. The FONC are

$$
\begin{aligned}
\partial y_t &: \quad -\lambda y_t + \omega_{1,t} - \frac{1-\alpha_1}{\beta}\omega_{1,t-1} - \kappa\omega_{2,t} = 0 \\
\partial \pi_t &: \quad -\pi_t - \frac{\sigma}{\beta}\omega_{1,t-1} + \omega_{2,t} - (1-\alpha_2)\omega_{2,t-1} = 0 \\
\partial i_t &: \quad \sigma\omega_{1,t} + \omega_{3,t} = 0,
\end{aligned}
$$

as well as the private sector behavioral constraints.

[17] $\omega_{3,t}$ is the Lagrange multiplier on the ELB constraint.

D.2 Solution method

The problem is to find a set of policy functions, $\{y(\mathbb{S}_t),\ \pi(\mathbb{S}_t),\ i(\mathbb{S}_t),\ \omega_1(\mathbb{S}_t),\ \omega_2(\mathbb{S}_t),\ \omega_3(\mathbb{S}_t)\}$, that solves the following system of functional equations

$$y(\mathbb{S}_t) = (1-\alpha_1)\mathbb{E}_t y(\mathbb{S}_{t+1}) - \sigma(i(\mathbb{S}_t) - \mathbb{E}_t\pi(\mathbb{S}_{t+1}) - r_t^n) \tag{D.1}$$

$$\pi(\mathbb{S}_t) = \kappa y(\mathbb{S}_t) + (1-\alpha_2)\beta\mathbb{E}_t\pi(\mathbb{S}_{t+1}) + e_t \tag{D.2}$$

$$i(\mathbb{S}_t) \geq 0 \equiv i_{ELB} \tag{D.3}$$

$$\omega_1(\mathbb{S}_t) = \lambda y(\mathbb{S}_t) + \frac{1-\alpha_1}{\beta}\omega_1(\mathbb{S}_{t-1}) + \omega_2(\mathbb{S}_t) \tag{D.4}$$

$$\omega_2(\mathbb{S}_t) = \pi(\mathbb{S}_t) + \frac{\sigma}{\beta}\omega_1(\mathbb{S}_{t-1}) + (1-\alpha_2)\omega_2(\mathbb{S}_{t-1}) \tag{D.5}$$

$$\omega_3(\mathbb{S}_t) = -\sigma\omega_1(\mathbb{S}_t) \tag{D.6}$$

Following the approach of Christiano and Fisher (2000), we decompose these policy functions into two parts using an indicator function: one in which the policy rate is allowed to be less than i_{ELB}, and the other in which the policy rate is assumed to be i_{ELB}. That is, for any variable Z,

$$Z(\cdot) = I_{\{R(\cdot)\geq i_{ELB}\}}Z_{unc}(\cdot) + (1 - I_{\{R(\cdot)\geq i_{ELB}\}})Z_{ELB}(\cdot). \tag{D.7}$$

The problem then becomes finding a set of a *pair* of policy functions, $\{[y_{unc}(\cdot),\ y_{ELB}(\cdot)],\ [\pi_{unc}(\cdot),\ \pi_{ELB}(\cdot)],\ [i_{unc}(\cdot),\ i_{ELB}(\cdot)],\ [\omega_{1,unc}(\cdot),\ \omega_{1,ELB}(\cdot)],\ [\omega_{2,unc}(\cdot),\ \omega_{2,ELB}(\cdot)],\ [\omega_{3,unc}(\cdot),\ \omega_{3,ELB}(\cdot)]\}$ that solves the system of functional equations above. This method can achieve a given level of accuracy with a considerable less number of grid points relative to the standard approach.

The time-iteration method starts by specifying a guess for the values policy functions take on a finite number of grid points. The values of the policy function that are not on any of the grid points are interpolated or extrapolated linearly. Let $X(\cdot)$ be a vector of policy functions that solves the functional equations above and let $X^{(0)}$ be the initial guess of such policy functions.[18] At the s-th iteration, given the approximated policy function $X^{(s-1)}(\cdot)$, we solve the system of nonlinear equations given by equations (D.1)-(D.6) to find today's $y_t, \pi_t, i_t, \omega_{1,t}, \omega_{2,t}$, and $\omega_{3,t}$ at each point of the state space. In solving the system of nonlinear equations, we evaluate the value of future variables that are not on the grid points with linear interpolation. The system is solved numerically by using a nonlinear equation solver, `dneqnf`, provided by the IMSL Fortran Numerical Library. If the updated policy functions are sufficiently close to the previously approximated policy functions, then the iteration ends. Otherwise, using the former as the guess for the next period's policy functions, we iterate on this process until the difference between the guessed and updated policy functions is sufficiently small ($\left\|vec(X^s(\delta) - X^{s-1}(\delta))\right\|_\infty <$ 1E-12 is used as the convergence criteria).

[18] For all models and all variables, we use flat functions at the deterministic steady-state values as the initial guess.

D.3 Solution accuracy

We assess the accuracy of our solution for the allocation under the optimal commitment policy through the following two residual functions:

$$\mathbb{R}_{1,t} = |y_t - (1-\alpha_1)\mathbb{E}_t y_{t+1} + \sigma(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n)|,$$
$$\mathbb{R}_{2,t} = |\pi_t - \kappa y_t - (1-\alpha_2)\beta \mathbb{E}_t \pi_{t+1} - e_t|,$$

where the residual $\mathbb{R}_{1,t}$ measures the difference between the chosen output gap today and today's output gap consistent with the optimization behavior of the household, as a percent deviation from steady-state output gap. Similarly, the residual $\mathbb{R}_{2,t}$ captures the same difference in inflation today.

It is common to simulate the economy once for a very long time, and report summary statistics of these residuals from the simulated path to quantify the solution accuracy (Maliar and Maliar (2015) and Nakata (2017), among others). However, such approach does not make sense in our model with an absorbing state; after a certain point in time, the economy is at the steady state forever in which the residuals are zero. We take the following approach to quantify the solution accuracy. We simulate 500,000 different economies. In each economy, the initial state is given by the crisis state in which $r_t^n = r_L^n$ and $e_t = e_L$. How long the crisis shock lasts depends on the realizations of the crisis shock. In each economy, we stop our simulation after 8 quarters from the lift-off quarter so as not to include observations in which the economy are very close to or at the steady state.[19] Table D.1 reports the average and the maximum of the residuals collected from 500,000 economies of varying lengths for three values of $\alpha_1 = \alpha_2 = 0,\ 0.5,\ 1$.

Table D.1: Solution Accuracy (Discounting in Euler Equation)

α_1	$\xi = [\log_{10}(\mathbb{R}_{k,t})]$	$\mathbb{E}[\xi]$	$\max[\xi]$
$\alpha_1 = \alpha_2 = 0$	$k=1$: Euler equation error	-11.0	-2.9
	$k=2$: Sticky-price equation error	-13.1	-5.7
$\alpha_1 = \alpha_2 = 0.5$	$k=1$: Euler equation error	-13.5	-3.3
	$k=2$: Sticky-price equation error	-15.6	-5.6
$\alpha_1 = \alpha_2 = 1$	$k=1$: Euler equation error	-19.8	-16.9
	$k=2$: Sticky-price equation error	-29.2	-18.4

E Sensitivity analysis (I)

In this section, we examine the implications of less powerful forward guidance on optimal policy when the weight on the output stabilization term in the central bank's objective function—given by λ—is higher than in the baseline.

We consider two alternative values for λ. The first value we consider—the value we focus on—is 1/64, which is substantially higher than the baseline and gives equal weights to the volatility of annualized inflation and the volatility of employment gap.[20] When the optimal

[19]The choice of 8 quarters is somewhat arbitrarily, but the economy typically have returned to the steady state 8 quarters after liftoff, regardless of how long the crisis shock has lasted.

[20]$1/64 = 1/16 * 1/4$. The first term, $1/16(= 1/4 * 1/4)$, is the term to take into account the fact that π_t in the model is a quarterly rate of inflation. The second-term, $1/4(= 1/2 * 1/2)$, translates the output gap volatility into the employment volatility, using the Okun's law with coefficient of 2.

policy analysis is conducted for policy purposes in central banks, it is common to put equal weights to the price stability and employment stability terms. See, for example, Yellen (2012), Reifschneider, Wascher, and Wilcox (2015), and Carney (2017). The second value we consider is 0.0045, which is five times as large as the baseline value and is lower than $1/64$.

Figure E.1: IRFs in the Model with Discounted Euler Equation ($\lambda = 1/64$)

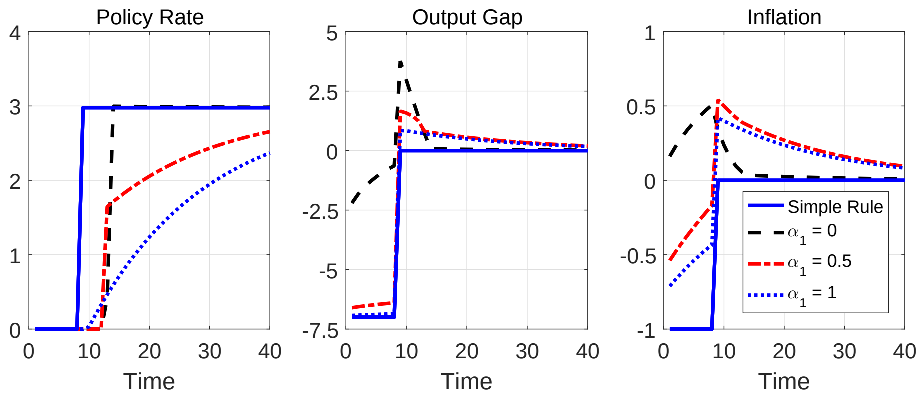


Figure E.1 shows the IRFs under the optimal commitment policy in the model with discounting in the Euler equation only under $\lambda = 1/64$. The IRFs with the standard Euler equation—shown by the dashed black lines—are qualitatively similar to those under the baseline λ shown in Figure 4. Quantitatively, the deviation of inflation from the target is larger, and the deviation of output from the target level is smaller, under $\lambda = 1/64$ than under the baseline calibration.

The effect of discounting on the optimal interest-rate path is qualitatively different under $\lambda = 1/64$ than under the baseline value of λ. In Figure E.3, the liftoff quarter is the same under $\alpha = 0.5$ as that under $\alpha = 0$. Under $\alpha = 1$, the liftoff quarter is earlier, albeit only by two quarters. Interestingly, even though the lift-off quarter is earlier under $\alpha = 1$ than—or, the same under $\alpha = 0.5$ as—under $\alpha = 0$, the speed of convergence to the steady state interest rate is slower the higher the discounting parameter is.

The left panel of Figure E.3 shows the non-monotonic effect of the discounting on the expected ELB duration under $\lambda = 1/64$; although the expected ELB duration initially increases as α increases from zero, an increase in α leads to a shorter expected ELB duration after some threshold value of α around 0.2. Note that, for any given α, a higher λ is associated with a shorter expected ELB duration.

Moving on to the effect of discounting in the Phillips curve, the effect of discounting on the optimal policy-rate path under $\lambda = 1/64$ is similar to that under the baseline λ shown in Figure E.2. As shown in the middle panel of Figure E.3, a higher discounting is associated with a longer expected ELB duration, though the effect of increasing the discounting parameter is negligible when alpha is sufficiently large. For any given α, a higher λ is associated with a shorter expected ELB duration.

Finally, the right panel of Figure E.3 shows the effect on the expected ELB duration of increasing the discounting parameters in the Euler equation and the Phillips curve at the same time. Qualitatively, the effect is similar to that under the baseline λ; although the expected ELB duration increases initially as α increases from zero, an increase in α leads to a shorter expected ELB duration after some threshold value of α. As in the model with discounting only in the Euler equation or the Philip curve, for any given α, a higher λ is

Figure E.2: IRFs in the Model with Discounted Phillips Curve ($\lambda = 1/64$)

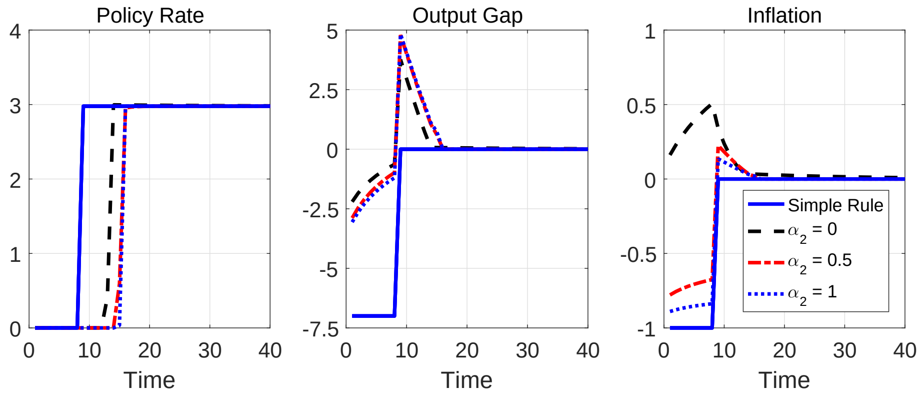
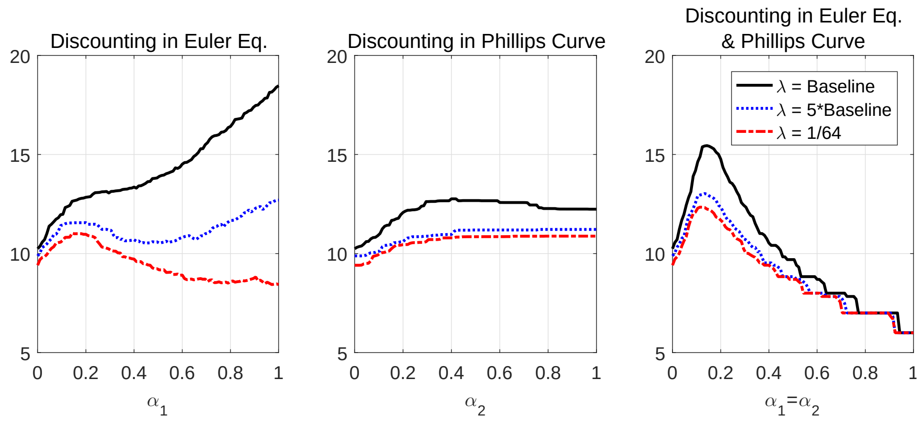


Figure E.3: Expected ELB Durations with various λs



associated with a shorter expected ELB duration.

F Sensitivity analysis (II)

Throughout the paper, we have examined the implications of less powerful forward guidance in a setup where, as we vary α, the size of the shocks are modified to keep the severity of the recession unchanged under the simple rule. In this section, we demonstrate the results from the alternative experiment in which the size of the shocks are kept constant as we vary α.

With the size of shocks unchanged, a decline in α makes the declines in output and inflation in the crisis state smaller, as shown in Figure F.1. For any given α, faced with a less severe recession, the central bank finds it optimal to keep the policy rate at the ZLB for a shorter duration in this alternative experiment than in the baseline experiment, as captured by the fact that black line is below the red line in the left panel of Figure F.5. Under the alternative setup—shown by the black line—an increase in α leads to a reduction in the expected ELB duration, whereas an increase in α leads to an increase in the expected ELB duration under the baseline setup—shown by the red line. The effect of increasing α on the IRFs are shown in Figure F.2.

Figure F.1: IRFs in the Model with Discounted Euler Equation under the Simple Rule: Keeping the Magnitudes of the Shocks Constant

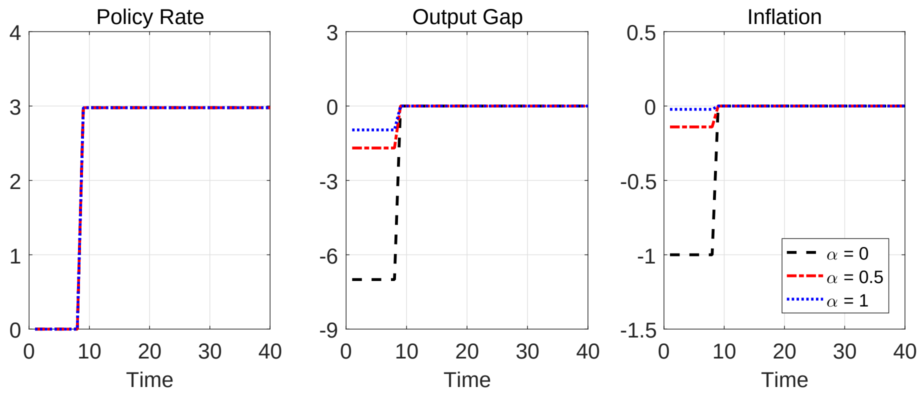
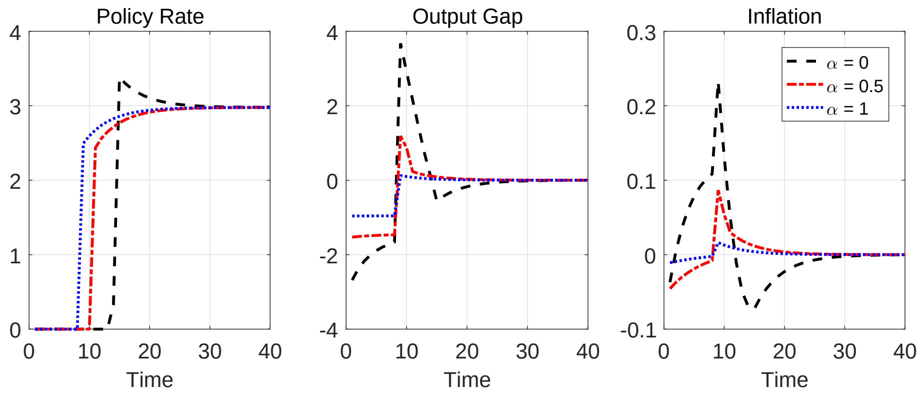


Figure F.2: IRFs in the Model with Discounted Euler Equation under the Optimal Commitment Policy:
Keeping the Magnitudes of the Shocks Constant



Moving on to the model with discounting only in the Phillips curve, a higher α is associated with a less severe recession under the simple rule, as in the model with discounting only in the Euler equation, and as shown in Figure F.3. For any given α, faced with a less severe recession, the central bank finds it optimal to keep the policy rate at the ZLB for a shorter duration in this alternative experiment than in the baseline experiment, as captured by the fact that black line is below the red line in the middle panel of Figure F.5. Under the alternative experiment—shown by the black line—an increase in α does not alter the expected ELB duration much, whereas an increase in α leads to an increase in the expected ELB duration under the baseline experiment—shown by the red line. The effect of increasing α on the IRFs are shown in Figure F.4.

Finally, the right panel of Figure F.5 shows the effect on the expected ELB duration of increasing the discounting parameter in both the Euler equation and the Phillips curve. Similarly to the model with the discounting only in the Euler equation, the expected ELB duration declines as α increases under the alternative experiment—shown by the black line.

Figure F.3: IRFs in the Model with Discounted Phillips Curve under the Simple Rule: Keeping the Magnitudes of the Shocks Constant

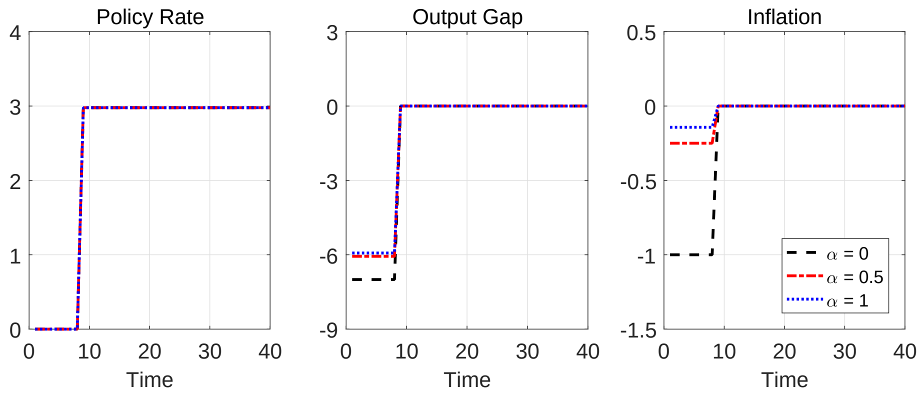
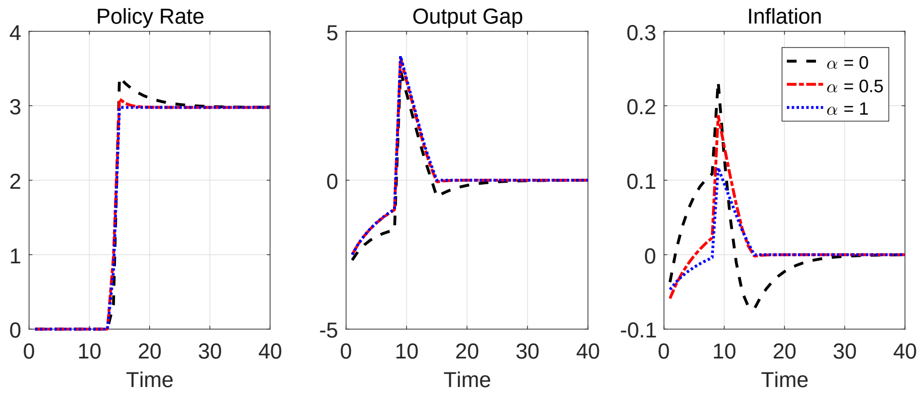


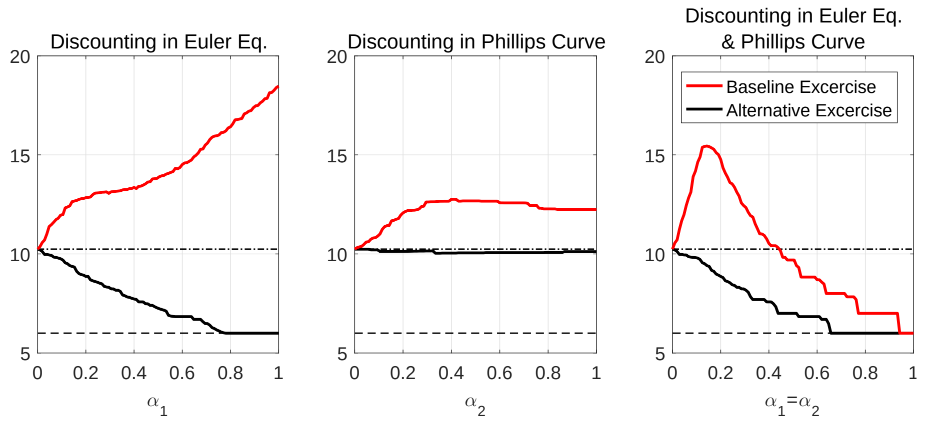
Figure F.4: IRFs in the Model with Discounted Phillips Curve under the Optimal Commitment Policy:
Keeping the Magnitudes of the Shocks Constant



G Analysis of optimal policy in a micro-founded model of less powerful forward guidance

In the main text, we analyzed the implication of less powerful forward guidance in a New Keynesian models with discounted Euler equation and Phillips curve without taking a stance on the structural model that generates discounting in these two equations. In this section, we characterize optimal commitment policy in a structural model of less powerful forward guidance. The structural model we examine is a New Keynesian model with overlapping generations developed by Piergallini (2006), Castelnuovo and Nisticó (2010), and Nisticó (2012). Recently, Del Negro, Giannoni, and Patterson (2015) show that the effect of anticipated monetary policy shock is smaller in this model than in the standard New Keynesian model.

Figure F.5: Expected ELB Duration:
Keeping the Magnitudes of the Shocks Constant



G.1 Model

G.1.1 Household

Every period j, a new cohort is born with mass p, and each cohort has a constant probability of dying—denoted by p—which does not depend on j. At time t, agents born in period j seek to maximize their discounted sum of future utility flows subject to budget constraints.

$$\max_{\{C_{j,t+s},L_{j,t+s}\}_{s=0}^{\infty}} \sum_{s=0}^{\infty}(\beta(1-p))^{t+s-1}\left[\prod_{u=0}^{s}\delta_{t+u-1}\right][\ln C_{j,t+s}+v\ln(1-L_{j,t+s})]$$

subject to

$$\begin{aligned}&P_{t+s}C_{j,t+s}+B_{j,t+s}+\int_0^1 Q^*_{t+s}(i)Z_{j,t+s}(i)di\\ =&W^*_{t+s}L_{j,t+s}+P_{t+s}T_{j,t+s}+\frac{1}{1-p}[B_{j,t+s}R_{t+s-1}+\int_0^1(Q^*_{t+s}(i)+D^*_{t+s}(i))Z_{j,t+s}(i)di]\end{aligned}$$

$C_{j,t}$ and $L_{j,t}$ are consumption and labor supply of cohort j at time t. $B_{j,t}$ and $Z_{j,t}(i)$ are respectively a nominal bond holding and the equity share issued by monopolistic firms, i, owned by cohort j at time t. $Q^*_t(i)$ is the nominal price of equity and $D^*_t(i)$ is the dividend. W^*_t is the nominal wage and $T_{j,t}$ is the lump-sum transfer. $*$ denotes the nominal price.

Each agent faces the constant death rate p for each period. As in Blanchard (1985), each agent has entered an annuity contract in which the fraction p of the same cohorts dying in each period leaves its wealth to those who remain alive. The wealth $[(B_{j,t}R_{t-1}+\int_0^1(Q^*_t(i)+D^*_t(i))Z_{j,t}(i)di]$ is divided by $1-p$ fraction of cohort members alive. By adding the annuity and his own wealth, each agent has $(\frac{p}{1-p}+1)[B_{j,t}R_{t-1}+\int_0^1(Q^*_t(i)+D^*_t(i))Z_{j,t}(i)di]$ for each period.

The discount rate at time t is given by $\beta\delta_t$ where δ_t is the discount factor shock altering the weight of future utility at time $t+1$ relative to the period utility at time t. The shock process is described as follows:

$$\delta_t = \begin{cases} \delta_H & \text{if} \quad 1 \leq t \leq \tau \\ 1 & \text{otherwise.} \end{cases}$$

Note that, unlike in the infinite-horizon model in the main text, there is no uncertainty. In particular, the duration of the shock, τ, is given.

For any variables X, aggregate variable X_t is defined by $X_t \equiv \sum_{j=-\infty}^{t} p(1-p)^{t-j} X_{j,t}$.

G.1.2 Firms

There is a final good producer and a continuum of intermediate goods producers indexed by $i \in [0,1]$. The final good producer purchases the intermediate goods $Y_{i,t}$ at $P_{i,t}$, produces the final good by CES technology, and sells it to households. The problem is summarized as:

$$\max_{\{Y_{i,t} \; i\in[0,1]\}} P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di$$

subject to the CES production function,

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

Intermediate goods producers use labor as an input of producing intermediate goods according to a linear production function($Y_{i,t} = L_{i,t}$) and sell them to the final good producer. Each firm faces a quadratic adjustment cost when he changes his price to maximize his profit. We assume that each firm receives a production subsidy τ^* so that economy is fully efficient in the steady state.

$$\max_{\{P_{i,t}\}} \sum_{t=1}^{\infty} \mathcal{F}_{1,t} \left[(1+\tau^*) P_{i,t} Y_{i,t} - W_t^* L_{i,t} - P_t \frac{\varphi}{2} \left(\frac{P_{i,t}}{P_{i,t-1}} - 1 \right)^2 Y_t \right]$$

subject to

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\theta} Y_t$$

where $\mathcal{F}_{1,t}$ is the aggregate stochastic discount factor. That is,

$$\mathcal{F}_{1,t} \equiv \sum_{j=-\infty}^{t} p(1-p)^{t-j} \mathcal{F}_{j,1,t}$$

with

$$\mathcal{F}_{j,1,t} \equiv \beta^{t-1} \frac{P_1 C_{j,1}}{P_t C_{j,t}}$$

The time zero price is assumed to be the same across firms (i.e. $P_{i,0} = P_0 > 0$).

G.1.3 Government policies

The supply of the government bond is assumed to be zero. With this assumption, the government budget constraint is given by

$$\tau^* P_t Y_t + P_t \sum_{j=-\infty}^{t} p(1-p)^{t-j} T_{j,t} = 0$$

We will consider two cases regarding how the central bank sets the short-term nominal interest rate. In the first case, the central bank sets the interest rate according to the following interest-rate feedback rule;

$$R_t = \max\{\frac{R_{ss}}{\delta_t}\Pi_t^{\Psi}, \quad 1\}$$

where $\Pi_t := \frac{P_t}{P_{t-1}}$ and,

$$R_{ss} := \frac{1}{\beta(1-\eta\frac{Q}{Y})}$$

R_{ss} is the steady state nominal interest rate and where $\eta = \frac{p}{1-p}\frac{1-\beta(1-p)}{(1+v)\beta}$. The intercept of the policy rule is time-varying and depends on δ_t in such a way that, in the absence of the ZLB constraint, the effect of variation in δ would be fully neutralized by a corresponding variation in the policy rate.

In the second case, the central bank choose the sequence of interest rates at time one in order to maximize the household's welfare subject to the private-sector equilibrium conditions.

G.1.4 Market clearing conditions

The market clearing conditions for the final good, labor, government bond and equity are given by

$$Y_t = C_t + \frac{\varphi}{2}\left(\Pi_t - 1\right)^2 Y_t$$
$$L_t = \int_0^1 L_t(i)\ di$$
$$B_t = 0$$
$$\int_0^1 Z_t(i)\ di = 1$$

G.1.5 Private-sector equilibrium conditions

Private sector equilibrium conditions are given by:

$$
\begin{aligned}
C_t &= \frac{p}{(1+v)(\omega_t - 1)} Q_t + \frac{(1-p)\omega_{t+1}}{\omega_t - 1} \frac{1}{R_t} \Pi_{t+1} C_{t+1} \\
Q_t &= \frac{1}{R_t} \Pi_{t+1} (Q_{t+1} + D_{t+1}) \\
vC_t &= W_t (1 - L_t) \\
D_t &= \frac{\theta}{\theta - 1} Y_t - W_t Y_t - \frac{\phi}{2} (\Pi_t - 1)^2 Y_t \\
C_t &= [1 - \frac{\phi}{2} (\Pi_t - 1)^2] Y_t \\
\Pi_t (\Pi_t - 1) &= \frac{1}{R_t} \Pi_{t+1}^2 (\Pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t} - \frac{\theta}{\phi} (1 - W_t) \\
Y_t &= L_t
\end{aligned}
$$

where

$$
\omega_t = 1 + \sum_{s=1}^{\infty} (1-p)^s \beta^s (\prod_{u=1}^{s} \delta_{t+u-1}))
$$

As death rate p rises, η also increases and households respond more to the asset price at time t. If $p = 0$ holds, $\eta = 0$ and this economy degenerates into a standard NK model. In this case, the wealth effect of an asset price Q_t does not directly affect the movement of consumption.

For notational simplicity, we will also write the consumption Euler equation as follows.

$$
C_t = \kappa_{1,t} Q_t + \kappa_{2,t} \frac{\Pi_{t+1} C_{t+1}}{R_t}
$$

G.2 Welfare

Welfare function at t_0 is defined as the weighted average of time-zero value functions of all cohorts, including those who are already born and those who are yet to be born. σ is the subjective discount factor of the social planner.[21] For expositional simplicity, we abstract

[21] The second term of the first line shows that the policy maker discounts time t instantaneous utilities of already-born agents back to birth dates, rather than the current period. This assumption looks odd, but, without the assumption, the problem of the time-inconsistency in preference arises, as noted by Calvo and Obstfeld (1988).

from the time-variation in the household's discount rate.

$$
\begin{aligned}
W_{t_0} &= \underbrace{\sum_{j=t_0+1}^{\infty} \sigma^j \left(p \sum_{t=j}^{\infty} [\beta(1-p)]^{t-j} u(j,t) \right)}_{unborn} + \underbrace{\sum_{j=-\infty}^{t_0} \sigma^j \left(p \sum_{t=t_0}^{\infty} [\beta(1-p)]^{t-j} u(j,t) \right)}_{already \quad born} \\
&= p \sum_{j=t_0+1}^{\infty} \left[\sum_{t=j}^{\infty} \left(\frac{\beta(1-p)}{\sigma} \right)^{t-j} u(j,t)\sigma^t \right] + p \sum_{j=-\infty}^{t_0} \left[\sum_{t=t_0}^{\infty} \left(\frac{\beta(1-p)}{\sigma} \right)^{t-j} u(j,t)\sigma^t \right] \\
&= p \sum_{t=t_0}^{\infty} \sigma^t \left[\sum_{j=-\infty}^{t} \left(\frac{\beta(1-p)}{\sigma} \right)^{t-j} u(j,t) \right] \\
&= \sum_{t=t_0}^{\infty} \beta^t u(C_t, L_t)
\end{aligned}
$$

The last equality follows from the following two assumptions. The first assumption is that the social planner can implement a distributional policy so that the aggregate consumption and labor supply becomes equal for each cohort at time t. That is, $C_{j,t} = C_t$, $L_{j,t} = L_t$, and $u_{j,t} = u_t$. The second assumption is that the subjective discount factor of social planner and that of each cohort are the same, that is $\sigma = \beta$.[22]

G.3 Ramsey Problem

At time $t = 1$, the government's problem is to find an allocation that maximizes the household welfare and price and policy variables that decentralize the allocation as an equilibrium.

$$\max \sum_{t=1}^{\infty} \beta^{t-1} (\prod_{s=1}^{t} \delta_{s-1}) [\ln C_t + v \ln (1 - Y_t)]$$

subject to

$$
\begin{aligned}
&C_t = \kappa_{1,t} Q_t + \kappa_{2,t} \frac{\Pi_{t+1} C_{t+1}}{R_t} \\
&Q_t = \frac{1}{R_t} \Pi_{t+1} (Q_{t+1} + D_{t+1}) \\
&v C_t = W_t (1 - Y_t) \\
&D_t = \frac{\theta}{\theta - 1} Y_t - W_t Y_t - \frac{\phi}{2} (\Pi_t - 1)^2 Y_t \\
&C_t = [1 - \frac{\phi}{2} (\Pi_t - 1)^2] Y_t \\
&\Pi_t (\Pi_t - 1) = \frac{1}{R_t} \Pi_{t+1}^2 (\Pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t} - \frac{\theta}{\phi} (1 - W_t) \\
&R_t \geq 1
\end{aligned}
$$

[22] Notice that $\sum_{z=0}^{\infty} \left(\frac{\beta(1-p)}{\sigma} \right)^z = \frac{1}{p}$, which is also used in the last equality.

The associated Lagrangian problem is given by:

$$
\begin{aligned}
\mathcal{L}_t = \max \ & \sum_{t=1}^{\infty} \beta^{t-1} (\prod_{s=1}^{t} \delta_{s-1}) \{ \ln C_t + v \ln (1 - Y_t) \\
& + \lambda_{1,t}(\kappa_{1,t} Q_t + \kappa_{2,t} \frac{\Pi_{t+1} C_{t+1}}{R_t} - C_t) \\
& + \lambda_{2,t}(\frac{1}{R_t} \Pi_{t+1}(Q_{t+1} + D_{t+1}) - Q_t) \\
& + \lambda_{3,t}(W_t(1 - Y_t) - vC_t) \\
& + \lambda_{4,t}(\frac{\theta}{\theta - 1} Y_t - W_t Y_t - \frac{\phi}{2}(\Pi_t - 1)^2 Y_t - D_t) \\
& + \lambda_{5,t}([1 - \frac{\phi}{2}(\Pi_t - 1)^2] Y_t - C_t) \\
& + \lambda_{6,t}(\frac{1}{R_t} \Pi_{t+1}^2 (\Pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t} - \frac{\theta}{\phi}(1 - W_t) - \Pi_t(\Pi_t - 1)) \\
& + \lambda_{7,t}(R_t - 1) \}
\end{aligned}
$$

FONCs are given by:

$$
\begin{aligned}
& C_t : \frac{\kappa_{2,t-1}}{R_{t-1}} \Pi_t \lambda_{1,t-1} + \delta_t \beta \frac{1}{C_t} - \delta_t \beta \lambda_{1,t} - \delta_t \beta v \lambda_{3,t} - \delta_t \beta \lambda_{5,t} = 0 \\
& Q_t : \lambda_{2,t-1} \frac{\Pi_t}{R_{t-1}} + \delta_t \beta \kappa_{1,t} \lambda_{1,t} - \delta_t \beta \lambda_{2,t} = 0 \\
& D_t : \lambda_{2,t-1} \frac{\Pi_t}{R_{t-1}} - \delta_t \beta \lambda_{4,t} = 0 \\
& Y_t : -\delta_t \beta \frac{v}{1 - Y_t} + \lambda_{6,t-1} \frac{1}{R_{t-1}} \Pi_t^2 (\Pi_t - 1) \frac{1}{Y_{t-1}} - \delta_t \beta \lambda_{3,t} W_t + \delta_t \beta \lambda_{4,t} (\frac{\theta}{\theta - 1} - W_t - \frac{\phi}{2}(\Pi_t - 1)^2) \\
& + \delta_t \beta \lambda_{5,t} [1 - \frac{\phi}{2}(\Pi_t - 1)^2] - \delta_t \beta \lambda_{6,t} \frac{1}{R_t} \Pi_{t+1}^2 (\Pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t^2} = 0 \\
& W_t : \lambda_{3,t}(1 - Y_t) - \lambda_{4,t} Y_t + \lambda_{6,t} \frac{\theta}{\phi} = 0 \\
& \Pi_t : \lambda_{1,t-1} \kappa_{2,t-1} \frac{C_t}{R_{t-1}} + \lambda_{2,t-1} \frac{1}{R_{t-1}} (Q_t + D_t) + \lambda_{6,t-1} \frac{1}{R_{t-1}} (3\Pi_t^2 - 2\Pi_t) \frac{Y_t}{Y_{t-1}} \\
& - \delta_t \beta \lambda_{4,t} \phi (\Pi_t - 1) Y_t - \delta_t \beta \lambda_{5,t} \phi (\Pi_t - 1) Y_t - \delta_t \beta \lambda_{6,t} (2\Pi_t - 1) = 0 \\
& R_t : -\lambda_{1,t} \kappa_{2,t} \frac{\Pi_{t+1} C_{t+1}}{R_t^2} - \lambda_{2,t} \frac{\Pi_{t+1}(Q_{t+1} + D_{t+1})}{R_t^2} - \lambda_{6,t} \frac{\Pi_{t+1}^2 (\Pi_{t+1} - 1)}{R_t^2} \frac{Y_{t+1}}{Y_t} + \lambda_{7,t} = 0, \\
& (\lambda_{7,t} \geq 0, \quad R_t - 1 \geq 0, \quad \lambda_{7,t}(R_t - 1) = 0)
\end{aligned}
$$

G.4 Calibration and solution method

We solve the model in its original nonlinear form using the modified Newton method developed by Juillard, Laxton, McAdam, and Pioro (1998). This method modifies the standard Newton algorithm to take advantage of the recursive structure common in infinite-horizon macroeconomic models. Nakata (2017) uses this method to solve optimal fiscal and monetary policy in New Keynesian model with the ZLB constraint. See the former paper for the general description of the method and the latter for how to apply the method to fully nonlinear New

Keynesian models with the ZLB constraint.

Parameter values are listed in Table G.1. The weight on leisure in the utility function is close to those values used in Castelnuovo and Nisticó (2010), Nisticó (2012), and Del Negro, Giannoni, and Patterson (2015). Price adjustment costs implies a slope of the Phillips curve that is consistent with Calvo parameter of 0.9. This adjustment cost parameter is a bit higher than what is commonly used in the literature and is chosen so that inflation does not decline too much in recessions. We consider 21 different values of p between 0 and 0.1. For each, p, the size of the discount rate shock is chosen so that the initial decline in consumption is 10 percent.

Table G.1: **Parameter Values**

Parameter	Description	Values
β	Discount factor	0.9925
v	Utility weight on $L_{j,t}$	1.25
p	Turnover rate	$[0, 0.1]$
θ	Relative price elasticity of demand	11
φ	Price adjustment cost	1200
Ψ	Coefficient of the Taylor rule	1.5
R_{ELB}	The effective lower bound	1
δ_H	Demand shock	Chosen so that C_1 declines 10% from the SS under the time varying Taylor rule

G.5 Results

Figure G.1 shows the dynamics of the economy for three different values of p under the simple rule. According to the figure, allocations are not noticeably different across different values of p. For all values of p, consumption declines 10 percent and inflation declines by 2 percentage points in period one. They gradually increase and, once the shock disappears at period 21, the economy is at the steady state. One unique feature of the model is that the steady state nominal interest rates depends on p. In particular, a higher p is associated with a higher steady state nominal interest rate.[23]

Note that, in this model, the magnitudes of the declines in consumption and inflation at the ZLB depend on the time, whereas they do not in the infinite-horizon model considered in Section 3 in the main text. This difference is driven by the difference in the shock process. In the model of this section, the shock is not stochastic; the duration of the shock is set to 20 periods. In the infinite-horizon model considered in Section 3 in the main text, the shock is stochastic and is governed by a Markov process.

Figure G.2 shows the dynamics of the economy for three different values of p under the optimal commitment policy. For any values of p, the policy rate is kept at the ZLB even after the shock disappears. Consistent with what we saw in models with discounted Euler equation and Phillips curve, this low-for-long policy creates overshooting of consumption and inflation, which in turn mitigates the decline in consumption and inflation at the beginning of recessions through expectations.

The additional duration of holding the policy rate at the ZLB is shorter when p is higher. With $p = 0$, the additional ZLB duration is 4 periods, whereas it is 3 periods under $p = 0.02$

[23] If we modify β as we modify p, the power of forward guidance will not get weakened.

Figure G.1: IRFs under the simple rule

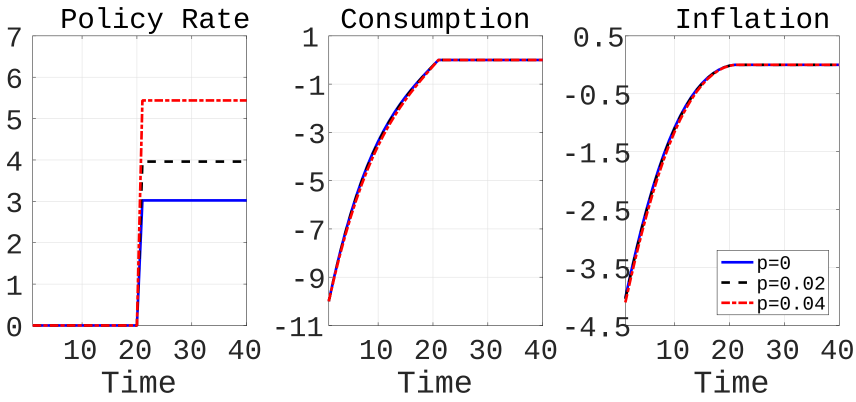
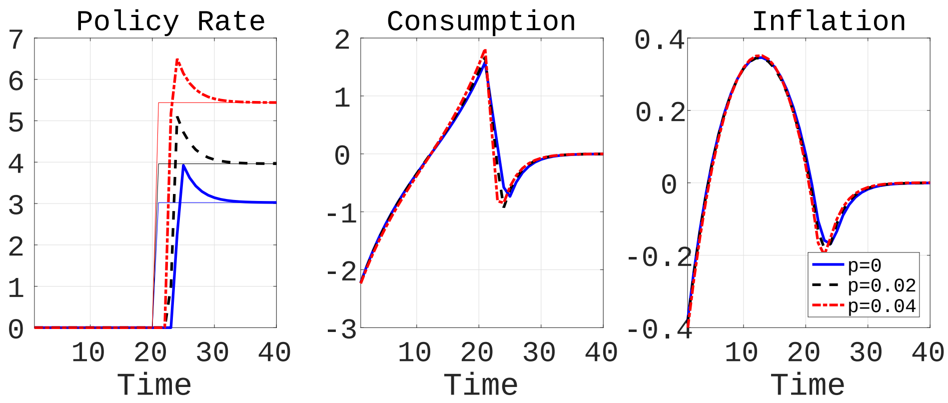


Figure G.2: IRFs under optimal policy



and $p = 0.04$. The allocations are similar, but there is slightly more overshooting in a model with a higher p.

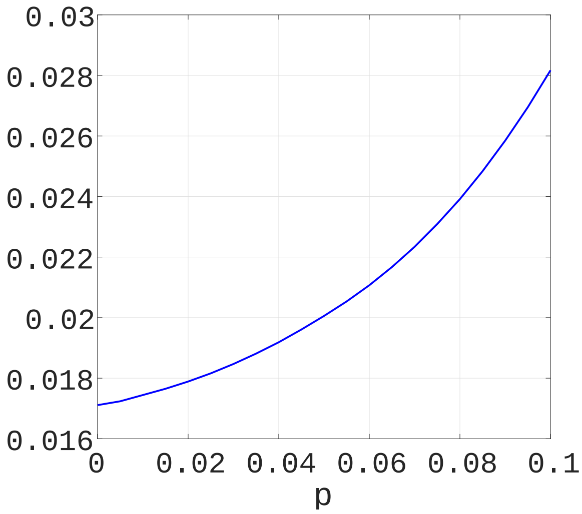
In this model, it would be misleading to use the additional ZLB duration as the measure of the extent to which the central bank uses forward guidance to stimulate economic activities at the ZLB, because the steady state nominal interest rates are different across different values of p. For example, when the central bank keeps the policy rate at the ZLB for one additional period in the model with $p = 0$, it is setting the interest rate 3 percentage points below the rate required to stabilize the economy. However, when the central bank keeps the policy rate at the ZLB for one additional period in the model with $p = 0.04$, it is setting the interest rate 5.5 percentage points below the rate required to stabilize the economy. Thus, by keeping the policy rate at the ZLB for one additional period, the central bank is providing more future policy accommodation under $p = 0.04$ than under $p = 0$.

To account for the difference in steady state nominal interest rates across different value of p, we use the following statistics—which we call "EA (Extra Accommodation)"—which is the cumulative difference between the path of policy rates under the simple rule (R_t^{SR}) and that under the optimal commitment policy (R_t^{OCP}))—as the measure of how actively the central bank uses forward guidance:

$$\text{EA} := \sum_{t=0}^{\infty}(R_t^{SR} - R_t^{OCP})$$

Figure G.3 shows how this measure of future policy accommodation varies with p. According to the figure, the central bank promises more future accommodation when p is higher—that is, when forward guidance is less powerful. This result is consistent with the analysis of the infinite-horizon model in the main text; the central bank finds it optimal to compensate the reduced power of forward guidance by more extensively using it.

Figure G.3: Extra accommodation



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