Innovation, Industry Equilibrium, and Discount Rates*

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Abstract

We develop a model to examine how discount rates affect the nature and composition of innovation within an industry. Challenging conventional wisdom, we show that higher discount rates do not discourage firm innovation when accounting for the industry equilibrium. Higher discount rates deter fresh entry—effectively acting as entry barriers—but encourage innovation through the intensive margin, which can lead to a higher industry innovation rate on net. Simultaneously, high discount rates foster explorative over exploitative innovation. The model rationalizes observed patterns of innovation cyclicality, and predicts that lower entry in downturns hedges innovating incumbents against higher discount rates.

Keywords: Vertical and horizontal innovation, creative destruction, time-varying discount rates, risk premia.

JEL Classification Numbers: G31; G12; O31

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Introduction

Since Schumpeter (1939), scholars have argued that innovation is key to understand the real economy. In recent years, the study of the determinants of corporate innovation has become particularly relevant, as firms' investment in research and development (henceforth, R&D) has increased dramatically.[1] Despite this growing interest, the literature has so far neglected the role of discount rates in explaining firms' R&D. This is surprising: Existing studies show that discount rates are key to explain other corporate decisions, such as physical investment (Lamont, 2000), initial public offerings (Pastor and Veronesi, 2005), buyouts and mergers (Malenko and Malenko, 2015; Haddad, Loualiche, and Plosser, 2017), or hiring and layoff decisions (Hall, 2017). Yet, discount rates should be arguably more significant in explaining R&D, as it is a long term investment, has an extended gestation period, and bears an uncertain outcome.

This paper seeks to fill this gap by developing a novel theoretical framework to study how discount rates affect corporate R&D, while stressing the importance of studying innovation in industry equilibrium. Corporate finance textbooks suggest that higher discount rates should penalize cash flows expected in the far future and, thus, should discourage investment, especially longer-term ones such as R&D. Yet, this line of reasoning neglects that a firm's R&D largely depends on the presence and decisions of competing firms, which are also affected by discount rates. As a starting point, we empirically document that higher discount rates need not reduce innovation, by focusing on the aggregate risk premium. As the aggregate risk premium increases, US firms invest a *larger* fraction of their assets or sales on R&D on average, controlling for firm fixed-effects—suggesting that discount rates have a positive impact on the intensive innovation margin. Simultaneously, however, the evidence suggests that higher discount rates discourage innovation in the extensive margin: A higher aggregate risk premium relates to significantly *fewer* firms, net of industry fixed-effects.[2] Figure 1 illustrates these patterns in the data.

[1] See Doidge, Kahle, Karolyi, and Stulz (2018), Brown, Fazzari, and Petersen (2009) or De Ridder (2023).

[2] Specifically, we observe both fewer innovating firms as well as fewer firms in innovating industries. We further elaborate on the empirical evidence in Section 5, and report methodological details in the Online Appendix OA.4.

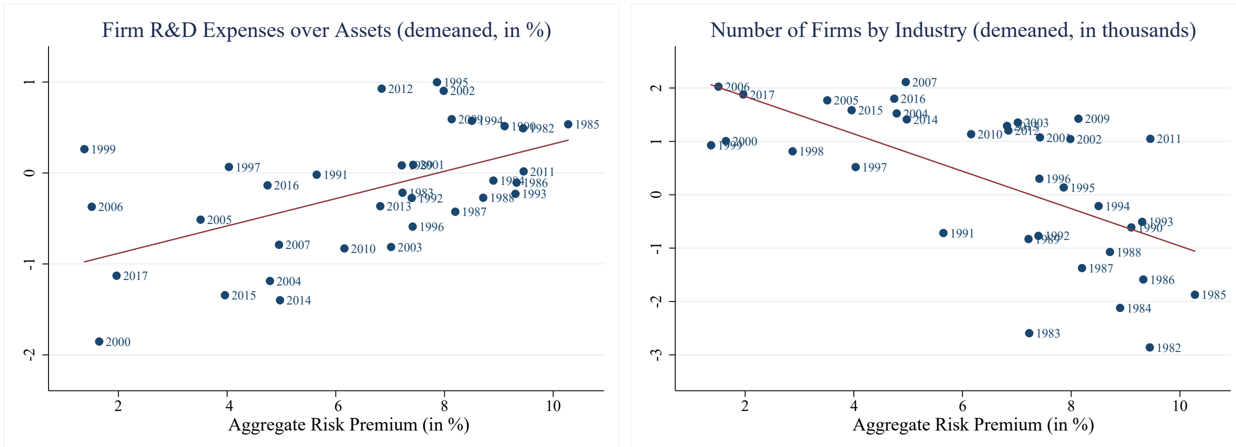

Figure 1: Firm-level R&D, Number of Firms, and Discount Rates. The left panel plots the average ratio of R&D over total assets of public US firms in Compustat, net of firm-fixed effects, against the aggregate risk premium, between 1982 and 2017. The right panel plots the average number of firms by industry-year, net of industry-fixed effects, for the same time period and set of industries, using data from the Business Dynamics Statistics (BDS) project covering both public and private firms.

Taking into account that firms do not innovate in isolation, we thus propose a model that characterizes how discount rates affect the composition of innovation within an industry. Crucially, we show that a higher *market price of risk*—the main common component of firms' discount rates in the cross section—significantly deters entry of new firms in the extensive margin and, simultaneously, encourages innovation by incumbents in the intensive margin. This mechanism of discount rates as an entry barrier rationalizes the evidence in Figure 1. Moreover, lower entry hedges innovating incumbents against the otherwise negative effect of higher discount rates on firm value. Considering that the market price of risk is countercyclical, our model shows that discount rate fluctuations help rationalize the cyclicality of R&D through its variation in the extensive margin: We reconcile the Schumpeterian view that firm-level innovation is countercyclical with the well-documented procyclicality of aggregate R&D. We also show that discount rates affect the type of innovation pursued by firms: Higher discount rates foster explorative over exploitative innovation.

Our novel theoretical framework allows us to prove key results analytically while incorporating endogenous entry and heterogeneous firms. We consider an industry in which firms are subject to two sources of systematic risk: a diffusion risk directly affecting firms' cash flows, and a jump risk associated with changes in the state of the economy. The market price of risk associated with the diffusion risk is state-contingent. Consistent with the evidence (e.g., Braguinsky, Ohyama, Okazaki, and Syverson, 2021; Manso, Balsmeier, and Fleming, 2021), we acknowledge that firms in the industry may pursue two types of innovation: vertical (or explorative)—which aims at major breakthroughs that improve the quality of technology—and horizontal (or exploitative)—which aims at creating new products. We further acknowledge that firms in the industry vary in their ability to innovate and produce (e.g., Akcigit, Hanley, and Serrano-Velarde, 2021; Bena, Fisher, Knesl, and Vahl, 2022).

In particular, we consider three types of firms: an initiator (or innovating producer), exploiters (or non-innovating producers), and entrants (or innovators). The initiator is the leading firm in the industry. It represents the latest successful innovator to advance the technology frontier via a vertical breakthrough, starting a bundle of products building on such breakthrough. Exploiters are firms that, building on the latest vertical breakthrough, have developed new products via horizontal breakthroughs, and solely focus on production. Lastly, entrants are startups on the sideline that invest in vertical and horizontal innovation, with the aim of becoming either initiators (upon a vertical breakthrough) or exploiters (upon a horizontal breakthrough). Vertical breakthroughs cast the threat of creative destruction on initiator and exploiters, causing their exit. Horizontal breakthroughs lead to partial displacement by making some of the existing products obsolete, eroding the initiator's and exploiters' revenues. Hence, the model accommodates two types of rivalries, which need not overlap (Bloom, Schankerman, and Van Reenen, 2013): The initiator competes with entrants over technological leadership, whereas it competes with exploiters over product lines.

To disentangle the strengths at play, we develop our analysis in steps and start by considering the case in which the market price of risk is constant and firms only invest in vertical innovation. When abstracting from industry dynamics, we confirm the conventional wisdom that a higher market price of risk discourages firm R&D. Yet, when allowing for endogenous industry dynamics with entry, this result is overturned. Specifically, we prove formally that a higher market price of risk discourages entry by new firms and yet encourages innovation by incumbents, consistent with the evidence in Figure 1. Furthermore, compounding these offsetting effects, the model predicts that the market price of risk has a non-monotonic effect on the industry-level rate at which new technologies emerge. A higher market price of risk can spur the advent of new technologies if the increase in R&D by active firms (the intensive margin) more than offsets the decline in the mass of entrants (the extensive margin).

Notably, our results are robust to allowing the initiator not only to innovate "in house" but also to acquire entrants that attain breakthroughs and preserve its technological leadership (Phillips and Zhdanov, 2013). In this case too, the extensive innovation margin decreases with higher aggregate discount rates, whereas in-house innovation increases. As discount rates increase and target firms become scarcer, acquisitions of innovating entrants simultaneously become less likely. Consistent with Haddad, Loualiche, and Plosser (2017), our model then shows that the frequency of acquisition increases as the market price of risk decreases.

Our core predictions continue to hold when allowing for horizontal innovation, in which case the initiator experiences profit erosion due to the introduction of competing product lines. As an additional implication, we show that that different types of innovation—horizontal or vertical—exhibit different sensitivity to discount rates. While the rate of vertical innovation increases with the market price of risk due to the ensuing lower threat of creative destruction, the rate of horizontal innovation is non-monotonic. First, as the market price of risk increases, the lower threat of creative destruction spurs horizontal innovation. Simultaneously, however, exploiters face a greater threat of exit due to the initiator's greater innovation rate, which reduces the reward and the incentives to invest horizontally. Overall, this second strength dominates when the market price of risk is sufficiently high. Hence, a greater market price of risk stimulates the more explorative type of innovation.

We next allow the market price of risk to vary over time, which enables us to empirically validate the predictions of our model by capturing the cyclicality of R&D at both the firm and aggregate levels. Consistent with the evidence in Lustig and Verdelhan (2012) and others, we acknowledge that the market price of risk is countercyclical and, thus, assume that the economy switches over two states, one with a low market price of risk (the good state or expansion) and the other with a high market price of risk (the bad state or recession). We show that active firms optimally set a higher innovation rate when the market price of risk is higher but, at the same time, fewer firms are active. That is, active firms face lower competition in innovation in bad states of the economy thanks to a lower threat of creative destruction and of product obsolescence which, in turn, encourage their investment in R&D. Our paper then reconciles the Schumpeterian view that firms should invest more in bad states of the economy with the evidence that R&D is procyclical at the aggregate level.[3]

Interestingly, the model predictions on R&D cyclicality are consistent with recent empirical studies. Babina, Bernstein, and Mezzanotti (2023) find that the intensive margin of innovation is resilient during downturns, whereas the extensive margin drops due to a substantial decline in patenting by entrepreneurs—aligned with our prediction that incumbents benefit from lower entry. Babina, Bernstein, and Mezzanotti (2023) also find that innovation shifts towards more impactful patents during downturns—hinting at greater exploration when the market price of risk increases, consistent with our model and the evidence in Manso, Balsmeier, and Fleming (2021). Furthermore, our results strengthen when acknowledging that firms' ability to finance entry deteriorates during recessions, an aspect that we embed in an extension, aligned with the empirical findings in Brown, Fazzari, and Petersen (2009) and Howell, Lerner, Nanda, and Townsend (2020).[4]

Our model also reveals that fluctuations in the market price of risk have the strongest impact on the extensive margin. On average, the mass of entrants is significantly larger and the rate of creative destruction is greater with time-varying market price of risk in comparison to an identical economy in which the market price of risk is fixed at the two-state average. Thus, fluctuations in the market price of risk lead to higher industry turnover, which spur new technologies—consistent with the Schumpeterian view of creative destruction.

Lastly, the main asset pricing implication of our model is that lower entry in downturns hedges innovating incumbents against the otherwise negative effect of higher discount rates on firm value. The negative impact of higher discount rates on firm-level R&D is more than offset by the positive effect of the lower rate of creative destruction, so that the gain from technological improvements extracted by active firms increases in downturns. Our model also shows that greater competition in innovation makes firms riskier—irrespective of whether these firms are innovating or not—but a firm's own R&D efforts make it safer.

[3]See, e.g., Griliches (1984), Comin and Gertler (2006), Barlevy (2007), or Fabrizio and Tsolmon (2014).

[4]The main mechanism of discount rates as entry barrier continues to hold if we look at the risk free rate instead. However, because the risk free rate is procyclical, the risk-free-rate variation alone would give rise to counterfactual cyclicality of R&D at both the firm and aggregate level. Indeed, the market price of risk is the main source of variation—and, thus, dynamics—in aggregate discount rates.

Related literature Our paper relates to the literature showing the significance of discount rates for various corporate decisions and aggregate dynamics (see the presidential address by Cochrane, 2011). In this strand, Pastor and Veronesi (2005) show that waves of initial public offerings are largely driven by declines in expected market returns. Malenko and Malenko (2015) and Haddad, Loualiche, and Plosser (2017) study the impact of discount rates on buyout activity. Chen (2010) develops a model in which fluctuations in aggregate risk premia rationalize empirical puzzles on debt financing and default decisions. Taking a macroeconomic perspective, Hall (2017) shows that the time variation in discount rates is a strong determinant of unemployment dynamics, and Di Tella and Hall (2021) find that spikes in aggregate risk premia explain business cycles. We contribute to this strand by showing that the level and fluctuations of the market price of risk have a first-order impact on R&D, challenging the conventional wisdom that higher discount rates discourage investment.

The model naturally relates to the theoretical literature on innovation. We propose a tractable quality ladder model with endogenous entry and heterogenous innovations, in the spirit of Akcigit and Kerr (2018) and Akcigit, Hanley, and Serrano-Velarde (2021). Unlike earlier contributions, firms in our model are subject to discount rate fluctuations. Previous models have studied how R&D relates to incentives schemes (Manso, 2011), ownership structure (Ferreira, Manso, and Silva, 2014), takeovers (Phillips and Zhdanov, 2013), financing frictions and cash holdings (Malamud and Zucchi, 2019; Lyandres and Palazzo, 2016), debt financing (Geelen, Hajda, and Morellec, 2022), and lobbying (Grotteria, 2023). We focus on discount rates while stressing the importance of studying R&D in industry equilibrium.

Our model implications also relate to the empirical literature on competition in innovation. In this strand, Kogan, Papanikolaou, Seru, and Stoffman (2017) measure how a firm's innovation affects its rivals; Manso, Balsmeier, and Fleming (2021) show that firms engage in more explorative innovations during downturns; and Braguinsky et al. (2021) study how firms grow by innovating vertically and horizontally. Our predictions on the cyclicality of R&D rationalize the evidence in Howell et al. (2020) and Babina, Bernstein, and Mezzanotti (2023). We contribute to this strand by showing that the aggregate risk premium—the main common source of variation in discount rates—significantly explains R&D.

More generally, our paper relates to models of corporate decisions in industry equilibrium.

Miao (2005) focuses on capital structure; Hackbarth and Miao (2012) elaborate on the link between mergers and industry dynamics; and Pindyck (2009) characterizes how uncertainty affects entry barriers. Asriyan, Laeven, Martin, Van der Ghote, and Vanasco (2023) show how heterogeneous quality of entrepreneurs, financial frictions, and imperfectly-elastic supply of capital make the impact of monetary policy ambiguous on economic activity. Liu, Mian, and Sufi (2022) propose a model with a duopoly of innovating firms to jointly explain the rising market concentration and falling productivity growth as the risk free rate decreases. While Liu, Mian, and Sufi (2022) do not allow for entry, our model does, which helps rationalize the observed cyclicality of R&D.[5] Relatedly, De Ridder (2023) and Corhay, Kung, and Schmid (2020a,b) stress the importance of firm entry in explaining aggregate trends.

The paper is organized as follows. Section 1 presents the model. Section 2 analyzes the model implications when the market price of risk is constant, whereas Section 3 allows for time-variation in the market price of risk. Section 4 analyzes the asset pricing implications. Section 5 concludes with a discussion of our supporting empirical evidence. Analytical proofs and technical developments are gathered in the Online Appendix.

1 The model

The economic environment Time is continuous, and the horizon is infinite. We consider a cluster of firms, or industry, which compete in innovation. Firms are subject to two sources of aggregate risk: a diffusion risk and a jump risk. These risks are both priced and affect the dynamics of the stochastic discount factor, denoted by ξ_t, which satisfies:

$$\frac{d\xi_t}{\xi_t} = -rdt - \eta(j_{t-})d\widetilde{B}_t + \sum_{j_t \neq j_{t-}} \left(e^{\theta(j_{t-},j_t)} - 1\right) d\widetilde{N}_t^{(j_{t-},j_t)}. \tag{1}$$

In this equation, r is the risk-free rate. $d\widetilde{B}_t$ is a standard Brownian motion representing the systematic source of diffusion risk, and $\eta(j_t)$ represents the associated market price of risk. $\tilde{N}_t^{(j_{t-},j_t)}$ is a compensated Poisson process with intensity $\tilde{\pi}_{j_{t-}}$, and $\theta(j_{t-}, j_t)$ is the associated

[5] In addition, Liu, Mian, and Sufi (2022) impose no leapfrogging among firms, whereas we allow for it. Our results are robust to allowing the market leader to endogenously avoid leapfrogging by acquiring startups (see Section 2.3).

risk adjustment.[6]

The jump risk represents switches in the state of the economy. The economy can be in two states $j = G, B$: a good (expansion) state G and a bad (recession) state B. The market price of the diffusion risk is state-contingent $\eta(j_t) \equiv \eta_j$, with $\eta_G < \eta_B$—that is, the market price of risk is countercyclical, as documented by Lustig and Verdelhan (2012) among others.[7] A switch in the state of the economy causes a jump in the stochastic discount factor, meaning that investors require a compensation for such risk. This compensation translates into a wedge between the transition intensity under the physical and risk neutral measure. Using the risk adjustment $\theta(j_{t-}, j_t) \equiv \theta_j$, the risk-neutral transition intensities satisfy $\pi_j = e^{\theta_j}\tilde{\pi}_j$ in each state. As in Bolton, Chen, and Wang (2013), we assume that $\theta_G = -\theta_B > 0$, which implies that the transition intensity from state G (respectively, B) to state B (G) is higher (smaller) under the risk-neutral probability measure than under the physical one. That is, risk averse agents expect the good (bad) state to be shorter (longer).

Innovation and firm types Consistent with the evidence, we acknowledge that firms may pursue two types of innovation: Vertical (or explorative) or horizontal (or exploitative).[8] Specifically, vertical innovation represents major breakthroughs in the quality of technology, denoted by q_t. Conversely, horizontal innovation builds on the latest vertical breakthrough and aims at introducing new products. As in Howitt (1999), horizontal innovation applied to a given quality level q_t eventually runs into diminishing returns to scale. In the following, we will exploit the concept of "technological cluster," representing the collection of products that stem from a given increase in the quality of technology.

We assume that the industry features three types of firms: an initiator, exploiters, and entrants. The initiator, denoted by U_j in each state j, represents the latest vertical innovator starting a new technological cluster. It manufactures products that build on the latest technological breakthrough and, meanwhile, continues to invest in innovation. Exploiters,

[6]The definition of our stochastic discount factor follows Bolton, Chen, and Wang (2013) and is microfounded in the model by Chen (2010).

[7]As shown by Campbell and Cochrane (1999), the countercyclicality of the market price of risk can be driven, for instance, by time-varying risk aversion.

[8]Consistent with Arora, Belenzon, and Sheer (2021), vertical innovation is more related to "research" whereas horizontal innovation to "development." Hsieh, Klenow, and Shimizu (2021) show that both are key to understand the patterns of growth.

denoted by X_j, are firms that have successfully developed new products via horizontal breakthroughs, and solely focus on production (i.e., they do not invest in innovation). Entrants, denoted by W_j, are startups on the sideline. They invest in vertical and horizontal innovation and have the potential to become the new initiator (through a vertical breakthrough) or an exploiter (through a horizontal breakthrough). We next describe these firm types in detail.

Initiator The latest vertical innovator improving the industry's quality level q_t becomes the initiator of a new technological cluster and drives the existing producers (that is, the previous initiator and exploiters) out of the market. Using this novel technology, the initiator manufactures a mass M_t of products. In each product line i, the firm faces the following demand function:[9]

$$p_{it} = \Gamma_j \left(\frac{Y_{it}}{q_t} \right)^{-\beta}, \tag{2}$$

where p_{it} represents the selling price associated to product i, Y_{it} represents quantity, and $\beta \in (0, 1)$ is the inverse of the price elasticity of demand.[10] Γ_j represents a demand-shift parameter, which varies with the state of the economy j. We normalize the cost of production to one in all product lines. Following previous models of innovation, we assume that all product lines exhibit the same demand function, and each product line is a monopoly until an entrant attains a breakthrough (see, e.g, Aghion and Howitt (1992), Howitt (1999), Acemoglu, Akcigit, Alp, Bloom, and Kerr (2018), among others). Hence, firms in our model do not compete in the same product market but still compete over product lines. Empirically, this setting is consistent with firms earning monopolistic rents from patented innovation as well as with recent evidence showing that firms compete by constantly launching new products (Baslandze, Greenwood, Marto, and Moreira, 2023).

The initiator earns revenues from producing the M_t goods and, at the same time, continues to invest in innovation. We denote by z_t the initiator's innovation intensity at time

[9] Our main results continue to hold when we assume that production decisions are not optimized—that is, the firm's expected revenues are exogenous and constant. That is, our results are not driven by the particular functional form of the firm's demand function.

[10] As we express p and Y as a function of time (as captured by the subscript t), we omit their dependence to the state of the economy (subscript j). Also, as all product lines face the same demand function, we drop the subscript i in the following.

t. We follow previous literature in capturing the key features of innovation: It is costly and has an uncertain outcome. That is, if the firm bears the following flow cost

$$\Phi(z,q,M) = \zeta \frac{z_t^2}{2} q_t M_t, \quad \zeta > 0, \tag{3}$$

it attains a breakthrough at Poisson rate ϕz_t, where ϕ is a positive constant. This specification implies that a breakthrough is more likely if the firm spends more on innovation. It also captures the idea that innovation is more costly if quality is greater or if the mass of current product lines is larger.[11] We assume that when the initiator attains a breakthrough, quality jumps by a factor $\lambda > 1$ and the mass of product lines jumps by a factor $\varphi > 1$. That is, because the initiator has specific "working" knowledge of the particular industry, an innovation increasing the quality of technology results in the creation of new products, in the spirit of Nelson (1959) and Akcigit, Hanley, and Serrano-Velarde (2021).

Absent breakthroughs by other firms, the cash flows of the initiator satisfy:

$$\begin{aligned} dC_t &= \left[Y_t\left(p_t - 1\right)M_t - \Phi(z,q,M)\right]dt + \sigma Y_t M_t d\widetilde{B}_t^U \\ &= \left[Y_t\left(p_t - 1\right)M_t - \Phi(z,q,M)\right]dt + \sigma Y_t M_t \left[\rho d\widetilde{B}_t + \sqrt{1-\rho^2}d\widetilde{B}_t^{U\perp}\right]. \end{aligned} \tag{4}$$

The first term represents the initiator's expected profits from production in the M_t product lines net of R&D expenditures. Throughout our analysis, we focus on cases in which this term is positive, so to avoid the degenerate case in which the initiator always makes losses in expectation. The second term represents the volatility of the initiator's cash flows, which increases with the firm's production rate. The parameter σ is a positive constant, and $\widetilde{B}^U$ is a standard Brownian motion under the physical probability measure. The Brownian motion $\widetilde{B}^U$ is correlated with the aggregate shock $\widetilde{B}$ by a factor $\rho \geq 0$. That is, $\widetilde{B}^U$ can be decomposed into the orthogonal components $\widetilde{B}_t$ and $\widetilde{B}_t^{U\perp}$ through ρ.

Consistent with Argente, Lee, and Moreira (2023), the initiator loses some of its product lines if an entrant attains a horizontal breakthrough. In this case, new products are launched,

[11] Scalability of the flow cost of innovation in quality or product lines is consistent with previous models of endogenous growth, see e.g. Akcigit and Kerr (2018), Acemoglu and Cao (2018), or Aghion, Akcigit, and Howitt (2014).

making the initiator's products obsolete. That is, horizontal breakthroughs cause profit erosion. Namely, if an entrant attains a horizontal breakthrough creating a mass ωM_{t-} of new products, the initiator's product lines drop from M_{t-} to $M_t = M_{t-}(1 - \omega\delta)$, and so do cash flows.[12] The parameter $\delta \in (0, 1]$ represents the degree of overlap between new and existing products. A greater overlap means that more of the initiator's products become obsolete due to an entrant's breakthrough.

In addition to partial displacement due to horizontal breakthroughs, the initiator is hit by creative destruction if an entrant attains a vertical breakthrough. When creative destruction hits, the successful entrant takes over the initiator's market position, and the initiator liquidates its assets and exits. We assume that liquidation is costly, as the initiator recovers just a fraction $\alpha \in [0, 1)$ of its value.

Entrants There is a continuum of entrants on the sideline, whose endogenous mass is denoted by μ. Entrants only invest in innovation and can be interpreted as startups. Because entrants do not have ongoing production—that is, differently from the initiator, they do not have working knowledge of specific products—they do not benefit from the synergy between vertical and horizontal R&D as the initiator does. Thus, entrants need to spend on vertical or horizontal innovation separately.

We denote an entrant's innovation rate targeting vertical breakthroughs by v_t at any time t. Similar to the initiator, v_t governs the Poisson rate of vertical breakthroughs—given by $\phi_v v_t$ with ϕ_v being a positive constant—and entails the flow cost:

$$\Phi_v(v, q, M) = \zeta_v \frac{v_t^2}{2} q_t M_t, \quad \zeta_v > 0. \tag{5}$$

When the entrant attains a vertical breakthrough, the industry's technological quality jumps by a factor $\Lambda > 1$, a new technological cluster is created, and the entrant becomes the new initiator.

In addition, we denote by h_t an entrant's innovation rate targeting horizontal break-

[12] As shown by Equation (4), the cash flows of the initiator scale up with its mass of product lines M_t.

throughs. When spending the amount

$$\Phi_h(h, q, M) = \zeta_h \frac{h_t^2}{2} q_t M_t, \quad \zeta_h > 0, \tag{6}$$

an entrant attains a horizontal breakthrough at a Poisson rate $\phi_h h$, with $\phi_h > 0$. The greater h_t is, the more likely the entrants will attain a breakthrough and create a mass of new products $M_{Xt} = \omega M_{t-}$, where $\omega \in [0, 1]$ and M_{t-} represents the mass of the initiator's products before the breakthrough. As M_t decreases as more products are introduced within a technological cluster, horizontal innovation run into diminishing returns to scale.[13] Once an entrant attains a horizontal breakthrough, it becomes an exploiter thereafter.

Entrants are exposed to random shocks—for instance, random outflows or windfalls in the development of new ideas or products. Specifically, entrants' cash flows satisfy:

$$dC_t^W = \left[-\frac{1}{2} \left(\zeta_v v_t^2 + \zeta_h h_t^2 \right) dt + \sigma_W d\widetilde{B}_t^W \right] M_t q_t \tag{7}$$

where $\sigma_W > 0$ and $\widetilde{B}_t^W$ is a standard Brownian motion under the physical measure correlated with the aggregate shock $\widetilde{B}_t$ by a factor $\rho_W \geq 0$.[14] Equation (7) implies that entrants have negative cash flows in expectation, consistent with the evidence that startups typically lack steady revenues. As entrants aim at improving the quality and expand the products launched by the initiator, their innovation costs and volatility depend on q_t and M_t.

At the outset, entrants face an entry cost $K_t = \kappa q_t M_t$ to start investing in innovation, which can then be interpreted as the cost of installing the firm's technological capital. The magnitude of the cost K_t varies over time due to technological improvements in q_t or due to the expansion or contraction of M_t. As a result, if the initiator or other entrants attain a breakthrough, an entrant needs to adjust its technological capital in proportion to the ensuing change in q_t and/or M_t, consistent with Luttmer (2007).

[13] Notably, because entrants aim to improve on the initiator's technology and products, their innovation cost is a function of current quality q_t and of the initiator's mass of product lines M_t.

[14] As for the initiator, we can decompose the Brownian motion $\widetilde{B}_t^W$ into the systematic source of risk and an orthogonal component, representing purely idiosyncratic risk.

Exploiters Entrants who successfully attain a horizontal breakthrough—then creating a mass of new products M_{Xt}—become the monopolistic producers in such new product lines. These firms, to which we refer as exploiters, give up on innovation and maximize their value by choosing the production quantity Y_{Xt} in their product lines. As the initiator, exploiters face the demand function (2) in each product line. An exploiter's cash flows are given by:

$$dC_t^X = Y_{Xt} (p_{Xt} - 1) M_{Xt} dt + \sigma_X Y_{Xt} M_{Xt} d\widetilde{B}_t^X, \tag{8}$$

where σ_X is a positive constant, and $\widetilde{B}_t^X$ is a standard Brownian motion under the physical probability measure that is correlated with the aggregate shock $\widetilde{B}_t$ by a factor $\rho_X \geq 0$.[15] Because the initiator and the exploiters both produce goods in the same industry, we assume that their exposure to aggregate risk is the same, so $\rho_X = \rho$. As for the initiator, an exploiter's cash flow volatility increases with its production rate.

As the initiator, exploiters lose a fraction $\omega\delta$ of their product lines when entrants attain horizontal breakthroughs. In addition, exploiters face the threat of exit if a vertical breakthrough improves the current quality q_t. When this happens, exploiters liquidate and recover just a fraction $\alpha_X \in [0, 1)$ of their value. Notably, the exploiters are subject to the threat of exit when either the initiator or the entrants attain a vertical breakthrough.

Industry equilibrium We consider an industry equilibrium in which: (1) the initiator maximizes its value by choosing its optimal production and innovation rate; (2) exploiters maximize their value by choosing their optimal production rate; (3) entrants maximize their value by choosing their optimal vertical and horizontal innovation rates; (4) the mass of active entrants makes the free-entry condition binding at any time.

In equilibrium, the rate of creative destruction Ψ_{vt} is derived endogenously as the rate at which entrants, on aggregate, attain a vertical breakthrough starting a new technological cluster. In turn, the equilibrium rate of horizontal displacement Ψ_{ht} is the rate at which entrants, on aggregate, attain horizontal breakthroughs, then creating new products.

[15] As for the other firms, the Brownian motion $\widetilde{B}_t^X$ can be decomposed into the orthogonal components $\widetilde{B}_t$ and $\widetilde{B}_t^{X\perp}$ through ρ_X, where $\widetilde{B}_t^{X\perp}$ is independent to the aggregate (priced) risk $\widetilde{B}_t$.

2 Constant market price of risk

To disentangle the forces at play, we start by considering the case in which there is only one state of the economy, in which the market price of risk is constant and denoted by η.[16]

2.1 Model solution

Initiator By Girsanov theorem, we derive the dynamics of cash flow under the risk neutral measure. Using standard arguments, the value of the initiator satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned} rU(q, M) = \max_{z,Y} \; & MY(p - 1 - \sigma\eta\rho) - \zeta\frac{z^2}{2}qM + \phi z\left[U(\lambda q, \varphi M) - U(q, M)\right] \\ & + \Psi_v\left[\alpha U(q, M) - U(q, M)\right] + \Psi_h\left[U(q, M(1 - \omega\delta)) - U(q, M)\right]. \end{aligned} \tag{9}$$

The left-hand side is the return required by risk-neutral investors. The right-hand side is the expected change in firm value on an infinitesimal time interval. Namely, the first two terms are the risk-adjusted expected cash flows net of R&D. The third term is the expected change in firm value due to a breakthrough by the initiator, which triggers an increase in quality and an expansion in the mass of products. The fourth term represents the effect of creative destruction triggered by entrants' vertical innovations (occurring at rate Ψ_v), in which case the initiator exits and recovers just a fraction α of its value. The last term is the effect of obsolescence triggered by entrants' horizontal innovations (occurring at rate Ψ_h), which erodes a fraction $\omega\delta$ of the initiator's product lines.

We conjecture that the value of the initiator scales with quality q_t and with the mass of product lines M_t, $U(q_t, M_t) = q_t M_t u$, where u represents the initiator's scaled value. Also, we define by $y \equiv Y_t/q_t$ the production quantity in each product line scaled by quality. Substituting these definitions into equation (9) and differentiating the resulting equation with respect to y gives the optimal production quantity and the associated selling price:

$$y(\eta) = \left(\frac{\Gamma(1-\beta)}{1+\sigma\eta\rho}\right)^{\frac{1}{\beta}} \quad \Rightarrow \quad p(\eta) = \Gamma y^{-\beta} = \frac{1+\sigma\eta\rho}{1-\beta}. \tag{10}$$

[16] As there is just one state, the demand shift parameter $\Gamma_j = \Gamma$ is constant too.

As illustrated by equation (4), the initiator's exposure to aggregate risk is endogenous as so is its production quantity. Equation (10) shows that if the market price of risk η is greater, the firm reduces its production quantity and increases the selling price. Hence, the firm effectively reduces its exposure to aggregate risk and, by increasing the selling price, it passes the higher price of risk on to the consumers.[17] By calculations, the quantity $\Upsilon(\eta) \equiv \beta \left(\frac{1-\beta}{1+\sigma\eta\rho}\right)^{\frac{1}{\beta}-1} \Gamma^{\frac{1}{\beta}}$ represents the initiator's risk-adjusted profits from production.

Differentiating the HJB equation with respect to z gives the optimal innovation rate:

$$z(\eta) = \frac{\phi\left(\lambda\varphi - 1\right)u(\eta)}{\zeta}, \tag{11}$$

where the numerator captures the gain from technological breakthroughs , whereas the denominator is the associated cost. The optimal innovation rate increases if R&D expenditures are more likely to translate into technological breakthroughs (higher ϕ), if the returns to innovation are greater (larger λ or φ), or if innovation is less costly (ζ is smaller).

Exploiter Consider now the dynamics of the exploiters. Recall that exploiters are entrants that have attained a horizontal breakthrough creating a mass of new product lines $M_{Xt} = \omega M_{t-}$. Exploiter value satisfies:

$$\begin{aligned} rX(q, M_X) = \max_{Y_X} \quad & M_X Y_X \left(p_X - 1 - \eta\rho\sigma_X\right) + \left(\Psi_v + \phi z\right)\left(\alpha_X X(q, M_X) - X(q, M_X)\right) \\ & + \Psi_h \left[X(q, (1-\omega\delta)M_X) - X(q, M_X)\right]. \end{aligned} \tag{12}$$

As for equation (9), the right-hand side is the expected change in exploiter value over an infinitesimal time interval. The first-term represents the exploiter's risk-adjusted expected profits. The second term represents the effect of vertical innovations by entrants (occurring at rate Ψ_v) or by the initiator (occurring at rate ϕz), which cause the exit of the incumbent exploiters. The third term represents the effect of horizontal innovations by entrants (occurring at rate Ψ_h), which cause the exploiters to lose a fraction of their product lines. Exploiters maximize their value by choosing their optimal production quantity Y_X.

[17] This result—coupled with the fact that the market price of risk increases in downturns, as we elaborate in Section 3—implies that firms' markups in our model are countercyclical.

We conjecture that the exploiter value function satisfies $X(q_t, M_{Xt}) = q_t M_{Xt}\tilde{x}$, where $\tilde{x}$ represents the exploiter value scaled by the industry's quality level q_t and by the mass of its active product lines M_{Xt}. We define $y_X \equiv Y_{Xt}/q_t$ as an exploiter's production quantity per active product line scaled by quality. When an exploiter starts production, its mass of product lines can be expressed as a function of the product lines of the initiator: $M_{Xt} = \frac{M_t}{1-\omega\delta}\omega$.[18] Thus, we can express the exploiter value as a function of the active product lines of the initiator M_t as follows:

$$X(q_t, M_{Xt}) = q_t M_{Xt}\tilde{x} = q_t M_t \tilde{x}\frac{\omega}{1-\omega\delta} = q_t M_t x. \tag{13}$$

To make the scaled value of the exploiter comparable to the other scaled quantities, we define $x = \tilde{x}\frac{\omega}{1-\omega\delta}$. Maximizing the ensuing scaled HJB equation with respect to y_X gives

$$y_X(\eta) = \left(\frac{\Gamma(1-\beta)}{1+\eta\rho\sigma_X}\right)^{\frac{1}{\beta}} \tag{14}$$

and the associated selling price is $p_X = \frac{1+\sigma_X\eta\rho}{1-\beta}$. Notably, the initiator and the exploiters choose a different production quantity due to the difference in their cash flow volatilities (see equation (10)), which in turn results in a different exposure to systematic risk.

Entrants Entrant's value $W(q, M)$ is a function of the current quality level and the product lines they try to improve on. Entrant value satisfies:

$$\begin{aligned} rW(q,M) = \max_{v,h} \quad & -qM\left(\eta\rho_W\sigma_W + \frac{\zeta_v}{2}v^2 + \frac{\zeta_h}{2}h^2\right) + \phi_v v\left[U(\Lambda q, M) - W(q,M)\right] \\ & + \phi_h h\left[X(q,\omega M) - W(q,M)\right] + \phi z\left[W(\lambda q, \varphi M) - W(q,M) - K(\lambda\varphi - 1)\right] \\ & + \Psi_v^-\left[W(\Lambda q, M) - W(q,M) - K(\Lambda - 1)\right] + \Psi_h^-\left[W(q, M(1-\omega\delta)) - W(q,M) + K\omega\delta\right]. \end{aligned} \tag{15}$$

The first term on the right-hand side represents an entrant's risk-adjusted expected outflow on any time interval. The second term represents the effect of a vertical breakthrough by the entrant occurring at rate $\phi_v v$, in which case it becomes the new industry initiator.

[18] Recall that right after a horizontal breakthrough leading to the emergence of a new exploiter, the initiator's product lines are $M_t = M_{t-}(1-\omega\delta)$.

The third term represents the effect of a horizontal breakthrough by the entrant occurring at rate $\phi_h h$, in which case it becomes an exploiter. The fourth, fifth, and sixth terms represent the effect of breakthroughs by the current initiator (occurring at rate ϕz), vertical breakthroughs by other entrants (occurring at rate Ψ_v^-), or horizontal breakthroughs by other entrants (occurring at rate Ψ_h^-), respectively. The fourth and fifth terms imply that, whenever the initiator or other entrants attain vertical breakthroughs, the entrant needs to catch up with the new technology, consistent with Luttmer (2007). Catching up requires an upgrade cost proportional to the size of the breakthrough—depending on whether the breakthrough is attained by the initiator or an entrant, it is, respectively, $K(\lambda\varphi - 1)$ or $K(\Lambda - 1)$. Conversely, as in Howitt (1999), horizontal breakthroughs erode the value of the initiator and of the exploiters, as their product lines are competed away. It follows that the entrants' perspective earnings from innovation fall, and the entrant responds by adjusting its capital downwards by $K\omega\delta$, as illustrated by the last term in equation (15).

Consistent with our previous analyses, we conjecture that the entrant value scales with $M_t q_t$, that is, $W(q_t, M_t) = wq_tM_t$ where we denote by w the scaled value of a perspective entrant. Differentiating the resulting scaled HJB equation with respect to v gives the optimal vertical innovation rate of entrants:

$$v(\eta) = \frac{\phi_v \left(\Lambda u(\eta) - w\right)}{\zeta_v}. \tag{16}$$

Similar to equation (11), the numerator of this expression captures the gain from a vertical breakthrough, whereas the denominator represents the associated cost. Because an entrant attaining a vertical breakthrough takes over the initiator's market position, the gain from a vertical breakthrough is driven by $u(\eta)$. In turn, differentiating the scaled HJB equation with respect to h gives the optimal horizontal innovation rate:

$$h(\eta) = \frac{\phi_h}{\zeta_h} \left(\omega x(\eta) - w\right). \tag{17}$$

Because an entrant attaining a horizontal breakthrough becomes an exploiter, the gain from such breakthroughs is driven by $x(\eta)$.

Endogenous industry-level quantities. We conclude the derivation of the model solution by pinning down the key quantities at the industry-level. Aggregating the rate of vertical innovation across active entrants μ gives the rate of creative destruction:

$$\Psi_v(\eta) = \mu(\eta)\, \phi_v v(\eta). \tag{18}$$

In turn, the rate of horizontal displacement obtains by aggregating the rate of horizontal innovation across the mass of active entrants:

$$\Psi_h(\eta) = \mu(\eta)\, \phi_h h(\eta). \tag{19}$$

In these expressions, $\mu(\eta)$ is endogenously determined so that the free-entry condition $w = \kappa$ holds. Notably, Ψ_{vt} and Ψ_{ht} affect and are affected by the initiator's and the exploiters' scaled value. Lastly, we pin down the aggregate rate at which new technological clusters endogenously arise, denoted by $\mathcal{I}(z, \Psi_v)$, which satisfies:

$$\mathcal{I}(z, \Psi_v) = \phi z(\eta) + \Psi_v(\eta) = \underbrace{\frac{\phi^2}{\zeta} u(\eta)(\lambda\varphi - 1)}_{\text{Initiator}} + \underbrace{\mu(\eta) \frac{\phi_v^2}{\zeta_v} \left[\Lambda u(\eta) - \kappa\right]}_{\text{Entrants}} \tag{20}$$

The first term represents the contribution of the initiator to the aggregate vertical innovation rate, whereas the second term is the contribution of entrants.

2.2 Model analysis

We analyze the model implications in steps. We start by considering simpler cases than the full model, for which we obtain analytical results. First, we analyze firms in isolation, instead of studying them in the industry equilibrium. Second, we allow for endogenous industry dynamics in two corner cases: an industry in which entrants engage in vertical innovation only, and an industry in which entrants invest in horizontal innovation only.

2.2.1 Exogenous industry dynamics

Suppose that the rate of creative destruction Ψ_v and the rate of horizontal displacement Ψ_h are exogenous and constant. In this case, the scaled value of the initiator continues to satisfy equation (9), but Ψ_v and Ψ_h are insensitive to the market price of risk η. The next proposition follows (see Appendix OA.1.1 for a proof).

Proposition 1 *For exogenous Ψ_v and Ψ_h, the initiator's innovation rate satisfies:*

$$z(\eta) = \frac{r + \Psi_v(1-\alpha) + \Psi_h\omega\delta - \sqrt{(r + \Psi_v(1-\alpha) + \Psi_h\omega\delta)^2 - 2\Upsilon(\eta)\frac{\phi^2}{\zeta}\left(\lambda\varphi - 1\right)^2}}{\phi\left(\lambda\varphi - 1\right)}, \quad (21)$$

which is a decreasing function of the market price of risk η.

By abstracting from endogenous industry dynamics—then neglecting that Ψ_v and Ψ_h are themselves functions of η in equilibrium—Proposition 1 shows that a greater market price of risk leads to a lower innovation rate. By discounting future breakthroughs and cash flows at a higher rate, a greater η decreases the initiator's optimal investment in innovation. This result is in line with the received wisdom that a greater market price of risk depresses long-term investment such as R&D.

2.2.2 Endogenous industry dynamics in corner cases

We now focus on two corner cases featuring endogenous industry dynamics.

Entrants only engage in vertical innovation If entrants invest in vertical innovation only, the industry features two types of firms: the initiator and the entrants.[19] The next proposition illustrates the sensitivity of the endogenous equilibrium quantities to η (see Appendix OA.1.2).

Proposition 2 *When entrants only invest in vertical innovation, the innovation rate of the initiator satisfies:*

$$z(\eta) = \frac{\phi(\lambda - 1)}{\zeta\Lambda}\left[\kappa + \sqrt{\frac{2\zeta_v(r\kappa + \eta\rho_W\sigma_W)}{\phi_v^2}}\right] \quad (22)$$

[19]In this case, there are no exploiters as entrants do not pursue horizontal innovation. The initiator is subject to creative destruction, but there is no horizontal displacement.

and the vertical innovation rate of active entrants satisfies:

$$v(\eta) = \sqrt{\frac{2(r\kappa + \eta\rho_W\sigma_W)}{\zeta_v}}. \tag{23}$$

Both $z(\eta)$ and $v(\eta)$ increase with η. At the same time, the mass of active entrants $\mu(\eta)$ as well as the rate of creative destruction $\Psi_v(\eta)$ decrease with η.

In contrast with Proposition 1 (where the initiator is considered in isolation), Proposition 2 shows that the optimal innovation rate of the initiator $z(\eta)$ and of active entrants $v(\eta)$ *increase* with η when accounting for the industry equilibrium. Proposition 2 also shows that the mass of entrants μ decreases with η, meaning that the market price of risk effectively acts as an entry barrier. Hence, a higher η bears two offsetting effects on the entrants' contribution to innovation, captured by the rate of creative destruction Ψ_v. First, active entrants invest more in innovation, as v increases with η. Second, the mass of active entrants shrinks. Proposition 1 illustrates that this second strength dominates, so Ψ_v decreases with η. Thus, the initiator is less threatened by exit and has greater incentives to invest in R&D.

Entrants only engage in horizontal innovation We next consider the case in which entrants only pursue horizontal innovation. As in the full model, there are three types of firms: initiator, entrants, and exploiters. Differently, the initiator is only subject to the threat of horizontal displacement. We prove the following result (see Appendix OA.1.3).

Proposition 3 *When entrants invest in horizontal innovation only, their investment in innovation satisfies*

$$h(\eta) = \sqrt{\frac{2(r\kappa + \eta\rho_W\sigma_W)}{\zeta_h}}, \tag{24}$$

which is an increasing function of η.

Similar to the case with vertical innovation only, horizontal innovation increases with η when accounting for endogenous displacement in the industry. The monotonicity of h with respect to η shown in Proposition 3 is noteworthy because, as discussed in Section 2.2.3, this result does not hold in the full model with both vertical and horizontal innovation. That is, these corner cases help us pin down the strengths in the full model, which we analyze next.

2.2.3 The full model

When entrants invest in both horizontal and vertical innovation, the rate of creative destruction and of horizontal displacement are jointly solved endogenously. While the richness of this case prevents us from obtaining closed-form solutions, we investigate it numerically.

Baseline parameterization Table 1 reports our baseline parameterization. We set the risk-free rate to 1%. Following previous contributions, we normalize $\phi = \phi_v = \phi_h$ to one.[20] We assume that the entrants' vertical R&D cost parameter ζ_v is ten times larger than the initiator's cost ζ, which is in the ballpark of Akcigit and Kerr (2018). We also assume that ζ_h is smaller than ζ_v to acknowledge that horizontal innovation is less costly than vertical innovation. The size of quality jumps $\lambda = 1.055$ and $\Lambda = 1.12$ are also in line with Akcigit and Kerr (2018). We set $\varphi = 1.14$, which is consistent with the estimates of Argente, Lee, and Moreira (2023) about the contribution of new products to sales growth. The inequality $\varphi > \lambda$ implies that the breakthroughs by the initiator is more exploitative than explorative, consistent with Gao, Hsu, and Li (2018) among others. We set δ to 0.2, which captures the overlap between existing and new innovations reported by the OECD (2015).[21] Furthermore, we set ω to 0.25, which implies that horizontal innovations lead to a 5% drop in the initiator's output, aligned with the estimates of Kogan et al. (2017).

We set $\beta = 0.13$, so that markups are consistent with the estimates by Hall (2018). Moreover, we normalize $\Gamma = 1$. We calibrate σ so that the initiator's cash flow volatility is about 11% (as in Malamud and Zucchi, 2019). We also assume that $\sigma_X < \sigma$ to acknowledge that, differently from initiators, exploiters do not have an active R&D program and, thus, their cash flow volatility is smaller.[22] In turn, we assume that the entrants' volatility is greater and equal to 20%—consistently, Begenau and Palazzo (2021) show that entrants exhibited greater volatility and R&D expenditures over time. We acknowledge that entrants are comparatively more exposed to idiosyncratic risk than actively-producing firms (initiator and exploiter)—consistently, we assume that $\rho = 0.55$ and $\rho_W = 0.2$. We set the exploiter's

[20] See, for instance, Akcigit and Kerr (2018) or Akcigit, Hanley, and Serrano-Velarde (2021).

[21] The degree of overlap is captured by the backward citation index, see OECD (2015). The report shows that, depending on the sector, the index ranges from slightly below 0.1 to slightly above 0.3.

[22] Recall that volatility for these firms is given by σX and $\sigma_X X_s$.

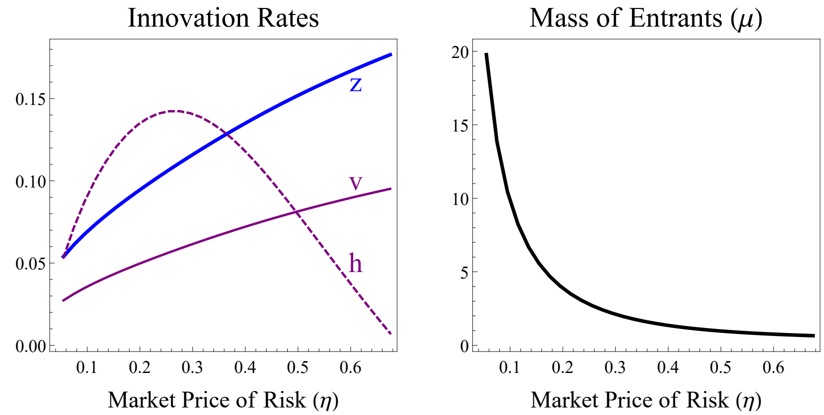

Figure 2: Innovation in the intensive and extensive margin. The left panel shows firm-level innovation rates (or intensive innovation margin) as a function of η: The initiator's innovation rate z, the vertical innovation rate of entrants v, and the horizontal innovation rate of entrants h. The right panel shows the mass of entrants (or extensive innovation margin) as a function of η.

recovery rate in liquidation to 0.85, consistent with Korteweg (2010). By setting a lower recovery rate for the initiator—which, differently from the exploiters, invests in R&D—we recognize that R&D entails asset intangibility, which leads to a greater value loss in liquidation. We set the magnitude of the entry cost to $\kappa_E = 0.015$, which gives a rate of creative destruction consistent with Acemoglu et al. (2018).

The equilibrium impact of the market price of risk on innovation Consistent with the analytical results in Proposition 2, Figure 2 shows that z and v increase with η. That is, when considering the industry equilibrium—and, thus, recognizing that a firm's incentives to invest in R&D depend on other firms' decisions—η has a positive effect on active firms' innovation rate aimed at starting new technological clusters. This result overturns the conventional wisdom that discount rates frustrate long-term investment such as R&D. At the same time, the figure also indicates that the mass of active entrants decreases with η—i.e., a higher η effectively acts as an entry barrier. Confirming the result in Proposition 2, Figure 3 shows that the declining pattern of μ more than offsets the increasing pattern of v in η—as a result, the rate of creative destruction Ψ_v decreases with η. A lower Ψ_v implies less competitive pressure on active firms, which spurs the more explorative R&D investment aimed at starting new technological clusters.

Focusing on horizontal innovation, Figure 2 also shows that h is hump-shaped in η.

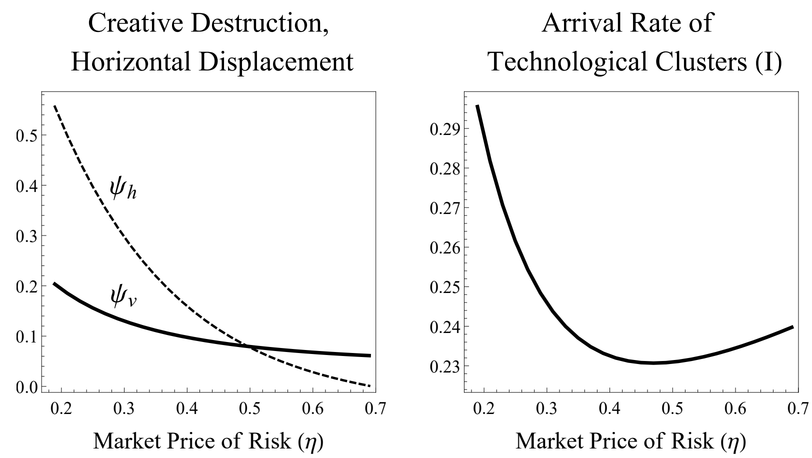

Figure 3: Industry-level quantities and the market price of risk. The figure shows the rate of creative destruction (Ψ_v), the rate of horizontal displacement (Ψ_h), and the endogenous arrival rate of new technological clusters ($\mathcal{I}$) as a function of η.

This result is in contrast with Proposition 3, showing that h increases with η if entrants only invest in horizontal innovation. In fact, the interaction between competition in the vertical and horizontal dimensions triggers nontrivial dynamics. At lower levels of η, h increases with η as the rate of creative destruction concurrently declines, and so does the associated liquidation risk of the exploiter—that is, the lower threat of creative destruction spurs horizontal innovation. However, as η increases further, the higher innovation rate of the initiator increases the exploiters' threat of exit, and reduces the entrants' incentives to invest in horizontal innovation.

These results illustrate that a higher market price of risk stimulates the more explorative type of innovation, i.e., vertical innovation. This can be seen at both the firm and industry level. At firm level, the right panel of Figure 2 illustrates that entrants shift resources from h to v when η is sufficiently large. At the industry level, Figure 3 shows that the rate of creative destruction Ψ_v is greater than the rate of horizontal displacement Ψ_h when η is sufficiently high, meaning that entrants invest more in vertical innovation, on aggregate.

A question then arises as to what is the net impact of the market price of risk on the rate $\mathcal{I}$ at which new technological clusters arise. As illustrated by equation (20), this quantity is the sum of the contribution of the initiator (ϕz) and of the entrants (Ψ_v). Because z increases whereas Ψ_v decreases with η, the sensitivity of $\mathcal{I}$ to η is ambiguous. Figure 3 shows that, under our baseline parameterization, $\mathcal{I}$ is U-shaped in η. That is, perhaps surprisingly, our model shows that an increase in the market price of risk can stimulate the advent of new technological clusters. This prediction contrasts with the textbook intuition that discount rates frustrate innovation. Our model suggests that the market price of risk importantly affects the composition of innovation within an industry—hence, a higher market price of risk needs not lead to a reduction in the industry innovation rate.

The interaction between vertical and horizontal innovation To further investigate the interaction between vertical and horizontal innovation, Table 2 exhibits the model's endogenous quantities in the cases in which entrants invest in either horizontal or vertical innovation only (as analyzed in Section 2.2.2) and in the full case, for different values of ω. Because horizontal innovation is less profitable when ω is smaller, the case with horizontal innovation only exists *if* ω is sufficiently large.

Introducing horizontal innovation—i.e., moving from the case with vertical innovation only to the full case—boils down to introducing the threat of profit erosion for active firms due to the emergence of competing product lines. Introducing horizontal innovation has an ambiguous effect on the mass of entrants μ. If horizontal innovation is sufficiently appealing (ω is larger), μ should increase. At the same time, horizontal innovation frustrates vertical innovation by making the initiator and exploiters more exposed to product obsolescence and, thus, profit erosion—a strength that reduces the mass of entrants μ. Table 2 suggests that, under our baseline parameterization, the second strength dominates and, thus, μ decreases when introducing horizontal innovation.[23]

Table 2 also shows that horizontal innovation decreases the initiator's R&D rate (z). The drop is wider if ω is larger, in which case horizontal breakthroughs trigger a sharper drop in the initiator's product lines. Similarly, the entrants' rate of vertical R&D v drops notably if ω is larger. Folding in the effects on μ and v, Table 2 shows that the rate of creative destruction decreases when introducing horizontal innovation—i.e., Ψ_v is lower in the full case than in the case with vertical innovation only. This result, together with the aforementioned impact on z, implies that horizontal innovation frustrates vertical innovation.

[23] In unreported results, we find that the first strength dominates if ω is unrealistically high, in which case the incentives to invest in horizontal innovation are disproportionately greater than those to invest in vertical innovation. These opposing strengths imply a tension regarding the effects of horizontal innovation (captured by ω) on entry.

Moreover, a greater emphasis on horizontal innovation leads to a lower industry turnover–on average, the initiator is expected to remain the technology leader for longer.

Consider now the effect of introducing vertical innovation, that is, moving from the case with horizontal innovation only to the full case. Introducing vertical innovation is akin to introducing the possibility of full displacement of firms—and, thus, exit—due to creative destruction. Table 2 shows that vertical innovation has a positive impact on the rate of horizontal displacement. Consistently, Braguinsky et al. (2021) show that vertical R&D has notable spillovers to horizontal R&D. In fact, the gain associated with vertical innovation—the prospect of becoming the next initiator—spurs an increase in the mass of entrants that, in turn, boosts the aggregate rate of horizontal displacement too. Yet, the rate of horizontal innovation h sharply drops: Because vertical innovation promises a greater upside potential (the perspective of becoming the new initiator), active entrants in the full case shift from horizontal to vertical innovation.

2.3 Allowing the initiator to take over entrants

In addition to innovating "in house," initiators often take over startups (e.g., Phillips and Zhdanov, 2013; Cunningham, Ederer, and Ma, 2021). In this section, we assess if our core predictions are confirmed when allowing the initiator to acquire entrants. For simplicity, we focus on the case with vertical innovation only.

We assume that the initiator bears a search cost $\zeta_s sqM$ for finding entrants that attain a breakthrough, where s denotes the search intensity and $\zeta_s > 0$ is a cost coefficient. The initiator acquires an entrant with (endogenous) probability $s/(1+s)$—i.e., the probability increases with the search intensity—at the endogenous cost A_t. With probability $1/(1+s)$, the initiator fails to take over the successful entrant, and the entrant becomes the new initiator. Furthermore, the entrant pays a setup cost G_t upon taking over the initiators' market position, and bears no cost if the entrant is acquired instead.

Under these assumptions, the value of the initiator satisfies the following HJB equation:

$$rU(q,M) = \max_{z,Y,s}\ MY(p-1-\sigma\eta\rho) - \frac{\zeta^2}{2}zqM - \zeta_s sqM + \phi z\left[U(\lambda q,\varphi M) - U(q,M)\right]$$
$$+\Psi_v\left[\frac{1}{1+s}\left(\alpha U(q,M) - U(q,M)\right) + \frac{s}{1+s}\left(U(\Lambda q,M) - U(q,M) - A\right)\right]. \tag{25}$$

The last term in this equation implies that, when an entrant attains a breakthrough, the initiator acquires it with probability $s/(1+s)$ at the cost A_t. Conversely, with probability $1/(1+s)$, the initiator does not acquire the successful entrant and exits. To solve the initiator's problem, we resort to scaled quantities (see Appendix OA.1.4) and define $a = A_t/(q_t M_t)$ and $g = G_t/(q_t M_t)$ as the scaled (time-invariant) acquisition cost and setup cost, respectively. Differentiating with respect to s yields the optimal search intensity:

$$s = \sqrt{\frac{\Psi_v\left[u(\Lambda-\alpha) - \Lambda a\right]}{\zeta_s}} - 1, \tag{26}$$

meaning that the initiator has greater incentives to search for a target either if the rate of creative destruction rises or if the cost of acquiring the target decreases.

In turn, maximizing the entrants' HJB equation with respect to v gives the expression for the optimal innovation rate of the entrant (see Appendix OA.1.4):

$$v = \frac{\phi_E}{\zeta_E}\left[\frac{\Lambda(u-g+as)}{1+s} - w\right]. \tag{27}$$

Differently from Proposition 2, the entrant's optimal innovation rate is now a function of the search intensity s and of the payoff a upon being acquired.

The initiator and the successful entrant negotiate over the terms of the deal. We consider a Nash bargaining solution where $b \in (0,1)$ corresponds to the bargaining power of the initiator. Solving for this bargaining game gives the equilibrium acquisition cost:

$$a = u - \frac{u}{\Lambda}\alpha(1-b) - bg, \tag{28}$$

which increases with the scaled value of the initiator (i.e., the gain upon a vertical breakthrough) and in the return to entrant's innovation Λ. Also, it decreases with the cost g.

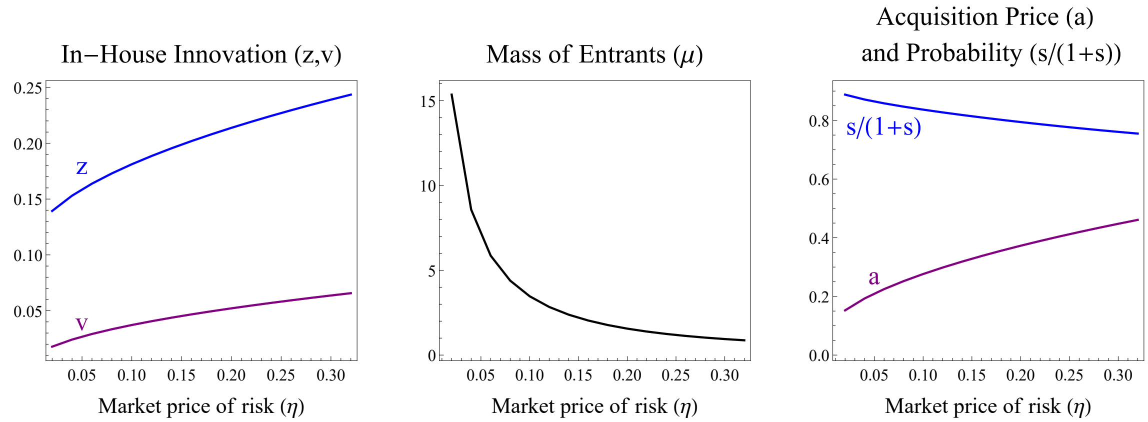

Figure 4: Allowing for takeovers. The figure shows the initiator and the entrants' in-house optimal innovation rates (z and v), the mass of entrants (μ), and the acquisition price and probability (a and $s/(1+s)$) as a function of η.

Figure 4 summarizes the main takeaways of this extension. The left and middle panels show that the core predictions of Proposition 2 are confirmed: The optimal innovation rates z and v increase with η, whereas the mass of entrants decreases. Additionally, the right panel shows that acquisitions become more likely as η decreases, in which case the mass of entrants (thus, the mass of potential targets) increases. This result is consistent with the evidence in Haddad, Loualiche, and Plosser (2017) that merger activity increases with lower discount rates. Finally, we find that the endogenous acquisition cost is increasing with η: As targets become relatively scarcer, the initiator pays a higher premium in equilibrium.

3 Time-varying market price of risk

The analysis so far reveals how the market price of risk affects the composition of innovation within an industry. Crucially, it rationalizes the motivating empirical evidence in Figure 1: A higher market price of risk reduces entry and stimulates R&D at the firm level. In this section, we allow the market price of risk to vary with the state of the economy, being η_G in the good state and $\eta_B > \eta_G$ in the bad state.[24] Accounting for this realistic feature allows us to formulate additional predictions on how discount rates affect the cyclicality of R&D.

[24] Derivations of the firm's optimal choices and of the industry equilibrium are in Appendix OA.2.

Corner cases. Before analyzing the full model with both vertical and horizontal innovation, it is worth considering the corner cases analogous to those in Section 2.2.2. We show the following results (see Appendix OA.2.2).

Proposition 4 *Assume that $\eta_B > \eta_G$. If entrants invest in vertical innovation only, the initiator's and active entrants' innovation rates respectively satisfy*

$$z_j(\eta_j, \eta_{j-}) = \frac{\phi}{\zeta}(\lambda\varphi - 1)\, u_j(\eta_j, \eta_{j-}),$$
$$v_j(\eta_j, \eta_{j-}) = \frac{\phi_v}{\zeta_v}\left[\Lambda u_j(\eta_j, \eta_{j-}) - \kappa\right],$$

and are countercyclical, $z_B > z_G$ and $v_B > v_G$. Conversely, the rate of creative destruction Ψ_{vj} and the mass of entrants μ_j are procyclical, $\Psi_{vG} > \Psi_{vB}$ and $\mu_G > \mu_B$. The extensive margin (μ_j) is more sensitive to variations in the market price of risk than the intensive margin (v_j). If, instead, entrants only invest in horizontal innovation, their optimal innovation rate satisfies

$$h_j(\eta_j, \eta_{j-}) = \frac{\phi_h}{\zeta_h}\left[\omega x_j(\eta_j, \eta_{j-}) - \kappa\right]. \tag{29}$$

and is countercyclical, $h_B > h_G$.

Proposition 4 illustrates how time variation in the market price of risk affects the industry equilibrium. It shows that the innovation rate of active firms is countercyclical—that is, it is greater when the market price of risk is larger. This holds in both corner cases with either vertical or horizontal innovation only. Proposition 4 also shows that the greater market price of risk in state B bears a negative impact on the *extensive* margin—i.e., the mass of active entrants declines. Moreover, the variation in the extensive margin is greater than the variation in the intensive margin. This implies that the procyclicality of μ then extends to the rate of creative destruction Ψ_v, which is procyclical too. In sum, in the good (bad) state, the mass of active entrants is larger (smaller), creative destruction is higher (lower), and incumbent firms reduce (increase) their R&D investment.

Innovation cyclicality and the market price of risk We next analyze the model with both vertical and horizontal innovation. On top of the parameters in Table 1, we assume that $\tilde{\pi}_G = 0.1$ and $\tilde{\pi}_B = 0.4$ under the physical measure, meaning that the good and the bad states are expected to last 10 and 2.5 years, respectively. Moreover, we set $\theta_G = -\theta_B = 0.08$, which implies that risk averse investors expect the good state to be shorter and the bad state to be longer than under the physical measure. Throughout the analysis, we consider two cases. First, we only allow η_j to vary across different states and set $\Gamma_G = \Gamma_B = 1$ (see equation (2)). Second, to acknowledge that variations in demand may impact R&D (e.g., Caballero and Hammour, 1994), we allow the demand function to vary. We keep $\Gamma_B = 1$ and set $\Gamma_G = 1.02$, so that the profit wedge between good and bad state is about 30%.[25]

Table 3 compares the endogenous quantities in the two states. We only vary η_j across states in the top panel, whereas we also let Γ_j vary in the middle panel. Consistent with Proposition 4, innovation by active firms is countercyclical—i.e., higher in state B, in which the market price of risk is larger. This result aligns with the Schumpeterian view that firms should invest more in innovation in recessions. In our model, this is the case both when abstracting (top panel) and when accounting for time-varying demand (middle panel). Hence, fluctuations in the market price of risk can alone generate countercyclical innovation.

Table 3 also shows that the mass of entrants is procyclical. That is, the higher market price of risk in the B state has the most detrimental effect on the extensive margin, by reducing the mass of entrants. At the industry level, the table also shows that the procyclicality of μ more than offsets the countercyclicality of firm-level innovation in both the vertical and horizontal dimension, so that Ψ_v and Ψ_h are both procyclical. Hence, active firms face greater competition in innovation—through greater creative destruction and horizontal displacement—when the market price of risk is lower in the good state. The aggregate innovation rate at which new technological clusters arise, $\mathcal{I}$, is also procyclical.

Our analysis then illustrates that variations in discount rates are an important driver of innovation cyclicality within an industry. Namely, when the market price of risk is high (in bad states of the economy), the mass of entrants should shrink, and active firms (initiator and entrants) should invest more in innovation, consistent with the evidence reported in Figure 1. These results provide novel theoretical grounds to the evidence on R&D cyclicality, by

[25]Such variation in profits across states is consistent with the change in total (detrended) earnings before interest, depreciation and amortization of R&D-active firms in Compustat between peaks and troughs.

posing the accent on the effect of discount rate fluctuations. Our paper can rationalize the observed procyclicality of innovation rates at the aggregate level—a pattern that has been consistently reported starting from Griliches (1984)—with the Schumpeterian view that firms should invest more in recessions than expansions. We predict that the procyclicality of aggregate R&D comes from the extensive margin, in line with the evidence in Brown, Fazzari, and Petersen (2009), Babina, Bernstein, and Mezzanotti (2023), and Howell et al. (2020).[26] Furthermore, Babina, Bernstein, and Mezzanotti (2023) find that, in downturns, the intensive margin of innovation is resilient whereas the extensive margin drops due to a substantial decline in patenting by entrepreneurs—consistent with our prediction that incumbents benefit from lower competition in downturns.

Table 3 also shows that entrants allocate relatively more resources to explorative innovation during recessions: The ratio of v over total investment in innovation $v + h$ is strictly higher in the B state under all specifications. The result echoes the prediction in our single state model that higher discount rates stimulate vertical innovation. The result also aligns with recent firm-level evidence: Manso, Balsmeier, and Fleming (2021) find that exploration strategies are more prevalent in recessions, whereas Babina, Bernstein, and Mezzanotti (2023) report that innovation leads to more impactful patents in downturns.

The impact of fluctuations in the market price of risk We next investigate the impact of *fluctuations* in the market price of risk vis-à-vis an environment in which η is fixed. To this end, the last two columns of Table 3 report the model endogenous quantities in the one-state model (as analyzed in Section 2) and their averages in the two-state case. To make the one- and the two-state cases comparable, we assume that the time-invariant market price of risk in the one-state is equal to the average in the two-state model.[27]

Table 3 shows that fluctuations in the market price of risk affect the firm-level innovation rate only slightly under our baseline parameterization, with the entrants' horizontal innovation rate being only modestly smaller in the two-state model, on average. In turn, fluctuations in the market price of risk have a considerable impact on the mass of entrants,

[26] As we elaborate later in this section, the evidence in Brown, Fazzari, and Petersen (2009) and Howell et al. (2020) further emphasize the role of financing frictions in explaining innovation cyclicality.

[27] We assume $\bar{\eta} = \frac{\eta_G \pi_B + \eta_B \pi_G}{\pi_B + \pi_G}$ in the one-state. In the middle panel, $\bar{\Gamma} = \frac{\Gamma_G \pi_B + \Gamma_B \pi_G}{\pi_B + \pi_G}$ in the one-state.

consistent with Proposition 4. Table 3 shows that the average mass of entrants in the two-state model is greater than its counterpart in the one-state. Moreover, the greater mass of entrants implies that the rate of creative destruction is greater, on average, in the two-states vis-à-vis the one-state model. As a result, the rate of arrival of new technological clusters $\mathcal{I}$ is greater in the two-state case, on average. In other words, our model predicts that fluctuations in the market price of risk induce a greater industry turnover, which fosters the emergence of new technological clusters. These patterns are robust to allowing for demand-shifts over the business cycle, as confirmed in the middle panel of Table 3. Thus, the model is consistent with the Schumpeterian view that creative destruction spurs innovation.[28]

Time-varying ability to finance entry As shown by Brown, Fazzari, and Petersen (2009) and Howell et al. (2020), entrants are exposed to shifts in the supply of finance over the business cycle.[29] We thus investigate the robustness of our results when allowing for time-variation in the entrants' ability to finance entry. We follow Malamud and Zucchi (2019) and assume that the entry cost includes a financial component. We denote this financing cost by κ_{Fj}, $j = G, B$, so that the cost of entry is $\kappa + \kappa_{Fj}$. Consistent with Brown, Fazzari, and Petersen (2009), we assume that $\kappa_{FG} < \kappa_{FB}$, i.e., the financing cost rises in the bad state of the economy.

The bottom panel of Table 3 shows that our results continue to hold and are amplified in magnitude in this case.[30] Compared to our baseline in which only η is time-varying, allowing for variation in the cost of financing leads to a greater gap in the mass of entrants between the two states. That is, the greater financing cost in state B makes the mass of entrants even more procyclical, which bolsters the procyclicality in the rate of creative destruction and the rate of horizontal displacement. As a result, the initiator increases its investment in state B compared to our baseline case, strengthening the countercyclicality in the initiator's innovation rate.[31] In line with the empirical evidence in Howell et al. (2020), our paper

[28] Figure OA.1 in the online appendix visually illustrates the sensitivity of equilibrium quantities to η_i.

[29] Conversely, they do not find a significant imprint of these shifts on mature, incumbent firms.

[30] We continue to use the same parameters as in our baseline calibration and, additionally, we normalize κ_{FG} to zero and assume that κ_{FB} increases the cost of entry by 4%.

[31] These results are consistent with Malamud and Zucchi (2019), who show that financing frictions slow down creative destruction, reducing the entrants' contribution to growth but increasing that of incumbents.

shows that the aggregate contribution of entrants to innovation is higher in good states of the economy—nonetheless, the firm-level investment of active startups is higher in bad states. Fluctuations in the market price of risk continue to spur greater creative destruction and foster the arrival of new technological clusters.

4 Asset pricing implications

We conclude by analyzing the model implications for valuations and firm-level risk premia. A heuristic derivation of risk premia involves a comparison of the HJB equations under the physical and risk-neutral measures, as in Bolton, Chen, and Wang (2013). In each j, the risk premium of the initiator ($\mathcal{R}_{U,j}$) and of the exploiter ($\mathcal{R}_{X,j}$) respectively satisfy:[32]

$$\mathcal{R}_{U,j} \equiv \rho\sigma\eta_j \frac{y_j}{u_j} + \tilde{\pi}_j \left(\mathrm{e}^{\theta_j} - 1\right) \frac{u_j - u_{-j}}{u_j}, \tag{30}$$

$$\mathcal{R}_{X,j} \equiv \rho\sigma_X\eta_j \frac{\omega}{1-\delta\omega} \frac{y_{X,j}}{x_j} + \tilde{\pi}_j \left(\mathrm{e}^{\theta_j} - 1\right) \frac{x_j - x_{-j}}{x_j}. \tag{31}$$

In these expressions, the first term is the risk premium associated with the diffusion risk, and the second term is the premium associated with changes in the state of the economy.[33]

Our first insight is that the resolution of the idiosyncratic uncertainty associated with breakthroughs within the industry affects firms' exposure to systematic risk. We thus add to previous work highlighting the impact of a firm's *own* idyosincratic shocks affect its exposure to systematic risk (i.e., Berk, Green, and Naik, 2004; Kumar and Li, 2016). Next, we develop the implications of the model about the impact of competition in innovation on asset prices.

The extensive innovation margin as a hedge Our model shows that lower entry in downturns hedges innovating incumbents against the otherwise negative effect of higher discount rates on firm value. We begin by focusing on the one-state case with vertical innovation only, for which we obtain analytical results.

[32] We do not elaborate on the corresponding risk premium of the entrant as it is likely unobservable by the econometrician (entrants should be interpreted as startups).

[33] The risk premium in the one-state case can be thus obtained by setting $\tilde{\pi}_j = 0$ in equation (30).

Corollary 5 *When entrants only invest in vertical innovation and η is constant, the risk loading of the initiator $\rho\sigma\frac{y(\eta)}{u(\eta)}$ decreases with η.*

Corollary 5 follows from Proposition 2. For higher levels of η, the initiator (that is, the innovating incumbent) faces a reduced threat of exit as the rate of creative destruction is lower. Thus, the lower rate of creative destruction hedges the initiator against a higher market price of risk. Consistently, the risk loading of the initiator, $\rho\sigma\frac{y(\eta)}{u(\eta)}$, falls as the market price of risk increases. The next corollary focuses instead on the case with time-varying η_j.

Corollary 6 *When entrants only invest in vertical innovation and the market price of risk is time-varying, the initiators' risk premium associated to changes in the state of the economy, $\tilde{\pi}_j\left(e^{\theta_j}-1\right)\frac{u_j-u_{-j}}{u_j}$, is strictly negative.*

When allowing the market price of risk to vary over time, the extensive innovation margin continues to act as a hedge, so that the scaled value of the initiator is countercyclical. Crucially, such countercyclicality implies that the risk premium associated with jump risk (i.e., the second term in equation (30)) is unconditionally negative.[34] The same result in Corollary 6 applies to the full model with both vertical and horizontal innovation.[35] Hence, in the two-state model, the extensive innovation margin acts as a hedge against cyclical fluctuations in discount rates.

Competition in the intensive margin and risk premia We now investigate how the interactions among active firms in the industry affect risk premia. We start by studying how incumbents' risk premia are affected by entrants' innovation rates in the corner cases, for which we obtain analytical proofs (see Appendix OA.3.1).

Proposition 7 *If entrants only engage in vertical innovation, the initiator's risk premium, $\mathcal{R}_{U,j}$, is increasing in both the frequency and size of entrants' innovations (ϕ_v and Λ). Similarly, if entrants only engage in horizontal innovation, the exploiter's risk premium, $\mathcal{R}_{X,j}$, is increasing in the frequency and size of entrants' innovations (ϕ_h and ω) and in the degree of overlap between the ensuing new products and those of the initiator (δ).*

[34] Corollary 6 follows from Proposition 4 and $\theta_G = -\theta_B > 0$, capturing that risk-averse agents expect recessions (respectively, expansions) to be longer (shorter), as in Bolton, Chen, and Wang (2013).

[35] In this case, results are numerical and available from the authors upon request.

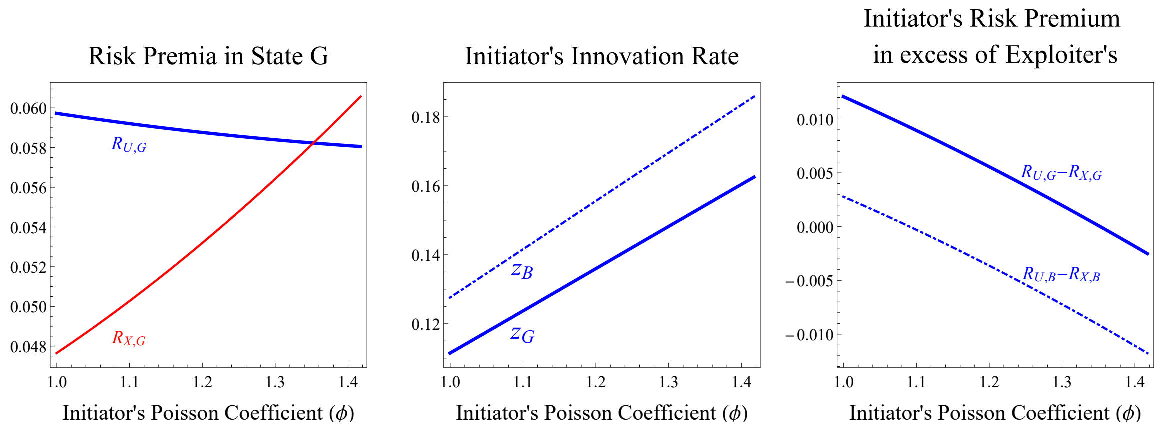

Figure 5: RISK PREMIA AND INNOVATION INTENSITY. The figure shows the risk premia of the initiator and of the exploiters in $j = G$, the initiator's optimal innovation rate, and the risk premium of the initiator in excess of the exploiters' as a function of the initiator's intensity ϕ.

Proposition 7 shows that a higher likelihood or size of entrants' breakthroughs—in either the vertical or horizontal dimension—results in higher risk premia for the initiator and exploiters, respectively. All else equal, if breakthroughs are more likely or more profitable, entrants invest more in R&D, which increases the threat of exit or horizontal displacement for incumbent firms. Hence, greater innovation rates by entrants make both the initiator and the exploiters riskier. Under our baseline calibration, this prediction extends to the full model with both vertical and horizontal innovation.

The full model also reveals that the risk premium of the initiator decreases with its own innovation—indicating that a firm's own innovation acts as insurance. The result is illustrated in the left panel in Figure 5. A higher breakthrough intensity for the initiator (captured by an increase in ϕ) boosts its incentives to invest in innovation (leading to a higher value of z_j), which in turn reduces its risk premium. Simultaneously, the middle panel in Figure 5 shows that an increase in ϕ leads to a higher risk premium for the exploiter, as it increases its threat of exit due to an initiator's breakthrough.

In sum, either when we look at interactions between entrants and incumbents or between the initiator and exploiters, the model reveals that innovation by competitors increases a firm's risk premium, whereas a firm's own innovation acts as insurance. These results are consistent with the patent race model by Bena and Garlappi (2020), in which the expected return of a firm decreases with its own innovation output, and increases with that of its rival. Bena and Garlappi (2020) consider a setting in which all firms innovate and without entry. By contrast, we consider an industry with entry as well as both innovating and non-innovating firms. In recent work, Bena et al. (2022) document that non-innovating firms earn lower returns than innovating firms on average. Relatedly, the right panel of Figure 5 shows that the initiator is riskier than exploiters in our baseline calibration (with $\phi = 1$), although this relation can flip if, for instance, the initiator's Poisson intensity ϕ increases.

5 Concluding Remarks

This paper highlights that discount rates are significant determinants of R&D dynamics. We develop a model showing that discount rates affect both the nature and composition of R&D within an industry. While corporate finance textbooks suggest that higher discount rates deter long-term investment, our model predicts that higher discount rates can encourage innovation in the intensive margin when accounting for the industry equilibrium. The underlying mechanism relates to the negative impact of higher discount rates on the extensive margin, which boosts the incumbents' value of innovation breakthroughs. Higher discount rates also spur the emergence of new technologies due to explorative innovation.

The opposing effects of aggregate discount rates on the extensive and intensive innovation margins are supported by the motivating evidence in Figure 1. Table 4 digs further into the data.[36] As in Figure 1, the independent variable is the aggregate risk premium, which captures the main common source of variation in discount rates. On the intensive margin, the first three columns in the top panel show firm-level panel regressions using Compustat data, and the dependent variable is either R&D-to-sales or R&D-to-assets.[37] Irrespective of the specification, we confirm that firm-level R&D is significantly and positively related to the aggregate risk premium, consistent with our prediction that higher discount rates encourage innovation in the intensive margin. The last column of Table 4—using data from the National Science Foundation (NSF)—shows that the positive relation also holds on

[36] See the Online Appendix OA.4 for details.

[37] In the context of our model, the returns to innovation of the initiator scale with both q (capturing the degree of technological advancement) and M (capturing the number of product lines). In a similar setting, Akcigit and Kerr (2018) scale R&D by sales, given that sales are increasing in both q and M.

aggregate, considering both public and private firms.

The bottom panel of Table 4 focuses instead on the extensive innovation margin. We use the number of firms as our dependent variable and, for robustness, consider multiple data sources.[38] Using industry-level data, the first two columns show that the number of firms is negatively related to the aggregate risk premium. The third and fourth columns assess whether the result also holds on aggregate. Irrespective of the specification, we confirm a negative and significant coefficient, aligned with the prediction that higher aggregate discount rates impact negatively the extensive margin, effectively acting as entry barriers.

Our paper also highlights that discount rate fluctuations help rationalize the documented cyclicality of R&D. The paper reconciles the Schumpeterian view that firms should invest more in innovation during recessions with the documented procyclicality of aggregate R&D. The underlying mechanism requires that the extensive innovation margin is more sensitive to discount rates than the intensive margin. Indeed, using aggregate data from NSF, Table 4 shows that the sensitivity of the number of innovating firms to the aggregate risk premium is larger in magnitude than the corresponding coefficient for R&D intensity at the firm level. In the spirit of Barlevy (2007), we further verify that firm-level growth in unscaled R&D relates positively to changes in the aggregate risk premium—consistent with our prediction that higher aggregate discount rates boost innovation in the intensive margin during downturns.[39]

In sum, by acknowledging that firms do not take decisions in a vacuum, our theoretical framework provides novel insights on how discount rates affect firm-level R&D that challenge conventional wisdom—and are yet consistent with the empirical evidence. The paper highlights the importance of studying corporate innovation in industry equilibrium. We hope that our findings stir further breakthroughs in the study of innovation, and more generally of any other corporate decisions, while accounting for endogenous industry dynamics.

[38]NSF reports innovating firms only, whereas BDS covers all firms (innovating or not) in each industry.

[39]For brevity, these results are reported in the Online Appendix OA.4.

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Table 1: Baseline parameters.

Parameter	Description	Value
r	Risk-free rate	0.01
η	Market price of risk (one state)	0.30
ϕ	Poisson coefficient (initiator)	1.00
ϕ_v	Poisson coefficient (entrant, vertical)	1.00
ϕ_h	Poisson coefficient (entrant, horizontal)	1.00
ζ	R&D cost coefficient (initiator)	0.60
ζ_v	R&D cost coefficient (entrant, vertical)	6.00
ζ_h	R&D cost coefficient (entrant, horizontal)	0.08
λ	Vertical jump (initiator)	1.055
Λ	Vertical jump (entrants)	1.12
φ	Horizontal jump (initiator)	1.14
ω	Horizontal jump (entrants)	0.25
δ	Obsolescence due to horizontal innovations	0.20
β	Inverse of price elasticity of demand	0.13
Γ	Demand shift parameter	1.00
α	Recovery in liquidation (initiator)	0.60
α_X	Recovery in liquidation (exploiter)	0.85
σ	Coefficient of cash flow volatility (initiator)	0.80
σ_X	Coefficient of cash flow volatility (exploiter)	0.70
σ_W	Cash flow volatility (entrant)	0.20
ρ	Correlation with aggregate shocks (initiator and exploiter)	0.55
ρ_W	Correlation with aggregate shocks (entrant)	0.20
κ	Entry cost	0.015

Table 2: INNOVATION IN THE VERTICAL AND HORIZONTAL DIMENSION. This table reports the model endogenous quantities for the corner case in which entrants invest in vertical innovation only, for the corner case in which entrants invest in horizontal innovation only, and in the full case featuring both vertical and horizontal innovation. The top panel illustrates the case in which $\omega = 0.25$ (as in the baseline parameterization), whereas the bottom panel illustrates the case in which ω is higher and equal to 0.45.

	Vertical only	Horizontal only	Full case (both)
$\omega = 0.25$ (Baseline)			
z	0.120	–	0.116
v	0.064	–	0.062
h	–	–	0.141
μ	2.559	–	2.112
Ψ_v	0.163	–	0.130
Ψ_h	–	–	0.297
$\omega = 0.45$			
z	0.120	0.248	0.071
v	0.064	–	0.037
h	–	0.551	0.451
μ	2.559	0.918	1.881
Ψ_v	0.163	–	0.069
Ψ_h	–	0.506	0.849

Table 3: EQUILIBRIUM QUANTITIES. This table reports the quantities of interest for the case in which the market price of risk varies over time (first to third column) as well as when assuming that there is just one state of the economy in which the market price of risk is fixed at its two-state average. In the top panel, we assume that only the market price of risk η_j varies across the different states; in the middle panel, we assume that η_j and the demand shift parameter Γ_j vary; in the bottom panel, we assume that η_j and the financing cost κ_{Fj} vary.

	State G	State B	Average (2 states)	One-state
Varying η_j only				
z	0.112	0.128	0.116	0.116
v	0.060	0.068	0.062	0.062
h	0.134	0.151	0.138	0.141
μ	2.873	0.170	2.260	2.107
Ψ_v	0.171	0.012	0.135	0.130
Ψ_h	0.385	0.026	0.304	0.296
$\mathcal{I}$	0.283	0.140	0.251	0.246
Varying η_j and Γ_j				
z	0.112	0.128	0.115	0.115
v	0.059	0.068	0.061	0.061
h	0.147	0.164	0.151	0.154
μ	3.224	0.160	2.529	2.341
Ψ_v	0.191	0.011	0.150	0.143
Ψ_h	0.475	0.026	0.373	0.360
$\mathcal{I}$	0.302	0.139	0.265	0.258
Varying η_j and κ_{Fj}				
z	0.112	0.130	0.116	0.116
v	0.059	0.069	0.062	0.062
h	0.133	0.145	0.136	0.139
μ	2.946	0.027	2.284	2.109
Ψ_v	0.175	0.002	0.136	0.130
Ψ_h	0.392	0.004	0.304	0.294
$\mathcal{I}$	0.287	0.132	0.252	0.246

Table 4: EVIDENCE ON R&D, NUMBER OF FIRMS, AND DISCOUNT RATES. The top panel focuses on the intensive margin of innovation. The first three columns consider Compustat firm-level panel tests with R&D-to-assets or R&D-to-sales as dependent variables. The fourth column, as well as all tests in the bottom panel, use data from the National Science Foundation (NSF) and Business Dynamics Statistics (BDS), covering public and private firms. In the bottom panel, focusing on the extensive margin, the data from NSF only surveys innovating firms, whereas BDS covers all firms. "Premium" denotes the aggregate market risk premium as defined in Haddad, Loualiche, and Plosser (2017). The period is 1982 to 2017. Further details on dataset construction and controls are provided in Appendix OA.4. All coefficients are standardized, and standard errors are reported in parentheses. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

INTENSIVE MARGIN	R&D/Assets	R&D/Assets	R&D/Sales	R&D/Sales
Premium	0.012***	0.005***	0.004**	0.290*
	(0.00)	(0.00)	(0.00)	(0.00)
Adjusted R^2	0.600	0.656	0.528	0.082
N	145,229	106,099	106,099	36
Controls	N	Y	Y	N
Fixed Effects	Y	Y	Y	N
Level	Firm	Firm	Firm	Aggregate
Source	Compustat	Compustat	Compustat	NSF

EXTENSIVE MARGIN	All Firms	All Firms	All Firms	Innovating Firms
Premium	-0.033***	-0.033***	-0.555***	-0.352*
	(0.01)	(0.00)	(0.14)	(0.16)
Adjusted R^2	0.962	0.201	0.288	0.086
N	8,424	8,424	36	36
Fixed Effects	Y	N	N	N
Level	Industry	Industry	Aggregate	Aggregate
Source	BDS	BDS	BDS	NSF

Online Appendix to "Innovation, Industry Equilibrium, and Discount Rates"

July 2023

OA.1 Proofs of the results in Section 2

Using Girsanov theorem, the risk-neutral dynamics of the cash flows of the initiator, exploiters, and entrants satisfy:

$$\begin{aligned} dC_t =& Y_t\left(p_t - 1 - \sigma\rho\eta\right) M_t dt - \Phi(z_t, q_t, M_t)dt + \sigma Y_t M_t dB_t^U, \\ dC_t^X =& Y_{Xt}\left(p_{Xt} - 1 - \eta\rho\sigma_X\right) M_{Xt} dt + \sigma_X Y_{Xt} M_{Xt} dB_t^X \\ dC_t^W =& \left[-\left(\frac{1}{2}\zeta_v v_t^2 + \frac{1}{2}\zeta_h h_t^2 + \eta\rho_W\sigma_W\right) dt + \sigma_W dB_t^W\right] M_t q_t, \end{aligned}$$

where B_t^k, $k = U, X, W$, are the Brownian motion describing the initiator, exploiter, and entrant shocks under the risk-neutral measure.

Substituting $U(q_t, M_t) = q_t M_t u$, into equation (9) gives the scaled HJB of the initiator:

$$ru = \max_{z,y}\; y^{1-\beta}\Gamma - y - \sigma\eta\rho y - \frac{z^2}{2}\zeta + \phi z\left(\lambda\varphi - 1\right) u - \Psi_v u(1-\alpha) - \Psi_h u\omega\delta \tag{OA.1}$$

and, plugging in this equation the optimal z, y, and p gives the valuation equation of the initiator:

$$\frac{\phi^2}{2\zeta}\left(\lambda\varphi - 1\right)^2 u^2 - (r + \Psi_v(1-\alpha) + \Psi_h\omega\delta)u + \Upsilon(\eta) = 0. \tag{OA.2}$$

Moreover, the scaled HJB of the exploiter satisfies:

$$rx = \max_{y_X \geq 0}\quad \frac{\omega}{1-\omega\delta} y_X\left(\Gamma y_X^{-\beta} - 1 - \eta\rho\sigma_X\right) - \left(\phi z + \Psi_v\right)\left(1 - \alpha_X\right)x - \Psi_h\omega\delta x. \tag{OA.3}$$

Substituting equation (14) into (OA.3) gives the valuation equation for the exploiter:

$$rx = \frac{\beta\omega}{1-\omega\delta}\left(\frac{1-\beta}{1+\sigma_X\rho\eta}\right)^{\frac{1}{\beta}-1}\Gamma^{\frac{1}{\beta}} - (\phi z + \Psi_v)x\left(1-\alpha_X\right) - \Psi_h\omega\delta x. \tag{OA.4}$$

Finally, the scaled HJB of the entrant satisfies:

$$rw = \max_{v,h} -\eta\rho_W\sigma_W - \frac{\zeta_v}{2}v^2 - \frac{\zeta_h}{2}h^2 + \phi_v v\left[\Lambda u - w\right] + \phi_h h\left[\omega x - w\right]$$
$$+ \phi z\left(\lambda\varphi - 1\right)\left(w-\kappa\right) + \Psi_v^-\left(\Lambda - 1\right)\left(w-\kappa\right) - \Psi_h^-(w-\kappa)\omega\delta. \tag{OA.5}$$

Substituting equations (16) and (17) back into the HJB equation (OA.5) gives:

$$rw = -\eta\rho_W\sigma_W + \frac{\phi_v^2}{2\zeta_v}\left[\Lambda u - w\right]^2 + \frac{\phi_h^2}{2\zeta_h}\left[\omega x - w\right]^2 + \phi z\left(\lambda\varphi-1\right)\left(w-\kappa\right)$$
$$+ \Psi_v^-\left(\Lambda-1\right)\left(w-\kappa\right) - \Psi_h^-\omega(w-\kappa). \tag{OA.6}$$

Using the free-entry condition $w = \kappa$, the above equation boils down to

$$rw = -\eta\rho_W\sigma_W + \frac{\phi_v^2}{2\zeta_v}\left[\Lambda u - w\right]^2 + \frac{\phi_h^2}{2\zeta_h}\left[\omega x - w\right]^2 \tag{OA.7}$$

and becomes a function of u and x. In turn, the value of the initiator u (equation (OA.2)) is a function of Ψ_v and Ψ_h, which are themselves endogenous functions of μ, u, and x. Similarly, the valuation equation of the exploiter x (equation (OA.3)) depends on z, Ψ_v, and Ψ_h. As a result, we solve the system of equations (OA.2), (OA.3), and (OA.7) to get the endogenous quantities μ, u, and x, which in turn we substitute into equations (11), (16), and (17) to get the optimal innovation rates z, v, and h. Finally, using the expressions for v and h, together with μ, we pin down Ψ_v and Ψ_h.

OA.1.1 Proof of Proposition 1

With exogenous industry dynamics, the scaled value of the initiator is given by:

$$u(\eta) = \frac{r + \Psi_v(1-\alpha) + \Psi_h\omega\delta - \sqrt{(r+\Psi_v(1-\alpha)+\Psi_h\omega\delta)^2 - 2\Upsilon(\eta)\frac{\phi^2}{\zeta}\left(\lambda\varphi-1\right)^2}}{\phi^2\left(\lambda\varphi-1\right)^2\zeta^{-1}}. \tag{OA.8}$$

The expression for $z(\eta)$ with exogenous industry dynamics follows by substituting equation (OA.8)—solved for a given (exogenous) Ψ_v and Ψ_h—into equation (11). Notably, $z(\eta)$ is a function of η through the function $\Upsilon(\eta)$, defined in Section 2.1, which decreases with η as

$$\Upsilon'(\eta) = -\rho\sigma \left(\frac{1-\beta}{1+\eta\rho\sigma}\right)^{\frac{1}{\beta}} \Gamma^{\frac{1}{\beta}} < 0.$$

Thus, $z(\eta)$ decreases with η too, and the result follows.

For completeness, we also report the scaled value of exploiters when Ψ_v and Ψ_h are exogenous:

$$x(\eta) = \frac{1}{[r + (\phi z + \Psi_v)(1-\alpha_X) + \Psi_h \omega\delta]} \frac{\beta\omega}{1-\omega} \left(\frac{1-\beta}{1+\sigma_X\rho\eta}\right)^{\frac{1}{\beta}-1} \Gamma^{\frac{1}{\beta}}. \tag{OA.9}$$

OA.1.2 Proof of Proposition 2

In the case with vertical innovation only, the value of the initiator satisfies:

$$\begin{aligned} rU(q,M) = \max_{z,Y}\ & MY(p-1-\sigma\eta\rho) - \zeta\frac{z^2}{2}qM + \phi z\left[U(\lambda q, \varphi M) - U(q,M)\right] \\ & + \Psi_v\left[\alpha U(q,M) - U(q,M)\right] \end{aligned} \tag{OA.10}$$

where the expressions for the optimal z, y, and p are given by equations (11) and (10).

The value of an entrant, denoted by $W(q,M)$, satisfies:

$$\begin{aligned} rW(q,M) = \max_{v\geq 0}\ & -qM\left(\eta\rho_W\sigma_W + \frac{\zeta_v}{2}v^2\right) + \phi_v v\left[U(\Lambda q, M) - W(q,M)\right] \\ & + \phi z\left[W(\lambda q, \varphi M) - W(q,M) - K(\lambda\varphi - 1)\right] + \Psi_v^-\left[W(\Lambda q, M) - W(q,M) - K(\Lambda - 1)\right] \end{aligned} \tag{OA.11}$$

where the terms admit a similar interpretation to equation (15). Using the scaling property and differentiating with respect to v gives the optimal innovation rate in equation (16). Plugging the optimal v back into the HJB gives:

$$rw = -\eta\rho_W\sigma_W + \frac{\phi_v^2}{2\zeta_v}\left[\Lambda u - w\right]^2 + \left[\phi z(\varphi\lambda - 1) + \Psi_v^-(\Lambda - 1)\right](w - \kappa). \tag{OA.12}$$

Using the free-entry condition $w = \kappa$, we solve the above equation with respect to u, which then gives

$$u(\eta) = \frac{1}{\Lambda}\left(\kappa + \frac{\sqrt{2\zeta_v(r\kappa + \eta\rho_W\sigma_W)}}{\phi_v}\right). \tag{OA.13}$$

By substituting u into (11) and (16), we obtain $z(\eta)$ and $v(\eta)$ as reported in Proposition 2, which are straightforward to show to be increasing with η.

We now prove the sensitivity of $\Psi_v(\eta)$ and $\mu(\eta)$ with respect to η. Scaling equation (OA.10) by q and M, substituting the optimal y and z, and solving with respect to $\Psi_v(\eta)$ gives:

$$\Psi_v(\eta) = \frac{1}{1-\alpha}\left(\frac{\phi^2}{2\zeta}u(\eta)\left(\lambda\varphi - 1\right)^2 - r + \frac{\Upsilon(\eta)}{u(\eta)}\right).$$

Differentiating with respect to η gives

$$\Psi_v'(\eta) = \frac{\Upsilon'(\eta)}{u(\eta)(1-\alpha)} - \left[\Upsilon(\eta) - \frac{\phi^2(\lambda\varphi-1)^2u^2(\eta)}{2\zeta}\right]\frac{u'(\eta)}{u^2(\eta)(1-\alpha)}. \tag{OA.14}$$

The first term is negative as $1-\alpha > 0$, and $\Upsilon'(\eta) < 0$ as shown in Appendix OA.1.1. The second term is also negative, as $u'(\eta) > 0$ (as is straightforward from equation (OA.13)) and the term in square brackets is positive when we consider values of η that rule out the degenerate case in which the initiator always makes losses in expectation—i.e., we consider values of η such that $y(p-1-\eta\sigma\rho) - \frac{\zeta}{2}\zeta^2 > 0$, as explained in Section 1. Indeed

$$y(p-1-\eta\sigma\rho) - \frac{\zeta}{2}z^2 = \Upsilon(\eta) - \frac{\phi^2(\lambda\varphi-1)^2u^2(\eta)}{2\zeta} > 0$$

is the term in brackets in (OA.14). Thus, Ψ_v decreases with η, as stated in Proposition 2.

As the last step, we differentiate $\mu(\eta) = \frac{\Psi_v(\eta)}{\phi_v v(\eta)}$ with respect to η, that gives

$$\mu'(\eta) = \frac{\Psi_v'(\eta)}{\phi_v v(\eta)} - \frac{\Psi_v(\eta)v'(\eta)}{\phi_v v^2(\eta)}. \tag{OA.15}$$

The first term is negative as $\Psi_v'(\eta) < 0$, as shown above. The second term is also negative, as $\Psi_v(\eta) > 0$ and $v'(\eta) > 0$. The claim in Proposition 2 then follows.

OA.1.3 Proof of Proposition 3

Assuming that entrants only innovate horizontally, their value satifies:

$$rW(q,M) = \max_{v,h} \quad -qM\left(\eta\rho_W\sigma_W + \frac{\zeta_h}{2}h^2\right) + \phi_h h\left[X(q,\omega M) - W(q,M)\right] \tag{OA.16}$$
$$+\phi z\left[W(\lambda q,\varphi M) - W(q,M) - K(\lambda\varphi - 1)\right] + \Psi_h^-\left[W(q,M(1-\omega\delta)) - W(q) + K\omega\delta\right]$$

where the terms admit a similar interpretation to equation (15). Using the same scaling property used in the full case and differentiating with respect to h, we obtain the optimal innovation rate reported in equation (17). Plugging this expression back into the HJB and imposing $w = \kappa$ gives:

$$rw = -\eta\rho_W\sigma_W + \frac{\phi_h^2}{2\zeta_h}\left[\omega x - w\right]^2. \tag{OA.17}$$

Solving this equation with respect to x gives

$$x(\eta) = \frac{1}{\omega}\left(\kappa + \frac{\sqrt{2\zeta_h(r\kappa + \eta\rho_W\sigma_W)}}{\phi_h}\right). \tag{OA.18}$$

By substituting x into (17) then gives the expression for $h(\eta)$ reported in Proposition 3.

In this case, differently from the full case, the initiator is not subject to creative destruction Ψ_v. Thus, the value of the initiator satisfies:

$$\frac{\phi^2}{2\zeta}\left(\lambda\varphi - 1\right)^2 u^2 - (r + \Psi_h\omega\delta)u + \Upsilon(\eta) = 0. \tag{OA.19}$$

In this case, the exploiters face the threat of exit only due to the initiator's breakthroughs (i.e., they are not subject to creative destruction due to the entrants' innovations). Thus, the value of the exploiters satisfies the following equation:

$$rx = \frac{\beta\omega}{1-\omega\delta}\left(\frac{1-\beta}{1+\sigma_X\rho\eta}\right)^{\frac{1}{\beta}-1}\Gamma^{\frac{1}{\beta}} - \phi z\left(1-\alpha_X\right)x - \Psi_h\omega\delta x. \tag{OA.20}$$

Now, recall that $\Psi_h = \mu\phi_v h$, where h satisfies the equation reported in Proposition 3. As a result, we can find v and μ by solving the system of equations (OA.19)-(OA.20) and, thus, the optimal innovation rate of initiators z as well as the rate of horizontal displacement Ψ_h.

OA.1.4 Proof of the results in Section 2.3

In scaled terms, equation (25) becomes

$$ru = \max_{z,y,s} y(p - 1 - \sigma\eta\rho) - \frac{\zeta^2}{2}z - \zeta_s s + \phi z\left[\lambda\varphi - 1\right]u + \Psi_v\left[-\frac{(1-\alpha)\,u}{1+s} + \frac{s}{1+s}\left(\Lambda u - u - \Lambda a\right)\right]. \tag{OA.21}$$

Differentiating this equation with respect to z and y, we get the same expression for the optimal innovation rate and production rate that we get in the baseline model. Additionally, differentiating the above equation and solving for s gives equation (26).

In turn, the entrants' HJB equation satisfies:

$$rW(q,M) = \max_{v} \quad -qM\left(\eta\rho_W\sigma_W + \frac{\zeta_v}{2}v^2\right) + \phi_v v\left[\frac{(U(\Lambda q, M) - W(q,M) - G)}{1+s} + \frac{s\,(A - W(q,M))}{1+s}\right]$$
$$+\phi z\left[W(\lambda q, \varphi M) - W(q,M) - K(\lambda\varphi - 1)\right] + \Psi_v^-\left[W(\Lambda q, M) - W(q,M) - K(\Lambda - 1)\right],$$

where the second term on the right-hand side specifies the outcome of a breakthrough for the entrant. With probability $1/(1+s)$, the entrant takes over the initiator's market position by paying the setup cost G_t. Conversely, with probability $s/(1+s)$, the entrant is acquired and receives the acquisition cost A_t. Exploiting the scaling property gives:

$$rw = \max_{v} \quad -\left(\eta\rho_W\sigma_W + \frac{\zeta_v}{2}v^2\right) + \phi_v v\left[\frac{1}{1+s}\left(\Lambda u - w - \Lambda g\right) + \frac{s}{1+s}\left(\Lambda a - w\right)\right]$$
$$+\phi z\left[\lambda\varphi w - w - \kappa(\lambda\varphi - 1)\right] + \Psi_v^-\left[\Lambda w - w - \kappa(\Lambda - 1)\right]. \tag{OA.22}$$

Maximizing with respect to v gives the optimal innovation rate in equation (27).

The initiator and the target entrant negotiate over the acquisition cost. The solution of their Nash bargaining solves:

$$\arg\max\left(\Lambda u-\alpha u-\Lambda a\right)^{b}\left(\Lambda a-\Lambda u+g\Lambda\right)^{1-b}. \tag{OA.23}$$

The first term is the incremental value to the initiator stemming from the acquisition as opposed to being kicked out of the industry. The second term is the incremental value for the entrant stemming from being acquired as opposed to becoming initiator. Solving (OA.23) gives the acquisition cost reported in equation (28). Note that the acquisition adds value to both parties if the setup cost g is sufficiently large, and the liquidation cost sufficiently low, i.e., $\frac{u}{\Lambda}\alpha < g$.

In the numerical implementation of this extension, we use our baseline parameterization with the only change that we assume that $\alpha = 0$. Additionally, we assume that the search cost is $\zeta_s = 0.001$ (meaning that searching for a target is much cheaper than innovating), the bargaining power of the initator is $b = 0.4$, and the setup cost $g = 0.65$.

OA.2 Proof of the results in Section 3

OA.2.1 Derivation of firm values and optimal investment rates

In the two-state model, all value functions and the endogenous quantities are a function of (η_j, η_{j-})—i.e., the market risk prices in the two states. For the ease of exposition throughout this appendix, we omit these arguments. Consider first the value of the initiator. Following standard arguments, the initiator's scaled HJB equation in each state j satisfies:

$$ru_j = \max_{z_j,y_j}\ \Gamma_j y_j^{1-\beta} - y_j - \frac{z_j^2}{2}\zeta - \sigma\eta_j\rho y_j + \phi z_j\left[\lambda-1\right]u_j - \Psi_{vj}u_j(1-\alpha) - \Psi_{hj}u_j\omega\delta + \pi_j\left[u_{j-}-u_j\right] \tag{OA.24}$$

where $\pi_j = \tilde{\pi}_j e^{\theta_j}$ is the transition intensity under the risk-neutral measure. The last term on the right-hand side captures the effect of a state switch, in which case firm value goes from

u_j to u_{j-}. Differentiating the above equation with respect to y_j gives:

$$y_j = \left(\frac{\Gamma_j(1-\beta)}{1+\sigma\eta_j\rho}\right)^{\frac{1}{\beta}} \quad \Rightarrow \quad p_j = \Gamma_j y_j^{-\beta} = \frac{1+\sigma\eta_j\rho}{1-\beta}.$$

Similarly, differentiating equation (OA.24) with respect to z_j gives the optimal innovation rate:

$$z_j = \frac{\phi}{\zeta}\left(\lambda\varphi - 1\right) u_j.$$

Plugging back the expressions for z_j and y_j into equation (OA.24) gives the value of the initiator.

Consider now the dynamics of the exploiters. Following arguments similar to those in Section 2, their scaled value satisfies the following equation:

$$rx_j = \max_{y_{Xj}\geq 0} \quad \frac{\omega}{1-\omega\delta} y_{Xj}\left(p_{Xj} - 1 - \eta_j\rho\sigma_X\right) - \left(\phi z_j + \Psi_{vj}\right)\left(1-\alpha_X\right) x_j - \omega\delta\Psi_{hj} x_j + \pi_j\left[x_{j-} - x_j\right],$$

where the last term on the right-hand side captures the effect of a state switch, in which case firm value goes from x_j to x_{j-}. Maximizing with respect to y_{Xj} gives:

$$y_{Xj} = \left(\frac{\Gamma_j(1-\beta)}{1+\eta_j\rho\sigma_X}\right)^{\frac{1}{\beta}}.$$

Finally, the scaled entrant value satisfy the following equation:

$$\begin{aligned} rw_j = \max_{v_j, h_j} - \eta_j\rho_W\sigma_W - \frac{\zeta_v}{2}v_j^2 - \frac{\zeta_h}{2}h_j^2 + \phi_v v_j\left[\Lambda u_j - w_j\right] + \phi_h h_j\left[\omega x_j - w_j\right] \\ + \phi z_j\left(\lambda\varphi - 1\right)\left(w_j - \kappa\right) + \Psi_{vj}^{-}\left(\Lambda - 1\right)\left(w - \kappa\right) - \Psi_{hj}^{-}(w_j - \kappa)\omega\delta + \pi_j\left[w_{j-} - w_j\right]. \end{aligned} \tag{OA.25}$$

The last term on the right-hand side captures the effect of a state switch. In each state, the optimal rate of vertical and horizontal innovation respectively satisfy:

$$v_j = \frac{\phi_v}{\zeta_v}\left[\Lambda u_j - w_j\right], \quad \text{and} \quad h_j = \frac{\phi_h}{\zeta_h}\left[\omega x_j - w_j\right]. \tag{OA.26}$$

In each state, the rate of creative destruction and the rate of horizontal displacement satisfy $\Psi_{vj} = \mu_j \phi_v v_j$ and $\Psi_{hj} = \mu_j \phi_h h_j$, and the free-entry condition $w_j = \kappa$ holds.

OA.2.2 Proof of Proposition 4

Vertical innovation Following steps as in Appendix OA.1.2, the initiator value in each j satisfies:

$$u_j = \frac{\kappa}{\Lambda} + \frac{1}{\Lambda}\sqrt{\frac{2\zeta_v(r\kappa + \eta_j \rho_W \sigma_W)}{\phi_v^2}}. \tag{OA.27}$$

As $\eta_B > \eta_G$, the initiator value is greater in $j = B$. Moreover, using equation (OA.27) gives

$$z_j = \frac{\phi(\lambda - 1)}{\zeta}\left[\frac{\kappa}{\Lambda} + \frac{1}{\Lambda}\sqrt{\frac{2\zeta_v(r\kappa + \eta_j \rho_W \sigma_W)}{\phi_v^2}}\right] \tag{OA.28}$$

as well as the optimal (vertical) innovation rate of active entrants:

$$v_j = \frac{\phi_v}{\zeta_v}\sqrt{\frac{2\zeta_v(r\kappa + \eta_j \rho_W \sigma_W)}{\phi_v^2}} = \sqrt{\frac{2(r\kappa + \eta_j \rho_W \sigma_W)}{\zeta_v}}. \tag{OA.29}$$

Hence, the first part of the claim in Proposition 4 follows.

Consider now the the rate of creative destruction. In the two states, it satisfies:

$$\Psi_{vj}(\eta_j, \eta_{j-}) = \frac{1}{1-\alpha}\left(\frac{\phi^2}{2\zeta}u_j\left(\lambda\varphi - 1\right)^2 - r + \frac{\Upsilon(\eta_j)}{u_j} + \pi_j \frac{(u_{j-} - u_j)}{u_j}\right). \tag{OA.30}$$

Let us start by considering the case in which Γ_j does not vary across states—i.e., $\Gamma_G = \Gamma_B \equiv \Gamma$. Now, express $\eta_B = \eta_G + \Delta$, with $\Delta \geq 0$. If $\Delta = 0$, $\eta_B = \eta_G$, and we are back to the one-state case, meaning that $\Psi_{vB} = \Psi_{vG}$—basically, there is no variation across the two states. Conversely, when $\Delta > 0$, $\Psi_{vB} \neq \Psi_{vG}$. To study the relative magnitude of creative destruction in the two states, we next define the function $F(\Delta) = \Psi_{vB}(\Delta) - \Psi_{vG}(\Delta)$ for $\Delta \geq 0$. As just discussed, $F(0) = 0$ holds. Using equation (OA.30), we study $F'(\Delta)$. Let us also express u_B and u_G as a function of Δ. By calculations, we find that

$$F'(\Delta) = -\frac{\rho\sigma}{(1-\alpha)u_B}\left(\frac{\Gamma(1-\beta)}{1+(\eta_G+\Delta)\rho\sigma}\right)^{\frac{1}{\beta}} - \frac{u_B'}{(1-\alpha)}\left(\frac{\pi_G}{u_G}+\frac{\pi_B u_G}{u_B^2}\right) - \left[\beta\left(\frac{(1-\beta)}{1+(\eta_G+\Delta)\rho\sigma}\right)^{\frac{1}{\beta}-1}\Gamma^{\frac{1}{\beta}} - \frac{\phi^2(\lambda\varphi-1)^2u_B^2}{2\zeta}\right]\frac{u_B'(\Delta)}{(1-\alpha)u_B^2}$$

with $u_B'(\Delta) = \frac{\zeta_v \rho_W \sigma_W}{\Lambda\phi_v\sqrt{2\zeta_v[r\kappa+(\eta_G+\Delta)\rho_W\sigma_W]}} > 0$. The term $\left[\beta\left(\frac{(1-\beta)}{1+(\eta_G+\Delta)\rho\sigma}\right)^{\frac{1}{\beta}-1}\Gamma^{\frac{1}{\beta}} - \frac{\phi^2(\lambda\varphi-1)^2u_B^2}{2\zeta}\right]$ is positive under our assumption that the initiator's expected net cash flow is positive. $F'(\Delta)$ is then negative. Thus, the function F is zero at $\Delta = 0$ and decreases for $\Delta > 0$, so that $\Psi_{vB}(\Delta) < \Psi_{vG}(\Delta)$ if $\eta_B > \eta_G$. That is, creative destruction is procyclical.

Consider now the case $\Gamma_G > \Gamma_B$. If $\Delta = 0$, then $u_B = u_G$, $z_B = z_G$, and $v_B = v_G$, as these quantities do not depend on Γ_j (see equations (OA.27), (OA.28), and (OA.29)). Consider again the function $F(\Delta)$ defined above. Let us first evaluate this function for $\Delta = 0$. Using equation (OA.30), we have that $F(0) = \frac{1}{(1-\alpha)u_B(0)}\beta\left(\frac{(1-\beta)}{1+\eta_G\rho\sigma}\right)^{\frac{1}{\beta}-1}\left(\Gamma_B^{\frac{1}{\beta}} - \Gamma_G^{\frac{1}{\beta}}\right)$, where we have used that $u_B(0) = u_G(0)$. As $\Gamma_G > \Gamma_B$ by assumption, then $F(0) < 0$. As $F'(\Delta) < 0$ following the steps above, then $\Psi_{vB}(\Delta) < \Psi_{vG}(\Delta)$ for the case $\Gamma_G > \Gamma_B$ too.

Recall that $\Psi_{vj} = \mu_{vj}\phi_v v_j$. As shown above, $\Psi_{vB} - \Psi_{vG} < 0$ and $v_B > v_G$. Thus, for $\Psi_{vB} - \Psi_{vG} = \phi_v[\mu_B v_B - \mu_{vG} v_G] < 0$ to hold, it must be that $\mu_B < \mu_G$. Thus, the mass of active entrants is also procyclical. Moreover, using the expression for v_j gives:

$$\Psi_{vB} - \Psi_{vG} = \phi_v\left[\mu_B\sqrt{\frac{2(r\kappa+(\eta_G+\Delta)\rho_W\sigma_W)}{\zeta_v}} - \mu_G\sqrt{\frac{2(r\kappa+\eta_G\rho_W\sigma_W)}{\zeta_v}}\right] \tag{OA.31}$$

The first square root is greater than the second, so $\sqrt{\frac{2(r\kappa+(\eta_G+\Delta)\rho_W\sigma_W)}{\zeta_v}} = A\sqrt{\frac{2(r\kappa+\eta_G\rho_W\sigma_W)}{\zeta_v}}$, with $A > 1$. Given $\mu_B < \mu_G$, we express $\mu_G = B\mu_B$ with $B > 1$. Hence

$$\Psi_{vB} - \Psi_{vG} = \phi_v\mu_B\sqrt{\frac{2(r\kappa+\eta_G\rho_W\sigma_W)}{\zeta_v}}\,[A-B]\,. \tag{OA.32}$$

As $\Psi_{vB} - \Psi_{vG} < 0$, then it must be that $A < B$, meaning that the mass of entrants μ_j (the extensive margin) varies more than v_j (the intensive margin) for a given variation in Δ.

Horizontal innovation Following steps similar to those in Appendix OA.1.3, we solve for the exploiter value:

$$x_j = \frac{\kappa}{\omega} + \frac{1}{\omega}\sqrt{\frac{2\zeta_h(r\kappa + \eta_j \rho_W \sigma_W)}{\phi_h^2}}. \tag{OA.33}$$

Using this expression into h_j gives

$$h_j = \sqrt{\frac{2(r\kappa + \eta_j \rho_W \sigma_W)}{\zeta_h}}. \tag{OA.34}$$

Because $\eta_B > \eta_G$, then $h_B > h_G$. The claims in Proposition 4 then follow.

Additional results Figure OA.1 investigates the sensitivity of equilibrium quantities to η_G and η_B. It shows that z increases with the magnitude of the market price of risk in the contemporaneous state j—consistent with the result in the one-state model—whereas it is quite insensitive to the market price of risk in the non-contemporaneous state j^-. By contrast, the mass of entrants μ and the arrival rate of new technological clusters $\mathcal{I}$ are notably sensitive to the market price of risk in both j and j^-—they decrease with the market price of risk in the contemporaneous state j but increase with the market price of risk in the non-contemporaneous state j^-. Specifically, an increase in the market price of risk shifts entrants from the contemporaneous to the non-contemporaneous state, then also affecting the arrival rate of new technologies. Figure OA.1 then sheds light on the importance of the market price of risk in transferring resources across states of the economy.

OA.3 Proof of the results in Section 4

OA.3.1 Proof of Proposition 7

The derivative of the risk premium of the initiator, $\mathcal{R}_{U,j}$, with respect to ϕ_v is given by:

$$\frac{\partial \mathcal{R}_{U,j}}{\partial \phi_v} = \frac{\eta_j \Lambda \rho \sigma y_j \sqrt{2\zeta_v(\eta_j \rho_W \sigma_W + \kappa r)}}{\left(\kappa\phi_v + \sqrt{2\zeta_v(\eta_j \rho_W \sigma_W + \kappa r)}\right)^2} + \frac{\kappa\sqrt{2\zeta_v}\left(\sqrt{(\eta_{-j}\rho_W\sigma_W + \kappa r)} - \sqrt{(\eta_j\rho_W\sigma_W + \kappa r)}\right)}{\left(e^{\theta_j} - 1\right)^{-1} \tilde{\pi}_j^{-1}\left(\kappa\phi_v + \sqrt{2\zeta_v(\eta_j \rho_W \sigma_W + \kappa r)}\right)^2},$$

which is positive, so $\mathcal{R}_{U,j}$ increases with ϕ_h.[1]

Next we calculate the derivative of $\mathcal{R}_{U,j}$ with respect to Λ, to obtain:

$$\frac{\partial \mathcal{R}_{U,j}}{\partial \Lambda} = \frac{\eta_j \phi_v \rho \sigma y_j}{\left(\kappa\phi_v + \sqrt{2\zeta_v(\eta_j \rho_W \sigma_W + \kappa r)}\right)},$$

which is strictly positive for any parameter value. It follows that $\mathcal{R}_{U,j}$ is increasing in Λ.

Consider next the risk premium of the exploiter, $\mathcal{R}_{X,j}$ defined in equation (31). Using the expression for x_j in equation (OA.33), we obtain:

$$\frac{\partial \mathcal{R}_{X,j}}{\partial \phi_h} = \frac{\sqrt{2}\kappa\tilde{\pi}_j\left(\sqrt{\zeta_h(\eta_j\rho_W\sigma_W + \kappa r)} - \sqrt{\zeta_h(\eta_{-j}\rho_W\sigma_W + \kappa r)}\right)}{\left(1 - e^{\theta_j}\right)^{-1}\left(\kappa\phi_h + \sqrt{2}\sqrt{\zeta_h(\eta_j\rho_W\sigma_W + \kappa r)}\right)^2} + \frac{\eta_j\sqrt{2\zeta_h(r\kappa + \eta_j\rho_W\sigma_W)}}{\left(\rho_X\sigma_X y_{Xj}\right)^{-1} x_j^2(1 - \delta\omega)\phi_h^2},$$

which is strictly positive for any parameter value, given $\eta_B > \eta_G$, $e^{\theta_G} - 1 > 0$ and $e^{\theta_B} - 1 < 0$. It follows that $\mathcal{R}_{X,j}$ is increasing in ϕ_h as stated in Proposition 7.

We next consider the derivative of $\mathcal{R}_{X,j}$ with respect to ω:

$$\frac{\partial \mathcal{R}_{U,j}}{\partial \omega} = \frac{\eta_j\rho\sigma_X y_{X,j}\left(\kappa\phi_h + \sqrt{2\zeta_h(\eta_j\rho_W\sigma_W + \kappa r)}\right)}{x_j^2\omega(1 - \omega\delta)^2\phi_h} + \frac{\eta_j\rho\sigma_X y_{X,j}}{x_j(1 - \omega\delta)},$$

[1] In $j = G$, $\eta_B > \eta_G$ and $e^{\theta_G} - 1 > 0$. In $j = B$, $\eta_B > \eta_G$ and $e^{\theta_B} - 1 < 0$.

which is strictly positive. Similarly, the derivative of $\mathcal{R}_{X,j}$ with respect to δ equals:

$$\frac{\partial \mathcal{R}_{U,j}}{\partial \delta} = \frac{\eta_j \rho \sigma_X \omega^3 y_{X,j} \phi_h}{(1-\delta\omega)^2 \left(\kappa\phi_h + \sqrt{2\zeta_h(\eta_j \rho_W \sigma_W + \kappa r)}\right)},$$

which is strictly positive, proving that $\mathcal{R}_{X,j}$ is increasing in δ. Proposition 7 then follows.

OA.4 Methodological details on the empirics

In this Appendix, we report methodological details underlying the construction of Figure 1 and Table 4 in the main text—representing our motivating evidence—as well as some additional evidence.

The sample period goes from 1982 to 2017, which correspond to the common years across the multiple datasets used in our tests. We download the measure of the aggregate risk premium from Professor Erik Loualiche's website, whose construction is described in Haddad et al. (2017). We use Compustat yearly data for the firm-level analyses of R&D investment (i.e., the intensive innovation margin). To study variation in the number of firms (i.e., the extensive innovation margin), we use the surveys on industrial R&D from the National Science Foundation (NSF) and the historical series reported by Business Dynamics Statistics (BDS) project from the US Census Bureau. We also use the aggregate time series of R&D over sales from the NSF. Unlike WRDS Compustat, the NSF and BDS datasets cover public and private firms, and have been widely used in the literature. We define industries at the 4-digit NAICS code level.

In the top panel of Table 4, we include firm-level regressions on R&D for which we use controls. We follow Fang et al. (2014) and include lagged firm-level characteristics typically used to explain innovation, including: the market-to-book assets ratio (or Q), asset tangibility, book leverage, firm size (logarithm of market value), return on assets, and the Kaplan-Zingales Index. We construct each of these controls using the same definitions as in Fang et al. (2014). In all firm-level regressions of Tables 4, we cluster standard errors at the firm level and use firm-level fixed-effects when indicated so.

Moving to the bottom panel of Table 4, NSF reports innovating firms only, whereas BDS covers all firms (innovating or not) by industry. When using BDS data, we restrict our analysis to the same set of industries covered in the firm-level Compustat tests on R&D—for consistency in the comparison with the firm-level evidence in Figure 1 and the top panel of Table 4. Results remain qualitatively unchanged if we remove this restriction. In all industry-level regressions in Table 4, we cluster standard errors at the industry level, and apply industry-level fixed effects if reported.

Table OA.1 provides additional evidence in support of the predictions of our model with time-varying risk premia. Consistent with the empirical approach to test the firm-level cyclicality of R&D in Barlevy (2007), we confirm that the firm-level growth in unscaled R&D relates positively to changes in the aggregate risk premium. Simultaneously, we find that growth in the number of firms by industry relates negatively to changes in the aggregate risk premium.[2]

[2]The industry-level regressions in Table OA.1 also rely on information from BDS. As for Table 4, in all firm-level regressions, we cluster standard errors at the firm level and use firm-level fixed-effects when reported. In turn, in the industry-level regressions in this table we cluster standard errors at the industry level, and apply industry-level fixed effects if reported.

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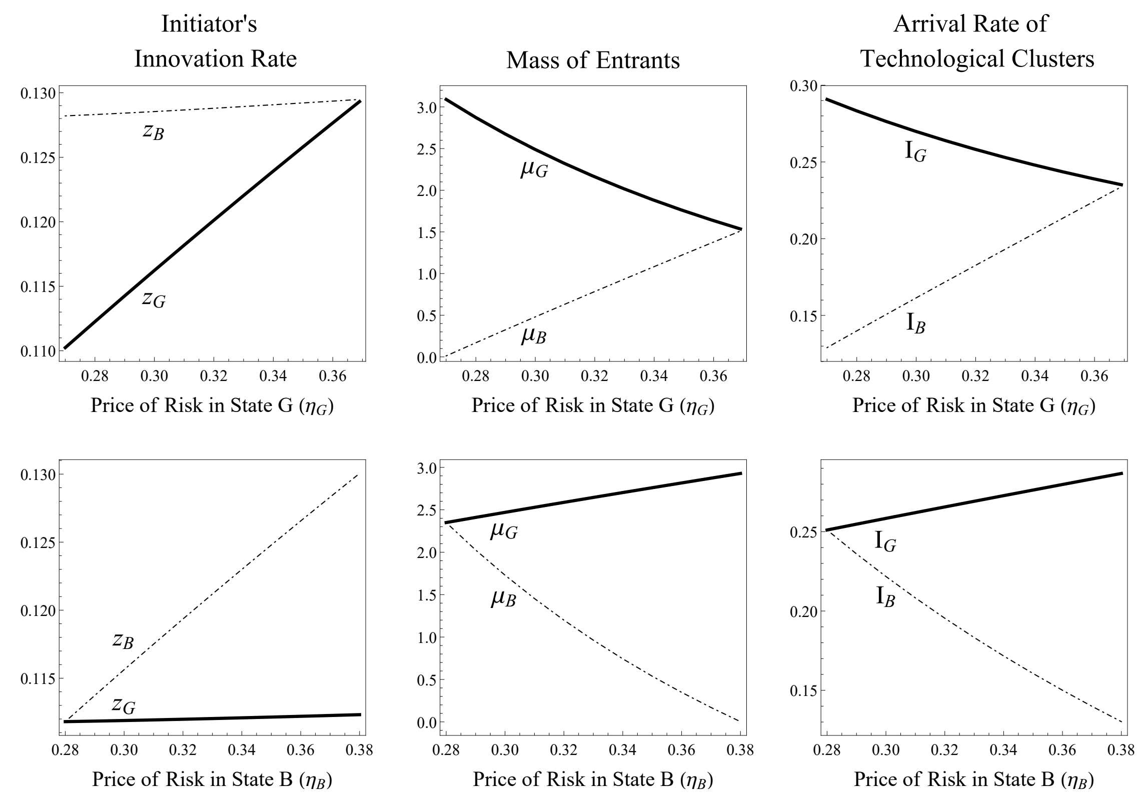

Figure OA.1: Time-varying extensive and intensive margins. The figure shows how the initiator's innovation rate (z_j), the mass of entrants (μ_j), and the arrival rate of technological clusters ($\mathcal{I}_j$) vary with the market price of risk in the contemporaneous (η_j) and non-contemporaneous (η_{-j}) state. The top (respectively, bottom) panel varies η_G (η_B).

Table OA.1: ADDITIONAL TESTS ON GROWTH RATES OF R&D AND GROWTH IN NUMBER OF FIRMS. This table provides evidence on the growth rates of R&D and the number of firms in our sample. "Firm -level R&D growth" is the annual growth rate in unscaled R&D expenses at the firm level. 'Growth in No. of firms" represents the growth in the total number of firms by industry-year. Premium growth is the annual growth rate in the aggregate risk premium. Details on dataset construction are provided in Appendix OA.4. The period is 1982 to 2017. All coefficients are standardized. Standard errors are reported in parentheses. . *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

	Firm-level R&D Growth	Firm-level R&D Growth	Growth in No. of Firms	Growth in No. of Firms
Premium Growth	0.012***	0.017***	-0.029***	-0.026***
	(0.01)	(0.01)	(0.01)	(0.01)
Adjusted R^2	0.012	0.062	0.355	0.019
N	101,441	100,340	8424	36
Source	Compustat	Compustat	BDS	BDS
Level	Firm	Firm	Industry	Aggregate
Fixed-Effects	Industry	Firm	Industry	None