Mixed frequency models with MA components

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Motivation

- Temporal aggregation generally introduces a moving average (MA) component in the model for the aggregate variable (see, e.g., Marcellino (1999)).
- A similar feature should be present in the mixed frequency models.
- We show formally that this is in general the case.

MA component

- The MA component is often neglected, likely to preserve the possibility of OLS estimation and on the grounds that it can be approximated by a sufficiently long autoregressive (AR) component.

- The effects of neglecting the MA component have been rarely explicitly considered. Few examples:
  - Luetkepohl (2006): VARMA models are especially appropriate in forecasting, since they can capture the dynamic relations between time series with a small number of parameters.
  - Dufour and Stevanovic (2013): a VARMA instead of VAR model for the factors provides better forecasts for several key macroeconomic aggregates.
  - Stock and Watson (2006), Leroux et al. (2017): ARMA(1,1) models predict well the inflation change.

- For mixed frequency models, there are no results available.

Contribution

- We analyze the relevance of the inclusion of an MA component in MIDAS and UMIDAS models, with the resulting specifications labeled, respectively, MIDAS-ARMA and UMIDAS-ARMA.
- We first compare the forecasting performance of the mixed frequency models with and without the MA component in a set of Monte Carlo experiments.
- Next, we carry out an empirical investigation, where we predict several quarterly macroeconomic variables using timely monthly indicators:
  - quarterly GDP deflator using monthly CPI inflation and the interest rate on 3-month T-Bills;
  - GDP growth with industrial production;
  - personal consumption growth with a consumption index.

Summary of the results

Monte Carlo findings:

- Short-term forecasting performance is better when including the MA component.
- The gains are higher the more persistent is the series.

Empirical findings:

- We obtain good results for inflation and personal consumption growth, with mean squared error (MSE) improvements up to 24% and 19% respectively.
- Adding the MA part to forecast GDP growth one year ahead ameliorates the MSE up to 10%.

Road map

- The rationale for an MA component in mixed frequency models.
- UMIDAS-ARMA and MIDAS-ARMA specification and estimation.
- Monte Carlo exercise.
- Empirical application.

The rationale for an MA component in mixed frequency models

- The DGP for the variable y and the N variables x is a $VAR(p)$ process:

$$\begin{pmatrix} \underset{1\times 1}{a(L)} & \underset{1\times N}{-b(L)} \\ \underset{N\times 1}{-d(L)} & \underset{N\times N}{C(L)} \end{pmatrix} \begin{pmatrix} \underset{1\times 1}{y_{t_m}} \\ \underset{N\times 1}{x_{t_m}} \end{pmatrix} = \begin{pmatrix} \underset{1\times 1}{e_{yt_m}} \\ \underset{N\times 1}{e_{xt_m}} \end{pmatrix}. \tag{1}$$

- The model can be written as:

$$a(L)y_{t_m} = b_1(L)x_{1t_m} + ... + b_N(L)x_{Nt_m} + e_{yt_m} \tag{2}$$

$$C(L)x_{t_m} = d(L)y_{t_m} + e_{xt_m} \tag{3}$$

$a(L) = 1 - a_1 L - ... - a_p L$, $b(L) = (b_1(L), ..., b_N(L))$, $b_j(L) = b_{j1} L + ... + b_{jp} L^p$, $j = 1, ..., N$, $d(L) = (d_1(L), ..., d_N(L))'$, $d_j(L) = d_{j1} L + ... + d_{jp} L^p$, $C(L) = I - C_1 L - ... - C_p L^p$, and the errors are jointly white noise.

The rationale for an MA component in mixed frequency models

Timing:

- x can be observed for each period t_m; $t_m = 1, ..., T_m$ is the high frequency (HF) time unit.
- y can be only observed every m periods; $t = 1, ..., T$ is the low frequency (LF) time unit.
- The HF time unit is observed m times in the LF time unit (if we are working with quarterly (LF) and monthly (HF) data, it is $m = 3$)
- L indicates the lag operator at t_m frequency, while L^m is the lag operator at t.

The rationale for an MA component in mixed frequency models

To derive the generating mechanism for y at mixed frequency (MF), we introduce:

- the aggregation operator $\omega(L) = \omega_0 + \omega_1 L + ... + \omega_{m-1} L^{m-1}$
- a polynomial $\beta(L)$, such that the product $h(L) = \beta(L)a(L)$ only contains powers of L^m.

Multiply both sides of (2) by $\omega(L)$ and $\beta(L)$ to get the **mixed frequency y:**

$$h(L^m)\omega(L)y_{t_m} = \beta(L)b_1(L)\omega(L)x_{1t_m} + ... + \beta(L)b_N(L)\omega(L)x_{Nt_m} + \beta(L)\omega(L)e_{yt_m}. \quad (4)$$

with $t_m = m, 2m, ..., T_m$

- IN GENERAL THERE IS AN MA COMPONENT!

Analytical example 1: VAR(1) with average sampling

- HF DGP:

$$\begin{pmatrix} y_{t_m} \\ x_{t_m} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} y_{t_m-1} \\ x_{t_m-1} \end{pmatrix} + \begin{pmatrix} e_{yt_m} \\ e_{xt_m} \end{pmatrix}. \tag{5}$$

- Focus on the dynamics of y_{t_m}:

$$(1 - aL)\, y_{t_m} = bLx_{t_m} + e_{yt_m}. \tag{6}$$

- Consider average sampling $\omega(L) = 1 + L + L^2$.
- Introduce $\beta(L) = \left(1 + aL + a^2 L^2\right)$, such that the product $h(L) = \beta(L)\,(1 - aL)$ only contains powers of L^3.

Analytical example 1: VAR(1) with average sampling

- Multiply both sides of equation (6) by $\omega(L)$ and $\beta(L)$:

$$\begin{aligned}\left(1-a^3L^3\right)\widetilde{y}_{t_m} &= \left(1+aL+a^2L^2\right)bL\left(1+L+L^2\right)x_{t_m} + \\ &\quad \left(1+(a+1)L+\left(a^2+a+1\right)L^2+\left(a^2+a\right)L^3+a^2L^4\right)e_{yt_m}. \end{aligned} \quad (7)$$

- MA(1) component!
- Error term: $u_{t_m} = (1+(a+1)L+(a^2+a+1)L^2+(a^2+a)L^3+a^2L^4)e_{yt_m}$.

Correlation in the error term $\left(u_{t_m}, u_{t_m-3}\right)$

a	0	0.2	0.5	0.9	0.99
$Corr\left(u_{t_m}, u_{t_m-3}\right)$	0	0.07	0.16	0.20	0.20

Analytical example 2: VAR(2) with point-in-time sampling

- Assume a VAR(2) as HF DGP:

$$\begin{pmatrix} y_{t_m} \\ x_{t_m} \end{pmatrix} = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \begin{pmatrix} y_{t_m-1} \\ x_{t_m-1} \end{pmatrix} + \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} \begin{pmatrix} y_{t_m-2} \\ x_{t_m-2} \end{pmatrix} + \begin{pmatrix} e_{yt_m} \\ e_{xt_m} \end{pmatrix}. \tag{8}$$

- The dynamic for y_{t_m} in this case is:

$$\left(1 - a_1 L - a_2 L^2\right) y_{t_m} = \left(b_1 L + b_2 L^2\right) x_{t_m} + e_{yt_m}. \tag{9}$$

- Consider point-in-time sampling: $\omega(L) = 1$.
- Find a polynomial $\beta(L)$ such that the product $h(L) = \beta(L)\left(1 - a_1 L - a_2 L^2\right)$ only contains powers of L^3:

$$\left(1 + a_1 L + \left(a_1^2 + a_2\right) L^2 - a_1 a_2 L^3 + a_2^2 L^4\right).$$

Analytical example 2: VAR(2) with point-in-time sampling

- Multiply both sides of equation (6) by $\omega(L)$ and $\beta(L)$:

$$\left(1-\left(a_1^3+3a_2a_1\right)L^3-a_2^3L^6\right)y_{t_m} = \left(1+a_1L+\left(a_1^2+a_2\right)L^2-a_1a_2L^3+a_2^2L^4\right)\left(b_1L+b_2L^2\right)x_{t_m}+ \left(1+a_1L+\left(a_1^2+a_2\right)L^2-a_1a_2L^3+a_2^2L^4\right)e_{yt_m}. \quad (10)$$

- MA(1) component!
- Error term $u_{t_m}=(1+a_1L+(a_1^2+a_2)L^2-a_1a_2L^3+a_2^2L^4)e_{yt_m}$

Correlation in the error term $\left(u_{t_m}, u_{t_m-3}\right)$

a_1, a_2	0,0	0.2,0.2	0.3,0.3	0.45, 0.45
$Corr\left(u_{t_m}, u_{t_m-3}\right)$	0	0.056	0.097	0.167

UMIDAS-ARMA and MIDAS-ARMA specification

- **UMIDAS-ARMA:**

$$y_{t_m} = \tilde{c}(L^m)y_{t_m-h_m} + \delta(L)x_{t_m-h_m+w} + u_{t_m} + q(L^m)u_{t_m-h_m}. \tag{11}$$

$t_m = m, 2m, ..., T_m$, h_m is the forecast horizon, w is the number of months with which x is leading y.

- **MIDAS-ARMA:**

$$y_{t_m} = \tilde{c}(L^m)y_{t_m-h_m} + \beta B(L,\theta)x_{t_m-h_m+w} + u_{t_m} + q(L^m)u_{t_m-h_m}, \tag{12}$$

where

$$\begin{aligned} B(L,\theta) &= \sum_{j=0}^{K} b(j,\theta)L^j, \\ b(j,\theta) &= \frac{exp(\theta_1 j + \theta_2 j^2)}{\sum_{j=0}^{K} exp(\theta_1 j + \theta_2 j^2)}. \end{aligned}$$

UMIDAS-ARMA and MIDAS-ARMA estimation

- **UMIDAS-ARMA:** two possible options:
  1. We can estimate the model as in the standard ARMA literature, by maximum likelihood or, to be coherent with the MIDAS literature, by NLS.
  2. We can adapt to the UMIDAS-ARMA model the GLS method proposed in Dufour and Pelletier (2008). In this latter case, we proceed as follows: first, we fit a long ARX process to the data, then the lagged innovations in the UMIDAS-AR model are replaced by the residuals of our first step regression. The residuals are, in other words, treated as observables in the second regression.
- **MIDAS-ARMA:** NLS as for standard MIDAS models.

Monte Carlo design

- Two experiments:
  1. DGP 1: HF VAR(1), with average sampling.
  2. DGP 2: HF VAR(2), with point-in-time sampling.
- Parameter combinations representing different degrees of persistence and correlation between HF ald LF variables.
- Two sample sizes: T = 50,100. Evaluation sample = 50. 500 replications.
- Recursive estimation.

Competing models

1. A MIDAS-AR model, with 12 lags in the exogenous HF variable and 1 lag in the AR component;
2. A MIDAS-ARMA model, as in the previous point but with the addition of an MA component;
3. A MIDAS-ARMA model, with only 3 lags in the exogenous HF variable and 1 AR lag;
4. A UMIDAS-AR model, with lag length selected according to the BIC criterion, where the maximum lag length is set equal to 12;
5. A UMIDAS-ARMA model, as in the previous point, with the addition of an MA component, estimated by the GLS method presented above (we will refer to this model as UMIDAS-ARMA-GLS)
6. The same model estimated by NLS (we will refer to this model as UMIDAS-ARMA-NLS);
7. The UMIDAS-ARMA-NLS, fixing at 3 the number of lags of the HF exogenous variable.

Results

- Adding an MA component to the MIDAS model generally helps.
- The gains are not very large but they are visible at almost all percentiles.
- The gains are larger with substantial persistence.
- Exception: gains are large with low persistence in the first DGP ($\rho = 0.1$), the result is mainly due to a deterioration in the absolute performance of the standard MIDAS model.
- The more parsimonious specification with 3 lags only of the HF variable is generally better, except when $\rho = 0.5$.
- Adding an MA component to the UMIDAS model is also generally helpful, though the gains remain small.
- NLS and GLS estimation yield comparable results, suggesting that the second can be preferable as it is simpler.

Results

- In general the MIDAS-ARMA specifications are slightly better than the UMIDAS-ARMA specifications, though the differences are minor.
- This pattern is in contrast with the findings in Foroni et al (2015) and suggests that adding the MA component to the MIDAS model helps somewhat in reducing the potential misspecification due to imposing a specific lag polynomial structure.
- Results are consistent across sample sizes, and the models do not seem sensitive to short sample sizes.

Results DGP1

Table: Monte Carlo simulations results: MSE(model) relative to MSE(MIDAS) - DGP: VAR(1) with average sampling, $T = 100$.

	PANEL (A): $\rho = 0.94, \delta_l = 1, \delta_h = 0$					
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	0.974	0.986	0.981	0.968	0.970	0.967
MIDAS-ARMA-3 (3)	0.966	0.981	0.969	0.962	0.959	0.966
UMIDAS-AR (4)	0.997	0.997	0.986	1.005	0.994	0.990
UMIDAS-ARMA-GLS (5)	0.971	0.973	0.979	0.974	0.968	0.973
UMIDAS-ARMA-NLS (6)	0.969	0.983	0.974	0.970	0.961	0.973
UMIDAS-ARMA-NLS-3 (7)	0.971	0.977	0.974	0.971	0.964	0.975
	PANEL (B): $\rho = 0.9, \delta_l = 1, \delta_h = 0$					
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	0.976	0.989	0.984	0.979	0.973	0.976
MIDAS-ARMA-3 (3)	0.975	0.983	0.988	0.969	0.978	0.971
UMIDAS-AR (4)	1.030	1.024	1.023	1.029	1.038	1.040
UMIDAS-ARMA-GLS (5)	1.014	1.019	1.019	1.019	1.019	1.026
UMIDAS-ARMA-NLS (6)	1.019	1.018	1.023	1.012	1.024	1.028
UMIDAS-ARMA-NLS-3 (7)	0.976	0.979	0.984	0.977	0.977	0.981

Results DGP1

Table: Monte Carlo simulations results: MSE(model) relative to MSE(MIDAS) - DGP: VAR(1) with average sampling, $T = 100$.

	PANEL (C): $\rho = 0.5, \delta_l = 0.1, \delta_h = 0$					
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	0.986	0.990	0.994	0.984	0.983	0.978
MIDAS-ARMA-3 (3)	1.184	1.197	1.178	1.174	1.202	1.176
UMIDAS-AR (4)	1.005	1.000	1.003	1.006	1.013	0.995
UMIDAS-ARMA-GLS (5)	1.000	1.012	0.994	0.998	0.992	0.991
UMIDAS-ARMA-NLS (6)	1.000	1.005	0.992	0.998	0.993	0.992
UMIDAS-ARMA-NLS-3 (7)	1.182	1.212	1.185	1.175	1.198	1.179
	PANEL (D): $\rho = 0.1, \delta_l = 0.1, \delta_h = 0$					
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	0.981	0.985	0.989	0.983	0.980	0.972
MIDAS-ARMA-3 (3)	0.833	0.848	0.846	0.834	0.827	0.828
UMIDAS-AR (4)	0.825	0.837	0.834	0.823	0.824	0.819
UMIDAS-ARMA-GLS (5)	0.832	0.841	0.844	0.830	0.831	0.834
UMIDAS-ARMA-NLS (6)	0.832	0.841	0.844	0.833	0.831	0.836
UMIDAS-ARMA-NLS-3 (7)	0.833	0.846	0.846	0.834	0.829	0.829

Results DGP2

Table: Monte Carlo simulations results: MSE(model) relative to MSE(MIDAS) - DGP: VAR(2) with point-in-time sampling, $T = 100$

	PANEL (A): $\rho_1 = 0.25, \rho_2 = 0.5, \delta_{l1} = 0.5, \delta_{l2} = 1, \delta_{h1} = 0, \delta_{h2} = 0$					
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	0.984	0.968	0.981	0.985	0.991	0.998
MIDAS-ARMA-3 (3)	0.980	0.981	0.982	0.968	0.981	0.999
UMIDAS-AR (4)	1.021	1.032	1.020	1.006	1.032	1.036
UMIDAS-ARMA-GLS (5)	0.991	0.988	0.997	0.979	1.005	1.014
UMIDAS-ARMA-NLS (6)	0.992	0.987	0.986	0.988	1.001	1.004
UMIDAS-ARMA-NLS-3 (7)	0.983	0.978	0.979	0.980	0.985	0.998
	PANEL (B): $\rho_1 = 0.125, \rho_2 = 0.5, \delta_{l1} = 0.125, \delta_{l2} = 0.5, \delta_{h1} = 0, \delta_{h2} = 0$					
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	0.956	0.955	0.960	0.963	0.949	0.959
MIDAS-ARMA-3 (3)	0.940	0.932	0.950	0.950	0.931	0.943
UMIDAS-AR (4)	0.938	0.921	0.938	0.945	0.929	0.946
UMIDAS-ARMA-GLS (5)	0.920	0.916	0.921	0.920	0.906	0.939
UMIDAS-ARMA-NLS (6)	0.921	0.927	0.922	0.926	0.908	0.939
UMIDAS-ARMA-NLS-3 (7)	0.943	0.921	0.950	0.947	0.932	0.948

Results DGP2

Table: Monte Carlo simulations results: MSE(model) relative to MSE(MIDAS) - DGP: VAR(2) with point-in-time sampling, $T = 100$

	PANEL (C): $\rho_1 = 0.05, \rho_2 = 0.1, \delta_{l1} = 0.5, \delta_{l2} = 1, \delta_{h1} = 0, \delta_{h2} = 0$					
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	1.007	1.010	1.003	1.007	1.010	1.025
MIDAS-ARMA-3 (3)	1.006	1.006	0.997	1.003	1.014	1.018
UMIDAS-AR (4)	1.014	1.007	1.016	1.005	1.006	1.030
UMIDAS-ARMA-GLS (5)	1.019	1.009	1.023	1.017	1.018	1.033
UMIDAS-ARMA-NLS (6)	1.015	1.000	1.014	1.007	1.014	1.026
UMIDAS-ARMA-NLS-3 (7)	1.007	0.998	1.004	1.006	1.016	1.023

Empirical applications

Three empirical applications with U.S. data:

1. Forecasting GDP deflator inflation using monthly CPI inflation and 3-month interest rate on T-Bill.
2. Forecasting quarterly GDP growth using monthly industrial production.
3. Forecasting real consumption growth using monthly industrial production and a index for consumption (the real personal consumption expenditures).

Forecasting exercise

- The total sample spans over 50 years of data, from the first quarter of 1960 to the end of 2015.
- The forecasts are computed on progressively expanding samples, with the evaluation period going from 1980Q1 to the end of the sample (roughly 35 years).
- We compute forecasts up to 4 quarters ahead.
- The forecasting object is the annualized growth rate.
- We focus on the case in which the first two months of the quarter are already available.
- We evaluate the forecasts both in terms of MSE and of MAE.
- We compare the forecasting performance relative to a standard MIDAS model with an autoregressive component and 12 lags of the explanatory variable.

Forecasting U.S. GDP deflator

	Explanatory variable: CPI inflation						Explanatory variable: 3-month Tbill					
	$h = 1$						$h = 1$					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12lags	0.65	1.00	NaN	0.61	1.00	NaN	1.04	1.00	NaN	0.75	1.00	NaN
MIDAS-ARMA-12lags	0.65	0.99	0.42	0.61	1.00	0.45	0.97	0.93	0.02	0.72	0.97	0.03
MIDAS-ARMA-3lags	0.61	0.94	0.09	0.59	0.97	0.17	0.96	0.92	0.00	0.71	0.95	0.00
UMIDAS-biclags	0.65	0.99	0.42	0.60	0.98	0.22	1.00	0.96	0.06	0.74	0.98	0.06
UMIDAS-ARMA-GLS-biclags	0.61	0.93	0.08	0.60	0.98	0.21	0.96	0.92	0.01	0.71	0.95	0.00
UMIDAS-ARMA-biclags	0.61	0.94	0.09	0.59	0.97	0.17	0.96	0.92	0.01	0.71	0.95	0.00
UMIDAS-ARMA-3lags	0.61	0.94	0.09	0.59	0.97	0.17	1.08	1.03	0.37	0.74	0.98	0.28
	$h = 4$						$h = 4$					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12lags	0.86	1.00	NaN	0.74	1.00	NaN	1.83	1.00	NaN	0.97	1.00	NaN
MIDAS-ARMA-12lags	1.08	1.25	0.00	0.84	1.13	0.00	2.28	1.25	0.01	1.10	1.14	0.00
MIDAS-ARMA-3lags	0.85	0.98	0.42	0.73	0.99	0.36	1.72	0.94	0.14	0.92	0.95	0.14
UMIDAS-biclags	1.00	1.16	0.00	0.80	1.07	0.01	1.65	0.91	0.02	0.91	0.94	0.06
UMIDAS-ARMA-GLS-biclags	1.26	1.46	0.00	0.87	1.17	0.00	1.64	0.90	0.06	0.93	0.96	0.16
UMIDAS-ARMA-biclags	1.06	1.23	0.01	0.81	1.09	0.02	2.01	1.10	0.19	0.99	1.03	0.35
UMIDAS-ARMA-3lags	0.98	1.13	0.06	0.77	1.03	0.20	2.14	1.17	0.04	1.07	1.11	0.05

Forecasting U.S. GDP growth

	Explanatory variable: Industrial production growth					
	$h = 1$					
	MSE			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12lags	4.06	1.00	NaN	1.58	1.00	NaN
MIDAS-ARMA-12lags	4.05	1.00	0.41	1.59	1.01	0.13
MIDAS-ARMA-3lags	4.27	1.05	0.07	1.60	1.01	0.22
UMIDAS-biclags	4.21	1.04	0.15	1.58	1.00	0.41
UMIDAS-ARMA-GLS-biclags	4.19	1.03	0.13	1.60	1.01	0.16
UMIDAS-ARMA-biclags	4.18	1.03	0.19	1.59	1.01	0.26
UMIDAS-ARMA-3lags	4.27	1.05	0.07	1.60	1.01	0.22
	$h = 4$					
	MSE			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12lags	9.14	1.00	NaN	2.11	1.00	NaN
MIDAS-ARMA-12lags	8.63	0.94	0.12	2.08	0.99	0.30
MIDAS-ARMA-3lags	8.26	0.90	0.03	2.02	0.96	0.05
UMIDAS-biclags	8.77	0.96	0.19	2.05	0.97	0.10
UMIDAS-ARMA-GLS-biclags	9.74	1.07	0.33	2.16	1.02	0.35
UMIDAS-ARMA-biclags	8.91	0.97	0.40	2.09	0.99	0.45
UMIDAS-ARMA-3lags	9.07	0.99	0.47	2.06	0.98	0.31

Forecasting U.S. real consumption growth

	Explanatory variable: Industrial production growth						Explanatory variable: Consumption index growth					
	$h = 1$						$h = 1$					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12lags	8.84	1.00	NaN	2.26	1.00	NaN	9.07	1.00	NaN	2.25	1.00	NaN
MIDAS-ARMA-12lags	7.45	0.84	0.00	1.88	0.83	0.00	8.07	0.89	0.06	1.95	0.87	0.00
MIDAS-ARMA-3lags	7.42	0.84	0.00	1.89	0.84	0.00	8.50	0.94	0.16	2.09	0.93	0.04
UMIDAS-biclags	8.63	0.98	0.02	2.23	0.99	0.03	8.85	0.98	0.14	2.24	1.00	0.48
UMIDAS-ARMA-GLS-biclags	7.14	0.81	0.00	1.84	0.81	0.00	7.75	0.85	0.00	1.95	0.87	0.00
UMIDAS-ARMA-biclags	7.30	0.83	0.00	1.88	0.83	0.00	8.61	0.95	0.32	1.99	0.89	0.02
UMIDAS-ARMA-3lags	7.47	0.84	0.00	1.91	0.85	0.00	7.63	0.84	0.00	1.92	0.86	0.00
	$h = 4$						$h = 4$					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12lags	9.56	1.00	NaN	2.37	1.00	NaN	9.38	1.00	NaN	2.37	1.00	NaN
MIDAS-ARMA-12lags	9.18	0.96	0.27	2.22	0.94	0.08	8.84	0.94	0.17	2.21	0.93	0.05
MIDAS-ARMA-3lags	8.31	0.87	0.00	2.10	0.88	0.00	8.08	0.86	0.00	2.04	0.86	0.00
UMIDAS-biclags	9.23	0.97	0.04	2.34	0.99	0.16	9.26	0.99	0.29	2.34	0.99	0.20
UMIDAS-ARMA-GLS-biclags	7.72	0.81	0.00	1.92	0.81	0.00	8.95	0.95	0.30	2.15	0.91	0.02
UMIDAS-ARMA-biclags	8.95	0.94	0.18	2.16	0.91	0.01	8.44	0.90	0.03	2.11	0.89	0.00
UMIDAS-ARMA-3lags	8.21	0.86	0.01	2.08	0.88	0.00	8.51	0.91	0.05	2.13	0.90	0.00

Summary of the evidence

- MSE and MAE ratios are often smaller than one for the MIDAS-ARMA and UMIDAS-ARMA models when compared with their versions without MA.
- The best improvement achieves 14% in terms of MSE for GDP deflator inflation when forecasted with the monthly CPI inflation, while it goes up to 24% in case of 3-month TBill.
- The improvements of the MA models when forecasting the real consumption growth with industrial production or the monthly consumption index growth are more uniform across horizons and top at 19% in terms of both MSE and MAE.
- Adding the MA part to predict the GDP growth with industrial production does not help at short horizons but improves the MSE up to 10% at four quarters ahead.
- In many cases the improvement in the forecasting performance is also statistically significant.
- There is no single model specification that systematically outstands all the others, though models with fewer lags of the explanatory variables seem generally better. The inclusion of the MA component likely compensates for the need of many lags.
- The improvements with the MA component, whenever present, are present at each forecast horizon.

Bias/Variance decomposition of MSE: summary of the evidence

$$MSE = \underbrace{(E(e))^2}_{\text{Bias}} + \underbrace{Var(e)}_{\text{Variance}}$$

with $e = y - \hat{y}$.

- We find that the MA part helps especially in reducing the bias, suggesting that the MA part is important to well approximate the conditional mean of y (the optimal forecast under the quadratic loss).
- When the models with the MA component are not performing well, this is due especially to the variance term, instead.

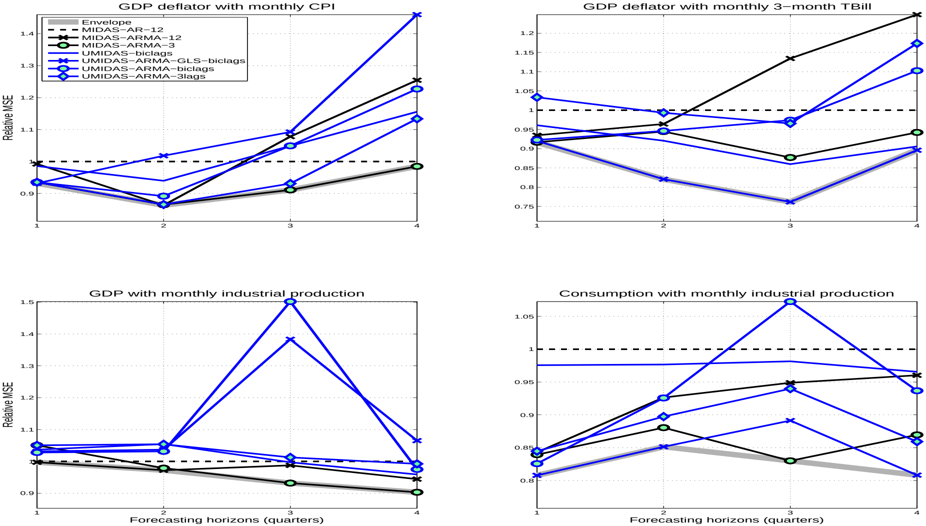
Bias/Variance decomposition of MSE

		Bias				Variance			
		$h=$ 1	$h=$ 2	$h=$ 3	$h=$ 4	$h=$ 1	$h=$ 2	$h=$ 3	$h=$ 4
GDP deflator with CPI inflation	MIDAS-AR-12lags	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	MIDAS-ARMA-12lags	1.04	0.89	0.91	0.98	0.98	0.88	1.15	1.35
	MIDAS-ARMA-3lags	0.93	0.78	0.72	0.85	0.94	0.92	1.03	1.07
	UMIDAS-biclags	0.80	0.78	0.80	0.99	1.02	1.01	1.18	1.21
	UMIDAS-ARMA-GLS-biclags	0.94	1.01	0.95	1.07	0.94	1.02	1.15	1.55
	UMIDAS-ARMA-biclags	0.93	0.80	0.77	0.87	0.94	0.95	1.19	1.37
	UMIDAS-ARMA-3lags	0.93	0.78	0.73	0.83	0.94	0.92	1.06	1.27
GDP deflator with TBill	MIDAS-AR-12lags	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	MIDAS-ARMA-12lags	1.11	1.02	0.96	1.16	0.91	0.95	1.20	1.21
	MIDAS-ARMA-3lags	0.97	0.99	0.91	0.91	0.91	0.94	0.89	0.99
	UMIDAS-biclags	0.93	0.95	0.89	0.94	0.97	0.93	0.88	0.91
	UMIDAS-ARMA-GLS-biclags	0.97	0.95	0.87	0.94	0.92	0.81	0.76	0.90
	UMIDAS-ARMA-biclags	0.95	1.00	0.96	1.06	0.92	0.94	0.99	1.10
	UMIDAS-ARMA-3lags	0.92	0.90	0.98	1.12	1.05	1.03	0.97	1.14

Bias/Variance decomposition of MSE

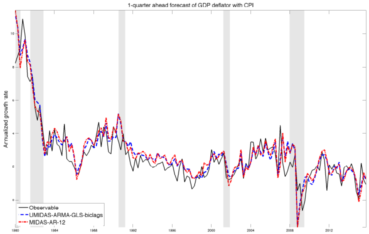
		Bias				Variance			
		$h=1$	$h=2$	$h=3$	$h=4$	$h=1$	$h=2$	$h=3$	$h=4$
GDP with Industrial Production	MIDAS-AR-12lags	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	MIDAS-ARMA-12lags	0.85	0.92	0.82	0.92	1.00	0.99	1.01	0.95
	MIDAS-ARMA-3lags	0.92	1.05	1.02	0.94	1.06	1.01	0.92	0.90
	UMIDAS-biclags	1.04	1.02	1.04	0.93	1.04	1.06	0.99	0.97
	UMIDAS-ARMA-GLS-biclags	0.96	1.06	1.31	1.07	1.03	1.10	1.36	1.06
	UMIDAS-ARMA-biclags	0.98	1.07	1.12	1.03	1.03	1.04	1.52	0.97
	UMIDAS-ARMA-3lags	0.92	1.11	1.11	0.92	1.06	1.06	1.00	1.00
Consumption with Industrial Production	MIDAS-AR-12lags	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	MIDAS-ARMA-12lags	0.61	0.68	0.79	0.69	1.00	1.08	1.06	1.17
	MIDAS-ARMA-3lags	0.51	0.63	0.61	0.80	1.03	1.04	0.99	0.97
	UMIDAS-biclags	0.99	0.99	1.02	1.00	0.98	0.98	0.96	0.95
	UMIDAS-ARMA-GLS-biclags	0.37	0.15	0.60	0.46	1.03	1.12	1.08	1.07
	UMIDAS-ARMA-biclags	0.49	0.58	0.65	0.80	1.02	1.12	1.30	1.07
	UMIDAS-ARMA-3lags	0.46	0.56	0.62	0.70	1.06	1.09	1.13	1.02

Relative MSE at different forecasting horizons

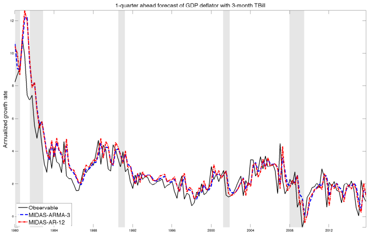


Out-of-sample performance: one-quarter ahead

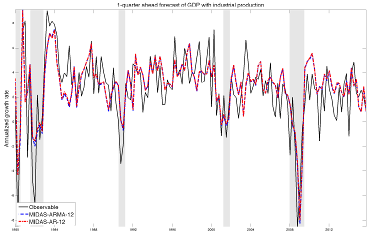
(a) GDP deflator with monthly CPI



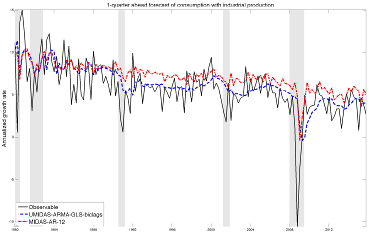
(b) GDP deflator with monthly 3m TBill



(c) GDP with monthly IP

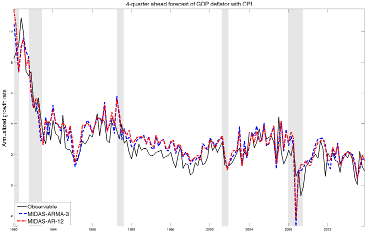


(d) Consumption with monthly IP

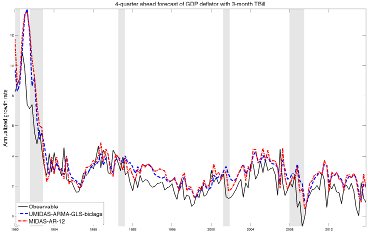


Out-of-sample performance: four-quarter ahead

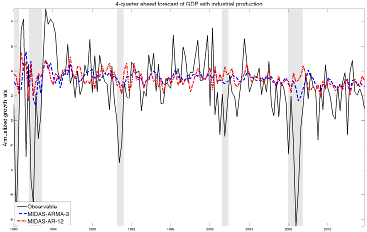
(a) GDP deflator with monthly CPI



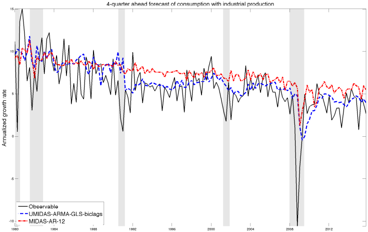
(b) GDP deflator with monthly 3m TBill



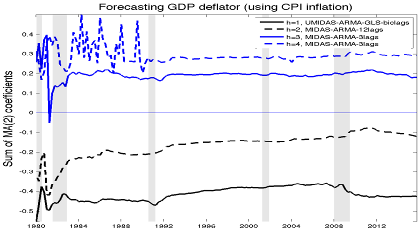
(c) GDP with monthly IP

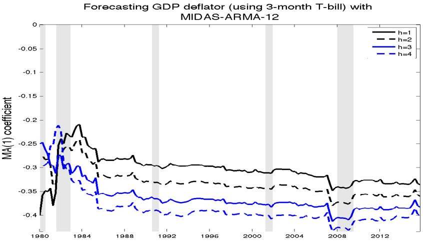


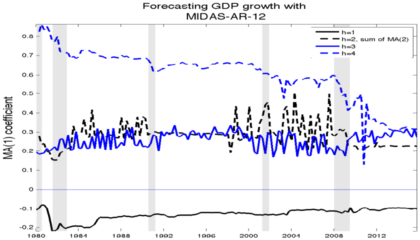
(d) Consumption with monthly IP

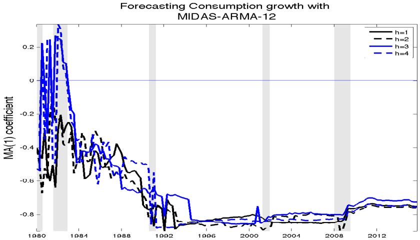


MA coefficients









Out-of-sample performance: summary of the evidence

- In most of the cases the relative MSE stays below 1 at the different forecasting horizons.
- The best results (indicated by the grey envelope line in the figure) are in general obtained by models with an MA component.
- MIDAS models perform well throughout the sample, both with and without an MA component, except for the striking case of real consumption, where the MA part improves substantially the forecasting performance.
- When looking at the estimated coefficients of the MA components, in all the cases the coefficients are quite different from zero.
- Except for the period of the early '80s, for most variables and models the estimated MA coefficients remain rather stable across the sample, although their magnitude (and in some cases their sign) change according to the forecast horizon (because of direct estimation).

Conclusions

- We start from the observation that temporal aggregation in general introduces a moving average component in the aggregated model.
- We show that a similar feature emerges when not all but only a few variables are aggregated, which generates a mixed frequency model.
- An MA component should be added to mixed frequency models, while this is generally neglected in the literature.
- We illustrate in a set of Monte Carlo simulations that indeed adding an MA component to MIDAS and UMIDAS models further improves their nowcasting and forecasting abilities, though in general the gains are limited and particularly evident in the presence of persistence.
- A similar pattern emerges in an empirical exercise based on actual data. The inclusion of an MA component can substantially improve the forecasting performance of GDP deflator growth and real personal consumption growth, while the results for GDP growth are more mixed.