# Mortgage Design in an Equilibrium Model of the Housing Market 

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#### Abstract

How can mortgages be redesigned to reduce housing market volatility, consumption volatility, and default? How does mortgage design interact with monetary policy? We answer these questions using a quantitative equilibrium life cycle model with aggregate shocks, realistic and priced long-term mortgages, and a housing market that clears in equilibrium, with a focus on designs that index payments to the aggregate state of the economy. If the central bank lowers interest rates, mortgages that index to short-term interest rates such as ARMs provide insurance benefits in a crisis by reducing payments, which smooths consumption, stimulates purchases by new homeowners, reduces default, and short circuits a price-default spiral. The welfare benefits of ARMs relative to FRMs in a crisis are large - equivalent to 1.27 percent of annual consumption in a crisis - because ARMs particularly help young, high LTV households who face severe liquidity constraints. ARMs do, however, have drawbacks if real rates rise in a downturn, so we evaluate several proposed mortgage designs that add ARM-like features to standard mortgages. We find that mortgage designs that front load payment reductions to provide maximal relief to constrained homeowners in the crisis perform better than designs that spread the benefit over the life of the mortgage. For instance, the best-performing design we consider is an FRM that can costlessly be converted to an ARM, providing concentrated relief to those who need it most when they need it most. By contrast, FRMs that can be refinanced underwater are less beneficial because they are priced off the long end of the yield curve and do not frontload the benefits of payment reductions.


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## 1 Introduction

The design of mortgages is crucial to both household welfare and the macroeconomy. Houses make up a majority of wealth for most households, and mortgages tend to be their dominant source of credit, so the design of mortgages has an outsized effect on household balance sheets (Campbell, 2013). Recent research has shown that these balance sheet effects can dramatically alter households' marginal propensities to consume because mortgages make home equity illiquid (e.g., Kaplan and Violante, 2014). Additionally, in the mid-2000s boom and subsequent bust, housing wealth extraction through the mortgage market boosted consumption in the boom and reduced consumption in the bust (e.g., Mian and Sufi, 2011). Mortgage debt also led to the wave of foreclosures that led to over six million households losing their homes, badly damaging household balance sheets and crippling the housing market (e.g., Guren and McQuade, 2015; Mian et al., 2015). Finally, in the wake of the recession, there has been increased attention paid to the role that mortgages play in the transmission of monetary policy to the real economy through household balance sheets (e.g., Auclert, 2016; Wong, 2016; Di Maggio et al., 2017; Cloyne et al., 2017).

We study how to best design mortgages in order to reduce household consumption volatility and default and to increase household welfare. There is considerable evidence that implementation frictions prevent financial intermediaries from modifying mortgages ex post in a crisis (see e.g., Agarwal et al., 2015; Agarwal et al., 2017). As a result, a better designed ex ante contract can likely deliver significant welfare benefits. We are furthermore motivated by the evidence that not just the level of household mortgage debt (e.g., loan-to-value or payment-to-income ratio), but also the design of such debt, can impact household outcomes including consumption and default. For example, Fuster and Willen (2015) and Di Maggio et al. (2017) study cohorts of borrowers with hybrid adjustable rate mortgages contracted in the years before the crisis. Exploiting heterogeneity in the timing of monthly payment reductions as mortgages transitioned from initial fixed rates to adjustable rates during the crisis, these papers show that downward resets resulted in substantially lower defaults and stabilized consumption. Similarly, studies that exploit quasi-random variation in housing market interventions in the Great Recession such as the Home Affordable Refinance Program (HARP) (Agarwal et al., 2015) and the Home Affordable Modification Program (HAMP) (Agarwal et al., 2017; Ganong and Noel, 2017) have found that monthly payment reductions significantly reduce default and increase consumption.

Such empirical evidence suggests that, given the cyclicality of interest rates, indexing mortgage payments to interest rates can improve household outcomes and welfare. We pursue this indexation question systematically using a quantitative equilibrium model featuring heterogeneous households, endogenous mortgage spreads, and endogenous house prices. Using this framework, we quantitatively assess a variety of questions related to mortgage design. How would consumption, default, home prices, and household welfare change if we were to change the design of mortgages in the economy, particularly in a deep recession and housing bust like the one experienced in the Great Recession? In a stochastic economy that transits between booms, recessions, and crises, how well do different indexed mortgages perform? What is the most effective simple form of indexation?

Designs wherein mortgage payments are higher in booms and lower in recessions do better than designs with fixed mortgage payments for risk and insurance reasons. But among such designs the
most effective ones in our quantitative analysis "front load" the payment reductions so that they are concentrated during recessions rather than spreading the payment reduction over the life of the mortgage. The reasons are twofold. First, homeowners who are liquidity constrained - primarily young households who have recently purchased a home and have little liquid savings - are less likely to default and better able to smooth consumption with payment reductions. Furthermore, in a downturn where the central bank reduces interest rates, many households would like to refinance but cannot due to down payment constraints, and indexation automatically provides them with relief. Second, renters who would like to buy in a crisis when homes are cheap but are highly constrained by their current income are more likely to buy when mortgages afford low initial payments. These buyers expand demand and help to put a floor under prices. Consequently, the benefit of different designs depends largely on how they deliver immediate payment reductions to highly constrained households, with the benefit mostly accruing to young homeowners.

Our model features overlapping generations of households subject to both idiosyncratic and aggregate shocks, making endogenous decisions over home purchases, borrowing, consumption, refinancing, and default. We consider different exogenous processes for the interest rate, reflecting alternative monetary policies. Competitive and risk-neutral lenders set spreads for each mortgage to break even in equilibrium, so lenders charge higher interest rates when a mortgage design hurts their bottom line. Equilibrium in the housing market implies that household decisions, mortgage spreads, and the interest rate process influence the equilibrium home price process. Household expectations regarding equilibrium prices and mortgage rates feed back into household decisions, and we solve this fixed-point problem in a rich quantitative model using computational methods based on Krusell and Smith (1998).

A key aspect of our analysis is that mortgage design affects household default decisions and hence home prices, which in equilibrium feeds back to household indebtedness. The quantitative implications of our model depend on accurately representing the link between home prices and default. Consequently, after calibrating our model to match standard moments and the empirical distributions of mortgage debt and assets, we evaluate its ability to quantitatively capture the effect of payment reductions on default by simulating the Fuster and Willen (2015) quasi-experiment in our model. Simulating quasi-experiments in our calibration procedure is an innovation that ensures that our model accurately captures the effects of changes in LTVs and interest rates as we alter mortgage design.

The calibrated model provides a laboratory to assess the benefits and costs of different mortgage designs. Our primary application is to a housing crisis impulse response, although we also consider the performance of different mortgages in stochastic simulations. We begin by comparing a world with all fixed rate mortgages (FRMs) against one with all adjustable rate mortgages (ARMs). While ARM and FRMs are not necessarily optimal contracts, they provide the simplest and starkest comparison for us to analyze the benefits of indexation. We find that in a counterfactual world with all adjustable rate mortgages instead of all fixed rate mortgages, house prices fall by 3.4 percentage points less, 33.5 percent fewer households default, and the overall welfare impact of a housing crisis is ameliorated by 1.27 percent of annual consumption in a crisis. Young, liquidity constrained households benefit to an even greater extent, with ARMs increasing their welfare by up to four
percent of one year consumption each year of the crisis relative to FRMs.
To understand why ARMs are so beneficial, note that as the central bank lowers short rates in response to the crisis, homeowners with low equity cannot refinance to take advantage of lower rates due to the minimum LTV constraint. Because the probability of a large and persistent negative income shock also rises in the recession, a fraction of these homeowners, in particular young households with little savings who have recently purchased, become liquidity constrained and default. These defaults increase the supply of homes on the market, further pushing down prices, which in turn leads to more default and prevents more homeowners from refinancing. This phenomenon generates a price-default spiral, amplifying the crisis through equilibrium feedbacks. Conversely, in a world with adjustable-rate mortgages, homeowners do not need to refinance to take advantage of lower interest rates: since the mortgage payment is pegged to the prevailing short rate in the market, payments fall automatically. This leads to less default by underwater homeowners, short-circuiting the default spiral and leading to a less severe housing crisis.

Furthermore, due to the restrictions and costs of refinancing, adjustable rate mortgages provide greater hedging benefits against declining labor income during the crisis. As a result, consumption falls by less during the crisis in a world with ARMs instead of FRMs. Additionally, ARM rates fall significantly more than FRM rates during the crisis since FRM rates are priced off the long end of the yield curve and thus are governed by the logic of the expectations hypothesis. Due to this, adjustable rate mortgages provide greater immediate insurance than fixed rate mortgages. In this way, ARMs "front load" payment reductions into the crisis states, which, as noted earlier, is a key element of effective indexed mortgage design. Finally, since ARM rates fall more than FRM rates, demand for housing by highly-constrained new home buyers rises more with ARMs, which further limits price declines.

One issue with indexation to short-term interest rates is that in an inflationary episode, interest rates can spike up while real income falls, with potentially catastrophic consequences. We consider a new mortgage design that partially protects from this scenario: a mortgage where borrowers have a fixed rate mortgage with a one-time option to convert to floating rate mortgage, as suggested in Eberly and Krishnamurthy, 2014. Of course, borrowers pay for the prepayment option with a higher average loan rate, which is offset somewhat by banks anticipating fewer defaults and losses in a crisis. Despite this cost, this mortgage delivers much better outcomes than a standard fixed rate mortgage: it realizes roughly 75 percent of the benefits of the all-ARM world when rates fall in a downturn, but only 35 percent of the downside in an inflationary episode in which rates rise in a housing bust.

We also consider a mortgage design where households own a fixed-rate mortgage and can refinance in a recession or crisis into another fixed-rate mortgage with equal principal regardless of their loan-to-value ratio. In the Great Recession, there were many households who were underwater on their mortgages and could not refinance to take advantage of lower mortgage rates. Indeed, this fact motivated the government's HARP program. The FRM with underwater refinancing delivers insurance benefits to households stuck at high rates, but far less than that of the option-to-convert to float design. The payment relief is limited because the new FRM is priced off of a 10-year bond and long-term rates fall less than short-term rates in the crisis. This means that the insurance
benefits of the payment reduction are spread over the life of the mortgage rather than concentrated in the crisis when home values are low, negative income shocks are more pervasive, and highly constrained households benefit the most from payment relief. Indeed, the consumption equivalent welfare gain relative to FRM for the FRM with underwater refinancing is a quarter of that of the Eberly-Krishnamurthy convertible mortgage. The comparison of these two designs leads us to conclude that the best designs are those that deliver immediate payment relief to liquidity constrained households. It also suggests that policies like HARP need to be combined with policies that push down long-term mortgage rates, such as the Fed's purchases of mortgage-backed securities, in order to be maximally effective.

Our analysis also calls attention to an important externality: when deciding their personal debt position, households do not internalize the impact of their debt choice on macro fragility. This has important consequences in our model. For instance, ARMs provide much more relief relative to FRMs if they are introduced when the crisis occurs rather than ex ante because households expect the central bank to provide insurance by reducing short rates in the ARM economy and take on more risk by levering up, undoing some of the insurance benefit. Similarly, an option ARM design, which Piskorski and Tchistyi (2010) argue is roughly optimal in normal times, performs poorly macroprudentially in a crisis because distressed households take advantage of the negative amortization option, creating a more fragile LTV distribution when a crisis hits. Finally, this externality can make quantitative easing policies counterproductive if they are overused. Indeed, we show that if a central bank is "doveish" and pursues quantitative easing too frequently, households will anticipate the relief and lever up to the extent that the crisis becomes deeper and default is worse than the economy without quantitative easing. These results highlight that policy makers must account for the fact that households do not share their macro-prudential concerns and may take on too much debt from a social planner's perspective when insurance is offered.

Finally, we find that monetary policy and mortgage design cannot be studied in isolation. Indeed, monetary policy efficacy depends on mortgage design, and mortgage design efficacy depends on monetary policy. We highlight this interaction by considering the performance of various mortgage designs under alternate monetary policies.

The remainder of the paper is structured as follows. Section 2 describes the relationship to the exiting literature. Section 3 presents our model, and Section 4 describes our calibration procedure. Section 5 compares the performance of ARM-only and FRM-only economies to develop economic intuition. Section 6 compares proposed mortgages that add state contingency to an FRM, and Section 7 considers the interaction of mortgage design with monetary policy. Section 8 concludes.

## 2 Related Literature

This paper is most closely related to papers that analyze the role of mortgages in the macroeconomy through the lens of a heterogeneous agents model. In several such papers, house prices are exogenous. Campbell and Cocco (2015) develop a life-cycle model in which households can borrow using long-term fixed- or adjustable-rate mortgages and face income, house price, inflation, and interest rate risk. They use their framework to study mortgage choice and the decision to default.

In their model, households can choose to pay down their mortgage, refinance, move, or default. Mortgage premia are determined in equilibrium through a lender zero-profit condition. Our modeling of households shares many structural features with this paper, but while they take house prices as an exogenous process, we crucially allow for aggregate shocks and determine equilibrium house prices. This critical feature of our model allows us to study the interaction of mortgage design with endogenous price-default spirals. A prior paper, Campbell and Cocco (2003), use a more rudimentary model without default and with exogenous prices to compare ARMs and FRMs and assess which households benefit most from each design. Similarly, Corbae and Quintin (2015) present a heterogeneous agents model in which mortgages are priced in equilibrium and households select from a set of mortgages with different payment-to-income requirements, but again take house prices as exogenous. They use their model to study the role of leverage in triggering the foreclosure crisis, placing particular emphasis on the differential wealth levels and default propensities of households that enter the housing market when lending standards are relaxed. Conversely, we focus on the impact of mortgage design and monetary policy on housing downturns, allowing for endogenous house price responses.

Other heterogeneous agent models of the housing market have endogenous house prices but lack aggregate shocks or rich mortgage designs. Kung (2015) develops a heterogeneous agents model of the housing market in which house prices are determined in equilibrium. His model, however, lacks aggregate shocks and household saving decisions. He focuses specifically on the equilibrium effects of the disappearance of non-agency mortgages during the crisis. By contrast, we include aggregate shocks and a rich set of household decisions that Kung assumes away. We also study a variety of mortgage designs and analyze how mortgage design interacts with monetary policy. Finally, Kaplan et al. (2016) present a life-cycle model with default, refinancing, and moving in the presence of idiosyncratic and aggregate shocks in which house prices are determined in equilibrium. Their focus, however, is on explaining what types of shocks can explain the joint dynamics of house prices and consumption in the Great Recession. They simplify many features of the mortgage contract for tractability in order to focus on these issues, while our paper simplifies the shocks and consumption decision in order to provide a richer analysis of mortgage design. ${ }^{1}$

Our paper also builds on a largely theoretical literature studying optimal mortgage design. Piskorski and Tchistyi (2010; 2011) consider optimal mortgage design from an optimal contracting perspective, finding that the optimal mortgage looks like an option ARM when interest rates are stochastic and a subprime loan when house prices are stochastic. Brueckner and Lee (2017) focus on optimal risk sharing in the mortgage market. These papers identify important trade-offs inherent in optimal mortgage design in a partial equilibrium settings. Concurrent research by Piskorski and Tchistyi (2017) studies mortgage design in a setting with equilibrium house prices and asymmetric information in a two-period model. The intuition they develop about the insurance benefits of state contingent contracts is complementary to our more quantitatively-focused analysis, although we abstract from the contracting frictions in their analysis. Our paper is also related to a literature advocating certain macroprudential polices design to ameliorate the severity of housing crises.

[^1]Mian and Sufi (2015) advocate for modifications through principal reduction, while Eberly and Krishnamurthy (2014) advocate for monthly payment reductions. Greenwald (2016) advocates for payment-to-income constraints as macroprudential policy to reduce house price volatility.

To calibrate our model, we draw on a set of papers which document empirical facts regarding household leverage and default behavior. Foote et al. (2008) provide evidence "double trigger" theory of mortgage default, whereby most default is accounted for by a combination of negative equity and an income shock as is the case in our model. Bhutta et al. (2010), Elul et al. (2010), and Gerardi et al. (2013) provide further support for illiquidity as the driving source of household default. Fuster and Willen (2015) and Di Maggio et al. (2017) show that downward rate resets lead to reductions in default and increases in household consumption, respectively. Agarwal et al. (2015), Agarwal et al. (2017), and Ganong and Noel (2017) study the HAMP and HARP programs and find similarly large effects of payment on default and consumption and limited effects of principal reduction for severely-underwater households. This micro evidence motivates our focus on mortgage designs with state-contingent payments, and we use Fuster and Willen's evidence to evaluate the quantitative performance of our model.

Finally, our research studies how mortgage design interacts with monetary policy and thus relates to a literature examining the transmission of monetary policy through the housing market. Caplin, Freeman, and Tracy (1997) posit that in depressed housing markets where many borrowers owe more than their house is worth, monetary policy is less potent because individuals cannot refinance. Beraja, Fuster, Hurst, and Vavra (2017) provide empirical evidence for this hypothesis by analyzing the impact of monetary policy during the Great Recession. Relatedly, a set of papers have argued that adjustable-rate mortgages allow for stronger transmission of monetary policy since rate changes directly affect household balance sheets (Calza et al., 2013; Auclert, 2016; Cloyne et al., 2017). Garriga et al. (2016) provide a model with long-term debt that features a yield curve and is related to our findings about the differential effects of mortgage designs that are priced off the short end and the long end of the yield curve. Di Maggio et al. (2017) show empirically that the pass-through of monetary policy to consumption is stronger in regions with more adjustable rate mortgages. Finally, Wong (2016) highlights the role that refinancing by young households plays in the transmission of monetary policy to consumption.

## 3 Model

This section presents an equilibrium model of the housing market with rich mortgage contracts that we subsequently use as a laboratory to study different mortgage design. Home prices and mortgage spreads are set in equilibrium. Short-term interest rates, on the other hand, are exogenous to the model and depend on an aggregate shock process. We are interested in understanding how the relationship between interest rates and the state of the economy affects the equilibrium. For ease of exposition, we present the model for the case of an FRM, but consider other designs when presenting our quantitative results.

### 3.1 Setup

Time is discrete and indexed by $t$. The economy consists of a unit mass of overlapping generations of heterogeneous households of age $a=1,2, \ldots, T$ who make consumption, housing, borrowing, refinancing and default decisions over their lifetime. Household decisions depend both on aggregate state variables $\Sigma_{t}$ and agent-specific state variables $s_{t}^{j}$, where $j$ indexes agents. Unless otherwise stated, all variables are agent-specific, and to simplify notation we suppress their dependency on $s_{t}^{j}$.

The driving shock process in the economy is $\Theta_{t}$, which is part of $\Sigma_{t} . \Theta_{t}$ follows a discrete Markov process over five states $\Theta_{t} \in\{$ Crisis With Tight Credit, Recession With Tight Credit, Recession With Loose Credit, Expansion With Tight Credit, Expansion With Loose Credit\} and is governed by a transition matrix $\Xi^{\Theta}$ described subsequently.

Each generation lives for $T$ periods. At the beginning of a period, a new generation is born and shocks are realized. Agents then make decisions, and the housing market clears. Utility is realized and the final generation dies at the end of the period. We make a timing assumption that households enter period $t$ with a state $s_{t}^{j}$ and choose next period's state variables $s_{t+1}^{i}$ in period $t$ given the period $t$ housing price $p_{t}$. Utility is based on period $t$ actions. However, agents who take out a new loan start receiving the interest rate prevailing at time $t$ immediately.

Households receive flow utility from housing $H_{t}$ and non-durable consumption $C_{t}:{ }^{2}$

$$
U\left(C_{t}, H_{t}\right)=\frac{C_{t}^{1-\gamma}}{1-\gamma}+\alpha_{a} H_{t}
$$

In the last period of life, age $T$, a household with terminal wealth $b$ receives utility:

$$
\frac{\left(C_{t}\right)^{1-\gamma}}{1-\gamma}+\alpha_{T} H_{t}+\psi \frac{(b+\xi)^{1-\gamma}}{1-\gamma} .
$$

For simplicity, we assume that households use their wealth to finance housing and end-of-life care after their terminal period. Consequently, the wealth $b$ is not distributed to incoming generations, who begin life with no assets. ${ }^{3}$

Households receive an exogenous income stream $Y_{t}$ :

$$
Y_{t} \equiv \exp \left(y_{t}^{a g g}\left(\Theta_{t}\right)+y_{t}^{i d}\right) .
$$

Log income is the sum of an aggregate component that is common across households and a household-specific idiosyncratic component. The aggregate component $y_{t}^{a g g}$ is a function of $\Theta_{t}$. The idiosyncratic component $y_{t}^{i d}$ is a discrete Markov process over a set $\left\{Y_{t}^{i d}\right\}$ with transition matrix $\Xi^{i d}\left(\Theta_{t}\right)$.

[^2]Households retire at age $R<T$. After retirement, households no longer face idiosyncratic income risk and keep the same idiosyncratic income they had at age $R$, reduced by $\rho \log$ points to account for the decline in income in retirement. This can be thought of as a social security benefit that conditions on terminal income rather than average life income for computational tractability, as in Guvenen and Smith (2014).

There is a progressive tax system so that individuals' net-of-tax income is $Y_{t}-\tau\left(Y_{t}\right)$. The tax system is modeled as in Heathcote et al. (2017) so that:

$$
\tau\left(Y_{t}\right)=Y_{t}-\tau_{0} Y_{t}^{1-\tau_{1}}
$$

Houses in the model are of one size, and agents can either own a house $\left(H_{t}=1\right)$ or rent a house $\left(H_{t}=0\right) .{ }^{4}$ There is a fixed supply of housing and no construction so the homeownership rate is constant. ${ }^{5}$ Buying a house at time $t$ costs $p_{t}$, and owners must pay a per-period maintenance cost of $m p_{t}$. With probability $\zeta$, homeowners experience a life event that makes them lose their match with their house and list it for sale, while with probability $1-\zeta$, owners are able to remain in their house.

The rental housing stock is entirely separate from the owner-occupied housing stock. Rental housing can be produced and destroyed at a variable cost $q$, so in equilibrium renting costs $q$ per period. Although this assumption is stark, it is meant to capture that while there is some limited conversion of owner-occupied homes to rental homes and vice-versa in practice, the rental and owner-occupied markets are quite segmented (Glaeser and Gyourko, 2009; Halket et al., 2015). This implies that most movements in house prices are accompanied by movements in the price-to-rent ratio. Indeed, in the data, the price-to-rent ratio has been nearly as volatile as price, and the recent boom-bust was almost entirely a movement in the price to rent ratio. Our modeling of the rental market also implies that changes in credit conditions will affect aggregate demand for housing as potential buyers enter or exit the housing market, in contrast to models with substantial conversion between renting and owning such as Kaplan et al. (2016).

A household's date $t$ mortgage balance is $M_{t} \geq 0$ and carries interest rate $i_{t}$. Mortgage interest is tax deductible, so that taxes are $\tau\left(Y_{t}-i_{t} M_{t}\right)$. In order to economize on state variables, the mortgage amortizes over its remaining life as in Campbell and Cocco (2003, 2015). This rules out mortgage designs with variable term lengths, but still allows for the analysis of mortgage designs that rely on state-dependent payments. The minimum payment on a mortgage for an agent who does not move or refinance at time $t$ is:

$$
M_{t} \frac{\left(i_{t}\left(1+i_{t}\right)^{T-a+1}\right)}{\left(1+i_{t}\right)^{T-a+1}-1} .
$$

[^3]The interest rate on the mortgage at origination is $i_{t}=i_{t}^{F R M}\left(\Theta_{t}\right)$, the exogenous FRM rate prevailing at time $t$, which the borrower keeps until a refinancing occurs. With adjustable rates, the borrower's current interest rate is $i_{t}^{A R M}\left(\Theta_{t}\right)$, the ARM rate at time $t . i_{t}^{F R M}$ and $i_{t}^{A R M}$ are determined based on a yield curve and lending spread for each mortgage type described in the calibration section below. ${ }^{67}$ The short interest rate $r_{t}\left(\Theta_{t}\right)$ is exogenous, stochastic, and a function of the state of the business cycle $\Theta_{t}$.

At origination, mortgages must satisfy a loan to value constraint :

$$
\begin{equation*}
M_{t+1}(a) \leq \phi p_{t} H_{t+1}(a), \tag{1}
\end{equation*}
$$

where $t+1$ is used for $M$ and $H$ because choices of mortgages and housing today determines the entering housing and mortgage balance tomorrow. $\phi$ parameterizes the maximum loan-to-value ratio.

Mortgages are non-recourse but defaulting carries a utility penalty of $d$ which is drawn each period iid from a uniform distribution over $\left[d_{a}, d_{b}\right] .{ }^{8}$ Defaulting households lose their house today and cannot buy a new house in the period of default due to damaged credit. The default goes on their credit record, and they are unable to purchase until the default flag is stochastically removed.

Each period, homeowners can take one of four actions in the housing market: take no action with regards to their mortgage and make at least the minimum mortgage payment $(N)$, refinance but stay in their current house $(R)$, move to a new house and take out a new mortgage $(M)$, or default $(D)$. Note that if a household refinances or moves to a new house, they must take out an entirely new mortgage which is subject to the LTV constraint in equation (1). Moving has a cost of $k_{m}+c_{m} p_{t}$ for both buying and selling, while refinancing has a cost of $k_{r}+c_{r} M_{t+1}$.

Homeowners occasionally receive a moving shock that forces them to move with probability $\zeta$. In this case, they cannot remain in their current house and either move or default, while agents who do not receive the moving shock are assumed to remain in their house and can either do nothing, refinance, or default.

Finally, regardless of whether they receive a moving shock $\zeta$, renters can either do nothing and pay their rent $(N)$ or move into an owner-occupied house $(M)$ each period.

### 3.2 Decisions and Value Functions

Consider a household at time $t$. This household enters the period with housing $H_{t} \in\{0,1\}$, a mortgage with principal balance $M_{t}$, and $S_{t}\left(1+r_{t}\right)>0$ in liquid savings (which has earned the risk free rate $r_{t}$ between $t-1$ and $t$ realized at $t$ ). The household may also have a default on its credit

[^4]record $D_{t}=\{0,1\}$. The state of the economy at time $t, \Theta_{t}$, is realized. The household receives income $Y_{t}$. The agent-specific state $s_{t}^{j}=\left\{S_{t}, H_{t}, M_{t}, i_{t}, Y_{t}, D_{t}, a_{t}\right\}$ is a vector of the household's assets, liabilities, income, credit record default status, and age. The vector of aggregate state variables $\Sigma_{t}$ includes the state of the economy $\Theta_{t}$, and $\Omega_{t}\left(s_{t}^{j}\right)$, the cumulative distribution of individual states $s_{t}^{j}$ in the population. The home price $p_{t}$ is a function of $\Sigma_{t}$.

The household faces two constraints. The first is a flow budget constraint:

$$
\begin{align*}
Y_{t}-\tau\left(Y_{t}-i_{t} M_{t}\right)+S_{t}\left(1+r_{t}\right)+M_{t+1} & =C_{t}+S_{t+1}+\left(1+i_{t}\right) M_{t}-p_{t}\left(H_{t+1}-H_{t}\right)  \tag{2}\\
& +q 1\left[H_{t}=0\right]+m p_{t} 1\left[H_{t}=1\right]+K(\text { Action })
\end{align*}
$$

where $K($ Action $)$ is the fixed or variable cost of the action the household takes. The left hand side of this expression is the sum of net-of-tax income, liquidated savings, and new borrowings. The right hand side is the sum of consumption, savings for the next period, payments on existing mortgage debt, net expenditures on owner-occupied housing, rental or maintenance costs, and the fixed and variable costs of the action that the household takes.

The second constraint addresses the evolution of a household's mortgage. Given a mortgage balance $M_{t}$, implicitly define $\Delta M_{t}$ as the change in the mortgage balance over and above the minimum payment:

$$
\begin{equation*}
M_{t+1}=M_{t}\left(1+i_{t}\right)-M_{t} \frac{\left(i_{t}\left(1+i_{t}\right)^{T-a+1}\right)}{\left(1+i_{t}\right)^{T-a+1}-1}+\Delta M_{t} \tag{3}
\end{equation*}
$$

The mortgage balance for the next period is equal to the current mortgage balance inclusive of all interest costs, minus payments equal to the minimum payment plus $\Delta M_{t}$. If $\Delta M_{t}$ is positive, the mortgage balance has risen relative to the minimum payment and the homeowner has extracted equity, and if $\Delta M_{t}$ is negative the mortgage balance has prepaid. Thus, households that do not move, refinance, or default face a constraint of $\Delta M_{t} \leq 0$. If a household moves, it pays off its mortgage balance and chooses a new mortgage balance $M_{t+1}$, subject to the LTV constraint (1). Finally, a household may also choose to default, in which case it loses its house today and cannot buy, so $M_{t}=H_{t}=M_{t+1}=H_{t+1}=0$. The household also receives a default on its credit record so $D_{t+1}=1$ and cannot again until its credit record is cleared, which occurs with probability $\lambda$.

We write the household's problem recursively. Denote $V_{a}\left(s_{t}^{j} ; \Sigma_{t}\right)$ as the value function for a household, and $V_{a}^{A}\left(s_{t}^{j} ; \Sigma_{t}\right)$ as the values when following action $A=\{N, R, M, D\}$. Then,

$$
V_{a}\left(s_{t}^{j} ; \Sigma_{t}\right)= \begin{cases}\zeta \max \left\{V_{a}^{D}\left(s_{t}^{j} ; \Sigma_{t}\right), V_{a}^{M}\left(s_{t}^{j} ; \Sigma_{t}\right)\right\}+ \\ (1-\zeta) \max \left\{V_{a}^{D}\left(s_{t}^{j} ; \Sigma_{t}\right), V_{a}^{R}\left(s_{t}^{j} ; \Sigma_{t}\right), V_{a}^{N}\left(s_{t}^{j} ; \Sigma_{t}\right)\right\} & \text { if } H_{t}>0 \\ \max \left\{V_{a}^{M}\left(s_{t}^{j} ; \Sigma_{t}\right), V_{a}^{N}\left(s_{t}^{j} ; \Sigma_{t}\right)\right\} & \text { if } H_{t}=0 \text { and } D_{t}=0 \\ V_{q}^{M}\left(s_{t}^{j} ; \Sigma_{t}\right) & \text { if } H_{t}=0 \text { and } D_{t}=1\end{cases}
$$

On the top line, if the household receives the moving shock with probability $\zeta$, it must decide whether to default on the existing mortgage and be forced to rent, or pay off the mortgage balance, in which case it can freely decide whether to rent or finance the purchase of a new home. On
the second line, if the household does not receive the moving shock, it decides between defaulting, refinancing, or paying the minimum mortgage balance. Finally, in the last two lines, a household that currently has no housing (currently a renter or just born) and does not have a default on their credit record can decide whether to purchase a house and take on a new mortgage or continue to rent. Renters with a default on their credit records $D_{t}=1$ cannot purchase.

We next define the value functions under each of the actions $A=\{N, R, M, D\}$. Households who continue to service their mortgage choose their mortgage payment, savings, and consumption to solve:

$$
\begin{gathered}
V_{a}^{N}\left(s_{t}^{j} ; \Sigma_{t}\right)=\max _{C_{t}, S_{t+1}, M_{t+1}} U\left(C_{t}, H_{t}\right)+\beta E_{t}\left[V_{a+1}\left(s_{t+1}^{i} ; \Sigma_{t+1}\right)\right] \text { s.t. (2), } \\
S_{t+1} \geq 0, \\
H_{t+1}=H_{t}, \\
i_{t+1}=i_{t}, \\
\Delta M_{t}<0 .
\end{gathered}
$$

Households who refinance make the same choices, but pay the fixed and variable costs of refinancing and face the LTV constraint rather than the $\Delta M_{t}<0$ constraint. They have value:

$$
\begin{gathered}
V_{a}^{R}\left(s_{t}^{j} ; \Sigma_{t}\right)=\max _{C_{t}, S_{t+1}, M_{t+1}}\left\{U\left(C_{t} H_{t}(a)\right)+\beta E_{t}\left[V_{a+1}\left(s_{t+1}^{i} ; \Sigma_{t+1}\right)\right]\right\} \text { s.t. (2), } \\
S_{t+1} \geq 0 \\
M_{t+1} \leq \phi p_{t} H_{t+1} \\
H_{t+1}=H_{t} \\
i_{t+1}=i_{t}^{F R M}
\end{gathered}
$$

Households who move choose their consumption, savings, and if buying, mortgage balance, as refinancers do, but also get to choose their housing $H_{t+1}$. They have value:

$$
\begin{gathered}
V_{a}^{M}\left(s_{t}^{j} ; \Sigma_{t}\right)=\max _{C_{t}, S_{t+1}, M_{t+1}, H_{t+1}}\left\{U\left(C_{t}, H_{t}\right)+\beta E_{t}\left[V_{a+1}\left(s_{t+1}^{a} ; \Sigma_{t+1}\right)\right]\right\} \text { s.t. (2), } \\
S_{t+1} \geq 0 \\
M_{t+1} \leq \phi p_{t} H_{t+1} \\
i_{t+1}=i_{t}^{F R M}
\end{gathered}
$$

Households who default lose their home but not their savings. The defaulting households choose
consumption and savings to solve:

$$
\begin{gathered}
V_{a}^{D}\left(s_{t}^{j} ; \Sigma_{t}\right)=\max _{C_{t}(a), S_{t+1}(a)}\left\{-d+U\left(C_{t}(a), H_{t}(a)\right)+\beta E_{t}\left[V_{a+1}\left(s_{t+1}^{a} ; \Sigma_{t+1}\right)\right]\right\} \text { s.t. } \\
S_{t+1} \geq 0 \\
H_{t}=M_{t}=H_{t+1}=M_{t+1}=0 \\
D_{t+1}=1
\end{gathered}
$$

In the final period, a household must liquidate its house regardless of whether it gets a moving shock, either through moving or defaulting:

$$
V_{T}\left(s_{t}^{j} ; \Sigma_{t}\right)=\max \left\{V_{T}^{N}\left(s_{t}^{T} ; \Sigma_{t}\right), V_{T}^{D}\left(s_{t}^{T} ; \Sigma_{t}\right)\right\} .
$$

### 3.3 Mortgage Spread Determination

We assume that mortgages are supplied by competitive, risk-neutral lenders with a one-time origination cost of $\kappa>0$. In the event of default, the lender forecloses on the home, sells it in the open market, and recovers a fraction $\Upsilon$ of its current value.

Define the net present value of the expected payments made by an age $a$ household with idiosyncratic state $s_{t}^{j}$ and aggregate state $\Sigma_{t}$, which is the value of the mortgage to a lender, as $\Pi_{a}\left(s_{t}^{j} ; \Sigma_{t}\right)$. This can be written recursively as:

$$
\begin{array}{r}
\Pi_{a}\left(s_{t}^{j} ; \Sigma_{t}\right)=\delta\left(s_{t}^{j} ; \Sigma_{t}\right) \Upsilon p_{t}+\sigma\left(s_{t}^{j} ; \Sigma_{t}\right) M_{t}\left(1+i_{t}\right)+  \tag{4}\\
\left(1-\delta\left(s_{t}^{j} ; \Sigma_{t}\right)-\sigma\left(s_{t}^{j} ; \Sigma_{t}\right)\right)\left[\begin{array}{c}
M_{t} \frac{\left(i_{t}\left(1+i_{t} t^{T-a+1}\right)\right.}{\left(1+i_{1}\right)^{T-a+1}-1}-\Delta M_{t}\left(s_{t, a} ; \Sigma_{t}\right) \\
+\frac{1}{1+r_{t}} E_{t}\left[\Pi_{a+1}\left(s_{t+1}^{J}, \Sigma_{t+1}\right)\right]
\end{array}\right],
\end{array}
$$

where $\Delta M_{t}\left(s_{t}^{j} ; \Sigma_{t}\right)$ is the prepayment policy function of the household implicitly defined by (3) and the household policy functions, $\sigma\left(s_{t}^{j}, \zeta ; \Sigma_{t}\right)$ is an indicator for whether a household moves or refinances, and $\delta\left(s_{t}^{j}, \zeta ; \Sigma_{t}\right)$ is an indicator for whether a household defaults. In the present period, the lender receives the recovered value in the event of a foreclosure, the mortgage principal plus interest in the event the loan is paid off, and the required payment on the mortgage plus any prepayments made by the borrower if the loan continues. The lender also gets the discounted expected continuation value of the loan at the new balance if the loan continues.

We assume that the interest rate paid by the borrower for a given type of loan is a spread over the short end of the yield curve for adjustable rate loans and the long end of the yield curve (10 year maturity) for fixed rate loans, where the long end is determined by the expectations hypothesis. For mortgages that allow borrowers to choose between an adjustable and fixed rate, we assume the same spread is used over each end of the yield curve.

For now, we assume that lenders determine a single spread $\pi$ for each type of loan that they charge to all borrowers. This pools risk across borrowers in different states but prices the mortgage so that if a mortgage design shifts risks from borrowers to lenders, the spread rises until the lenders
are compensated. In a future draft, we intend to price mortgages for each aggregate state $\Theta_{t}$ and idiosyncratic income. The single spread captures captures the first-order differences between mortgage designs in the extent to which different designs hurt lenders on average. Our single spread assumption implies that there is cross-subsidization in mortgage pricing. While the degree of crosssubsidization implied by our model is unrealistic, there is ample evidence of cross-subsidization in GSE mortgage pricing. Indeed, Hurst et al. (2016) document that GSE mortgage rates for otherwise identical loans do not vary spatially.

The condition for the lenders to break even that determines the spread $\pi$ is:

$$
\begin{equation*}
E\left[E_{\Omega_{t}^{\text {orig }}}\left[\frac{1}{1+r_{t}} \Pi_{a}\left(s_{t+1}^{j} ; \Sigma_{t+1}\right)-M_{t}\right]\right]=\kappa \tag{5}
\end{equation*}
$$

where $\Omega^{\text {orig }}$ is state distribution of newly originated mortgages. This equates the average value of future loan payments net of the loan principal to the lender's origination cost. We calibrate the model under all FRMs and determine $\kappa$ from the economy's equilibrium. We then price all other mortgages given this $\kappa$.

### 3.4 Equilibrium

A competitive equilibrium consists of decision rules over actions $A=\{N, R, M, D\}$ and state variables $C_{t}, S_{t+1}, M_{t+1}, H_{t+1}$, a price function $p\left(\Sigma_{t}\right)$, a mortgage spread $\pi$ for each mortgage type, and a law of motion for the aggregate state variable $\Sigma_{t}$. Decisions are optimal given the home price function and the law of motion for the state variable. At these decisions, the housing market clears at price $p_{t}$, the risk-neutral lenders break even on average according to (5), and the law of motion for $\Sigma_{t}$ is verified.

Given the fixed supply of homes, market clearing simply equates supply from movers, defaulters, and investors who purchased last period with demand from renters, moving homeowners, and investors. Let $\eta\left(s_{t}^{j}, \zeta ; \Sigma_{t}\right)$ be an indicator for whether a household moves and $\delta\left(s_{t}^{j}, \zeta ; \Sigma_{t}\right)$ be an indicator for whether a household defaults.Movers and defaulters own $H_{t}\left(s_{t}^{j} ; \Sigma_{t}\right)$ housing, while buyers purchase $H_{t+1}\left(s_{t}^{j} ; \Sigma_{t}\right)$ housing. The housing market clearing condition satisfied by the pricing function $p\left(\Sigma_{t}\right)$ is then:

$$
\begin{gather*}
\int \delta\left(s_{t}^{j}, \zeta ; \Sigma_{t}\right) H_{t}\left(s_{t}^{j} ; \Sigma_{t}\right) d \Omega_{t}+\int \eta\left(s_{t}^{j}, \zeta ; \Sigma\right) H_{t}\left(s_{t}^{j} ; \Sigma_{t}\right) d \Omega_{t}  \tag{6}\\
=\int \eta\left(s_{t}^{j}, \zeta ; \Sigma_{t}\right) H_{t+1}\left(s_{t}^{j} ; \Sigma_{t}\right) d \Omega_{t},
\end{gather*}
$$

where the first line side is supply which includes defaulted homes and sales and the second line is demand.

### 3.5 Solution Method

Solving the model requires that households correctly forecast the law of motion for $\Sigma_{t}$ which drives the evolution of home prices. Note that $\Sigma_{t}$ is an infinite-dimensional object due to the distribution
$\Omega_{t}\left(s_{t}^{j}\right)$. In general, this infinite-dimensional object is impossible to handle computationally. To simplify the problem, we follow the implementation of the Krusell and Smith (1998) algorithm in Kaplan et al. (2016). We focus directly on the law of motion for home prices and assume that households use a simple $\operatorname{AR}(1)$ forecast rule that conditions on the state of the business cycle today $\Theta_{t}$ and the realization of the state of the business cycle tomorrow $\Theta_{t+1}$ for the evolution of $p_{t}$ :

$$
\begin{equation*}
\log p_{t+1}=f_{\left(\Theta_{t}, \Theta_{t+1}\right)}\left(\log p_{t}\right) \tag{7}
\end{equation*}
$$

where $f_{\left(\Theta_{t}, \Theta_{t+1}\right)}$ is a function for each realization of $\left(\Theta_{t}, \Theta_{t+1}\right)$. We parameterize $f(\cdot)$ as a linear spline. ${ }^{9}$ Expression (7) can be viewed either as a tool to compute equilibrium in heterogeneousagent economies, following Krusell and Smith (1998), or as an assumption that households and investors are boundedly rational and formulate simple forecast rules for the aggregate state. To verify that the decision rule is accurate, we both compute the $R^{2}$ for each $\left(\Theta_{t}, \Theta_{t+1}\right)$ realized in simulations and follow Den Haan (2010) by comparing the realized price with the 15, 30, 45, and 100 -year ahead forecasts given the realized process of aggregate shocks to verify that the forecast rule does a good job of computing expected prices many periods into the future and that small errors do not accumulate.

The model cannot be solved analytically, so a computational algorithm is used. First, the household problem is solved using the forecast rule by discretization and backward induction. Given the household policy functions, the spread is adjusted so that the lender breaks even on average, and the household problem is resolved. This is repeated until the spread converges. Given the household policy functions and the spread, the model is simulated for many periods with the home price determined by (6) and the $\mathrm{AR}(1)$ forecast regression (7) is run in the simulated data for each $\left(\Theta_{t}, \Theta_{t+1}\right)$. Finally, the forecast rule is updated based on the regression, and the entire procedure is repeated until the forecast rules converges to an approximate solution.

## 4 Calibration

Our calibration proceeds in three steps. First, we select the aggregate and idiosyncratic shocks to reflect modern business cycles in the United States. Second, we exogenously calibrate a number of parameters to standard values in the macro and housing literature or to match moments in the data. We pay particular attention to quantitatively matching the distribution of assets and mortgage debt in the population, which is important to accurately capturing the number of individuals who would be affected by different mortgage designs at the margin and appropriately aggregating their decisions when computing the housing market equilibrium Third, we choose the default cost to match the fraction of the housing stock foreclosed upon over a simulated crisis to data from the Great Recession. As a test of our calibrated model's ability to capture the effect of payment reductions, which is crucial for the counterfactuals we consider, we compare the model to new quasi-experimental evidence about the effect of debt on default. Our model does a good job in

[^5]Table 1: Model Parameters in Baseline Parameterization

| Param | Description | Value | Param | Description | Value |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $T$ | Years in Life | 45 | $c_{m}$ | Variable Moving Cost as \% of Price | 3.0\% |
| $R$ | Retirement | 35 | $k_{m}$ | Fixed Moving Cost | 0.1 |
| $\rho$ | Log Income Decline in Retirement | 0.35 | $c_{r}$ | Variable Refi Cost as \% of Mortgage | 1.0\% |
| $\tau_{0}$ | Constant in Tax Function | 0.8 | $d_{a}$ | Default Cost Dist Lower Bound | 35.0 |
| $\tau_{1}$ | Curvature Tax Function | 0.18 | $d_{b}$ | Default Cost Dist Upper Bound | 45.0 |
| $\gamma$ | CRRA | 3.0 | $k_{r}$ | Fixed Refi Cost | 0.04 |
| $\xi$ | Terminal Wealth Multiplier | 1.0 | $q$ | Rent | 0.20 |
| $\psi$ | Terminal Wealth Shifter | 500 | $m$ | Maint Cost as \% of Prices | 2.5\% |
| $a$ | Utility From Homeownership | 7.0 | $\zeta$ | Prob of Moving Shock | 1/9 |
| $\beta$ | Discount Factor | 0.96 | $\lambda$ | Prob Default Flag Removed | 1/3 |
| $\Upsilon$ | Foreclosure Sale Recovery \% | 0.654 |  | Homeownership Rate | 65.0\% |
| $\phi_{\text {loose }}$ | Max LTV, Loose Credit | 95.0\% | $\phi_{\text {tight }}$ | Max LTV, Tight Credit | 80.0\% |
| $r$ | Short Rate | [ $0.26 \%, 1.32 \%, 3.26 \%]$ (crisis, recession, expansion) |  |  |  |
| $i^{\text {ARM }}$ | ARM Interest Rate | [3.01\%, 4.07\%, 6.01\%] |  |  |  |
| $i^{\text {FRM }}$ | FRM Interest Rate | [ $4.96 \%, 5.48 \%, 5.66 \%]$ (expectations hypothesis) |  |  |  |
| $Y^{\text {agg }}$ | Aggregate Income | [0.0976, 0.1426, 0.1776] |  |  |  |

See text for transition matrix for $\Theta_{t}$ and $Y_{t}^{i d}$.
Note: This table shows baseline calibration for a fixed rate mortgage. Average income is normalized to one. There are five aggregate states, $\Theta_{t} \in\{$ Crisis With Tight Credit, Recession With Tight Credit, Recession With Loose Credit, Expansion With Tight Credit, Expansion With Loose Credit\}, but we assume that income and monetary policy are the same in a recession with loose or tight credit and in an expansion with loose or tight credit. The tuples of interest rates reflect the interest rate in a crisis, recession, and expansion, respectively.
matching this evidence.
Throughout, we calibrate to the data using a model in which all loans are fixed rate mortgages to reflect the predominant mortgage type in the United States and credit constraints are at their pre-downturn level. Table 1 summarizes the variables and their calibrated values. $\kappa$, the fixed origination cost for the lender, is backed out from the FRM equilibrium under the baseline monetary policy and imposed in solving for the model's equilibrium for other mortgages and monetary policies. The calibration is annual.

### 4.1 Aggregate and Idiosyncratic Shocks

We consider an economy that occasionally experiences crises akin to what occurred in the Great Recession. To trigger such a downturn, we combine a deep and persistent recession - which lowers aggregate income and leads to more frequent negative idiosyncratic shocks - with a tightening of credit in the form of a tightening downpayment constraints. Several papers argue that tightening credit helped amplify the bust and model this as a tightening LTV constraint (e.g., Faviliukis et al., 2017 and Justiniano et al., 2017). We consequently assume that credit always tightens in a crisis and then stochastically reverts to being loose in expansions. Since there is insufficient data to evaluate how monetary policy differs in various credit regimes, we assume that income and monetary policy are identical in recessions with high and low credit and expansions with high and low credit. This implies that the transition matrix between the five aggregate states $\Theta_{t} \in\{$ Crisis

With Tight Credit, Recession With Tight Credit, Recession With Loose Credit, Expansion With Tight Credit, Expansion With Loose Credit\} can be represented as a transition matrix between three states \{Crisis, Recession, Expansion\} along with a probability that credit switches form tight to loose in the tight expansion state.

We calibrate the Markov transition matrix between crisis, recession, and expansion based on the frequency and duration of NBER recessions and expansions. We use the NBER durations and frequencies to determine the probability of a switch between an expansion and crisis or recession, and we assume that crises happen every 75 years and all other NBER recessions are regular recessions. We assume that every time the economy exits a crisis or recession it switches to an expansion and that crises affect idiosyncratic income in the same way as a regular recession but last longer and involve a larger aggregate income drop, with a length calibrated to match the average duration of the Great Depression and Great Recession. A regular recession reduces aggregate income by 3.5 percent, while a crisis reduces it by 8.0 percent, consistent with Guvenen et al.'s (2014) data on the decline in log average earnings per person in recessions since 1980. For the probability of reverting to loose credit from tight credit in an expansion, we choose $2.0 \%$, so that when credit tightens it does so persistently but credit loosens quickly enough that a large number of crises begin in the loose credit state. Our results are not sensitive to perturbing this target.

We calibrate short rates and mortgages rates during expansions and recessions to historical real rates from 1985-2007. ${ }^{1011}$ We find that short rates equal $1.32 \%$ on average during recessions and $3.26 \%$ during expansions. For the crisis state, we assume that the real short rate is $3.0 \%$ less than during expansions, or $0.26 \% .^{12}$ The short rate faced by households that save, $r$ in the model, is $0.5 \%$ above the short rate in the data, reflecting higher rates of returns on savings that are illiquid for the one-year duration of a period in our model. We set the calibrated ARM spread over the short rate in the data to $2.75 \%$, its historical average margin over the short Treasury rate in the Freddie Mac Primary Mortgage Market Survey. The FRM interest rate is set using the expectations hypothesis on the ARM rate with a term of 10 years. ${ }^{13}$ We maintain these mortgage rates as we vary the mortgage contract to put different contracts on the same footing. In practice, mortgage design affects monetary policy, as we discuss in Section 7, and so with a different mortgage design

[^6]the Central Bank may set different interest rates.
For the idiosyncratic income process, we match the countercyclical left skewness in idiosyncratic income shocks found by Guvenen et al. (2014). Left skewness is crucial to accurately capturing the dynamics of a housing crisis because the literature on mortgage default has found that large income shocks are crucial drivers of default. To incorporate left skewness, we calibrate log idiosyncratic income to follow a Gaussian $\operatorname{AR}(1)$ with an autocorrelation of 0.91 and standard deviation of 0.21 following Floden and Linde (2001) in normal times but to have left skewness in the shock distribution in busts. We discretize the income process in normal times by matching the mean and standard deviation of shocks using the method of Farmer and Toda (2016), which discretizes the distribution and optimally adjusts it to match the mean and variance of the distribution to be discretized. For the bust, we add the standardized skewness of the 2008-9 income change distribution from Guvenen et al. (2014) to moments to be matched, generating a shock distribution with left skewness. This gives a distribution with a negative mean income change in busts and leads to income being too volatile, so we shift the mean of the idiosyncratic shock distribution in busts to match the standard deviation of aggregate log income in the data. In doing so, we choose the income distribution of the newly born generations to match the lifecycle profile of income in Guvenen et al. (2016). ${ }^{14}$ We normalize the income process so that aggregate income is equal to one.

### 4.2 Other Calibration Targets

Having set the parameters that determine aggregate and idiosyncratic shocks as well as interest rates, we then set a number of parameters to standard values in the literature or to directly to match moments in the data.

We assume households live for 45 years, roughly matching ages 25 to 70 in the data. Households retire after 35 years, at which point idiosyncratic income is frozen at its terminal level minus a 0.35 $\log$ point retirement decrease. The tax system is calibrated as in Heathcote et al. (2017), with $\tau_{0}=0.80$ and $\tau_{1}=0.18$.

For preference parameters, we use a discount factor of $\beta=0.96$ and a CRRA of $\gamma=3.0$. The bequest function parameters are chosen so that consumption is smooth at the end of life.

Moving and refinancing involve fixed and variable costs. We set the fixed cost of moving equal to 10 percent of average annual income, or $\$ 5,000$. The proportional costs, paid by both buyers and sellers, equal 3 percent of the house value to reflect closing costs and realtor fees. Refinancing involves a fixed cost of 4 percent of average annual income, or $\$ 2,000$, as well as variable cost equal to 1 percent of the mortgage amount to roughly match average refinancing costs quoted by the Federal Reserve.

Renters pay a rent of $q=0.20$ to match a rent-to-income ratio of $20 \%$. Homeowners must pay a maintenance cost equal to $2.5 \%$ of the house value every year. We assume that homeowners move an average of every 9 years as in the American Housing Survey. The homeownership rate is set to

[^7]match the long-run average homeownership rate of 65 percent in the United States.
$\Upsilon$, the fraction of the price recovered by the bank after foreclosure, is set to 64.5 percent. This combines the 27 percentage point foreclosure discount in Campebell, Giglio and Pathak (2011) with the fixed costs of foreclosing upon, maintaining, and marketing a property, estimated to be 8.5 percent of the sale price according to Andersson and Mayock (2014). ${ }^{15}$

We assume that the maximum LTV at origination under loose credit is $95.0 \%$, corresponding to the highest spike in the distribution of LTV at origination in the Great Recession, and under tight credit is $80 \%$, which is the conforming loan limit LTV. This generates crises with a tightening of credit that feature a price decline similar to what we observed in the Great Recession.

We calibrate $a$, the utility benefit of owning a home, to match a mean price to income ratio for homeowners in stochastic simulations of 4 , which is the mean value in the SCF. We finally calibrate $\bar{d}$, the average default cost, so that in simulations of the impulse response to a housing downturn akin to the Great Recession described below we match that 8.0 percent of the housing stock was foreclosed upon from 2006 to 2013 (Guren and McQuade, 2015). ${ }^{16}$

### 4.3 Lifecycle Patterns and Distributions Across the Population

The model does a good job matching the lifecycle patterns and overall distribution of debt and assets in the Survey of Consumer Finances for 2001, 2004, and 2007. Figure 1 shows the lifecycle patterns, while Figure 2 shows the distributions across the population. In both figures, the pooled SCF data for 2001 to 2007 is in dashed lines and the model analogues are in solid lines.

Panel A of Figure 1 shows the homeownership rate over the lifecycle. The model slightly underestimates the homeownership rate of the very young and over-estimates the homeownership rate of the middle aged.

Panel B of Figure 1 shows the mean, median, 10th percentile, and 90 th percentile of the loan to value ratio (LTV) by age, and panel A of Figure 2 shows the distribution of LTV across the population. On the whole, the model does very well at capturing the overall LTV distribution and the rate at which the mean, median, and 90th percentile homeowner pays down their mortgage. This translates into an LTV distribution that closely matches the data, although we have a few too many homeowners who have paid off their mortgage and slightly too few homeowners with high LTVs, reflecting the fact that we impose a 95.0 percent maximum LTV constraint on all households to reflect 95th percentile LTV of the young in the SCF, while in practice a few homeowners obtain loans with an even higher LTV. The fact that we match the LTV distribution in the data implies that when a crisis hits, our model will have a fraction of underwater and low equity homeowners that is close to the data for the housing bust.

Panel C of Figure 1 shows the mean, median, 10th percentile, and 90th percentile of the payment to income ratio (PTI) by age, and panel B of Figure 2 shows the distribution of PTI across the

[^8]Figure 1: Lifecycle Patterns: SCF vs. Model


Note: This figure compares the baseline calibration of the model with all FRMs (solid lines) to SCF data from 2001 to 2007 (dashed lines) in panels A-D. Panels E and F are constructed based only on the model. Panel A shows the homeownership rate. Panel B shows the mean, median, 10th percentile, and 90 th percentile of loan to value ratios for homeowners, and panel C shows the same statistics for the payment to income ratio. Panel D shows the mean, median, 10th percentile, and 90 th percentile of liquid assets along with median total wealth. Panel E shows the refinance rate, and Panel F shows consumption and investment in the model.
population. We do reasonably well for the PTIs of the young, although the old have PTIs that are too high. This is the case both because income falls in retirement and because the old who still have a mortgage in the data have a loan that amortizes past age 70. By contrast, our model assumes that they have a loan that must be paid off at age 70, leading to high payments at the end of life. Because most of the equilibrium effects in our model come from the purchase, refinance, and default decisions of the young who have relatively high LTVs (panel B), the fact that our calibration is off for elderly homeowners does not dramatically affect market equilibrium. Overall, our PTI distribution has slightly too much mass at zero PTIs, but the key feature for our analysis of a crisis is that we do a good job matching the upper tail of PTIs.

Panel D of Figure 1 shows the mean, median, 10th percentile, and 90th percentile of the liquid wealth and the median of total wealth by age, and panels C and D of Figure 2 show the distributions of total and liquid wealth in the population. The model does a reasonably good job matching median total wealth over the lifecycle and liquid wealth at young ages. Agents in the model accumulate more liquid assets in retirement than in the data. Again, this is not a significant issue, as the old do not play a crucial role in the housing market in our model. The old accumulate too many assets

Figure 2: Distributions Across Population: SCF vs. Model


Note: This figure compares the baseline calibration of the model with all FRMs (solid lines) to SCF data from 2001 to 2007 (dashed lines). Panel A shows loan to value ratios in 10 percentage point bins for homeowners, and panel B shows the payment to income ratio for homeowners in 0.025 bins. Panel C shows total wealth relative to mean income in bins of 0.2 , while panel D shows liquid wealth relative to mean income in bins of 0.2 . In all figures, the model and data are binned identically.
and so the total and liquid wealth distributions are skewed a bit to the right and there are too few people with zero assets, but we do a good job matching the distribution of liquid and total wealth for the young, who tend to have lower assets. Finally, the data has a thicker right tail of very wealthy individuals. Our model is designed to capture the impact of credit constraints and mortgages on housing markets, so capturing the extremely wealthy is not relevant for our exercise.

Finally, Panels E and F of Figure 1 show the fraction of owners refinancing and income and consumption over the life cycle, respectively. Most refinancing is of the cash out variety because the FRM rate does not fluctuate dramatically due to the expectations hypothesis. Cash out refinancing is low until retirement, at which point it jumps so that agents can smooth their consumption. Again, the refinancing of the old is not crucial to our results, and we plan to address it in a future draft. Income follows a standard lifecycle profile, and consumption is smoother than income and increasing as individuals age, consistent with buffer stock models of consumption.

### 4.4 Calibration Evaluation Using Quasi-Experimental Evidence on Default

To evaluate the extent to which our model quantitatively captures the impact of payment reductions, we compare our model to quasi-experimental evidence from Fuster and Willen (2015). Fuster

Figure 3: Fit to Fuster and Willen (2015) Natural Experiment


Note: The data from Fuster and Willen come from column 1 of Table A. 1 in their paper, which is also used in Figure 2 a. The model estimates come form comparing a $2 / 1 \mathrm{ARM}$ to a $1 / 1 \mathrm{ARM}$ in our model.
and Willen study a sample of homeowners who purchased ALT-A hybrid adjustable-rate mortgages during 2005-2008 period and quickly fell underwater as house prices declined. Under a hybrid ARM, the borrower pays a fixed rate for several years (typically five to ten) and then the ARM "resets" to a spread over the short rate once a year. These borrowers were unable to refinance because they owed the bank more than their house's value, and so when their rates reset to reflect the low short rates after 2008, they received a large and expected reduction in their monthly payment.

Fuster and Willen provide two key facts for our purposes. First, they show that even for ALT-A borrowers - who have low documentation and high LTVs relative to the population - at 135 percent LTV the average default hazard prior to reset was only about 24 percent. ${ }^{17}$ The fact that so many households with significant negative equity do not default implies that there are high default costs. It is also consistent with a literature that finds evidence for a "double trigger" model of default whereby both negative equity and a shock are necessary to trigger default, as is the case for most default in our model.

Second, Fuster and Willen use an empirical design that compares households just before and after they get a rate reset and show that these borrowers experience substantial declines in their default rate at reset. In particular, the hazard of default for a borrower receiving a 3.0 percent rate reduction fell by about 55 percentage points, equivalent to going from a 145 percent LTV to 95 percent LTV.

We evaluate the extent to which our model can match Fuster and Willen's estimates by simulating their rate reset quasi-experiment within our model. In particular, we compare the crisis default

[^9]behavior of agents in our model with a $2 / 1$ ARM that will reset next period with the behavior of an agent with a $1 / 1$ ARM that has reset this period. This corresponds to the treatment and control used by Fuster and Willen. We assume that these borrowers are an infinitesimal part of the market, so we can consider them in partial equilibrium, and we compute their default rates at different LTVs with the $2 / 1$ ARM and $1 / 1$ ARM. To deal with the fact that the ALT-A sample used by Fuster and Willen is not representative of the population, we roughly match the assets, age, and income of the homeowners we consider to households with hybrid ARMs that have yet to reset in the 2007 Survey of Consumer Finances. ${ }^{18}$ Finally, we assume that homeowners have a fixed rate corresponding to the FRM rate in the boom and reset to the ARM rate in the crisis for calculating the baseline default hazard. For calculating the effect of the rate reset on default, we consider a one percent, two percent, and three percent interest rate decline by adjusting the initial fixed rate of the hybrid ARM.

Figure 3 compares the calibrated model with the findings of Fuster and Willen (2015). Panel A shows the impact of rate reductions on default in the model and Fuster and Willen's estimates. Overall, the fit is quite good, although at very large rate reductions the model slightly underpredicts the reduction in the default hazard resulting from an interest rate decline. The right panel shows the baseline default rate under the $2 / 1$ ARM at various LTVs relative to the default rate at 135 percent LTV. LTV reductions have modestly larger effects than in the data until one gets below 100 percent, at which point the default hazard falls off in the model but not in Fuster and Willen's data. ${ }^{19}$

Based on the close fit to the Fuster and Willen quasi-experimental evidence, our model should do a good job of reflecting the benefits of principal and payment reductions.

## 5 ARM vs. FRM: The Economics of State Contingent Mortgages

Having created and calibrated a laboratory to study mortgage design and its interaction with monetary policy, we now use our model to assess various mortgages. We primarily focus on a crisis scenario that combines a housing bust and a deep recession as in the Great Recession. This allows us to analyze mortgage designs proposed to address the problems revealed by the Great Recession, which is the focus of the recent literature. Additionally, the equilibrium feedbacks that our model features are most interesting and potent in a downturn with a price-default spiral. In this section we focus on conventional monetary policy that reduces real rates in a crisis, and we consider alternate monetary policies, including the case in which the central bank raises rates in a bust, in section 7 .

To analyze a housing downturn, we simulate an impulse response where in the five years prior

[^10]Figure 4: FRM vs. FRM $\rightarrow$ ARM: Housing Market Outcomes


Note: The figure shows the impulse response to a simulated downturn preceded by an expansion with loose credit under FRM and FRM $\rightarrow$ ARM. In the downturn, the maximum LTV falls from 95.0 percent to 80 percent and the economy falls into a deep downturn for an average of 5.6 years.
to the downturn the economy is in an expansion with loose credit and when the crisis hits the economy falls into a deep recession and the LTV constraint tightens. We assume that the crisis lasts at least three years, after which the economy stochastically exits according to the transition matrix so that the average crisis length is 5.66 years. We study the impulse responses of prices, default rates, and consumption to the resulting downturn, which we compute by averaging together 100 simulations with random shocks prior to the five-year expansion and subsequent to the first three years of the crisis. We also compare FRMs and ARMs in stochastic simulations.

To analyze the effect of mortgage design in such a crisis, we first compare a world with all fixed rate mortgage borrowers to a world with all adjustable rate mortgage borrowers. This provides us with most of the economic intuition regarding the benefits of adding state contingency to mortgages. In Section 6, we consider more complex mortgage designs.

### 5.1 Economic Intuition: FRM vs. ARM

Our baseline case is a world in which the only available mortgage to home purchasers is a fixed-rate mortgage. The results are illustrated by the blue lines in Figure 4.

The model with all FRMs generates a housing crisis in the model of a similar magnitude to the experience in the United States between 2006 and 2012. Panel A of Figure 4 shows that prices fall by about a third, which closely matches the peak to trough decline in national repeat sales house price indices. ${ }^{20}$ Panel B shows the fraction of homeowners who have negative equity, under 10 percent equity, and under 20 percent equity. At the depths of the crisis, nearly 50 percent of homeowners are underwater. Approximately 70 percent of homeowners have less than 20 percent equity when prices are at their lowest and are unable to refinance into a lower interest rate mortgage given the tightened LTV constraint. Panel C shows the combination of negative equity and recession leads to substantial default (recall that matching the fraction of the housing stock that defaulted from 2006 to 2013 in this impulse response is a calibration target). Finally, consumption falls by approximately 12 percent due to the sudden and persistent decline in income and the large number of constrained households. The decline is slightly higher than the decline in the data. ${ }^{21}$

We examine the differential impacts of adjustable-rate mortgages through two experiments. In the first, we assume that home purchasers take out fixed-rate mortgages pre-crisis, but that when the crisis hits, all mortgages are converted to adjustable-rate mortgages. Because the central bank lowers the short rate in the crisis and this is fully passed through to households under ARM, mortgage payments fall dramatically. This experiment, which we call the FRM $\rightarrow$ ARM counterfactual (FARM in figure labels), is a useful thought experiment to understand the mechanisms at work in our model because it holds fixed the state of the economy when the crisis hits and isolates the ex-post effect of adjustable-rate mortgages on the severity of the crisis. In the second experiments, we consider a world with ARMs both before and after the crisis hits, allowing for ex-ante behavior and a different distribution of households across states at the beginning of the crisis.

### 5.1.1 Ex-Post Effect of Switching From FRM to ARM

We present the results of the FRM $\rightarrow$ ARM counterfactual in the purple lines in Figure 4. Relative to the baseline case, the housing crisis is less severe, house prices fall by 4.1 percentage points less, and 60.2 percent fewer households default, as indicated by the green line in panel C, which shows the cumulative difference in the default rates between the FRM and FRM $\rightarrow$ ARM worlds.

The intuition for these effects has to do with the extent to which homeowners can take advantage of lower interest rates in a crisis. In the baseline case, 70 percent of homeowners have less than 20 percent equity and cannot refinance to take advantage of these lower rates. Because of left-skewness of the income shock distribution in the crisis, a significant fraction of these homeowners experience a simultaneous drop in their income. These homeowners become liquidity constrained and those

[^11]Figure 5: FRM vs. FRM $\rightarrow$ ARM: Default and Renter Demand By Age


Note: The figure shows the mass of households of each age defaulting and purchasing after renting last period in the first period of a simulated crisis under FRM and FRM $\rightarrow$ ARM along with the difference between the two lines. To ensure that the differences are entirely due to the policy functions, we compare FRM to $\mathrm{FRM} \rightarrow \mathrm{ARM}$ and evaluate the policy functions assuming the price is the realized FRM price.
with little savings default because they would have to cut their consumption substantially - and in many cases to zero - to make their mortgage payment, which causes them to be willing to bear the utility cost of default. Default increases the supply of homes on the market, further pushing down prices, which in turn leads to more default. This phenomenon is the canonical price-default spiral.

In the ARM world, homeowners do not need to refinance to take advantage of the lower interest rates. Because the mortgage payment is pegged to the prevailing short rate in the market, payments fall automatically. They also fall by more than under FRM because the short end of the yield curve adjusts by more under the expectations hypothesis than the long end. Many defaulters under an FRM find that they would rather cut their consumption and avoid the costs of default. This leads to less default, short-circuiting the default spiral and causing a less severe housing crisis.

Additionally, because FRMs are priced off of long-term rates, which fall via the expectations hypothesis, buying becomes more affordable for young first-time homebuyers, who increase their demand in the ARM world more than the FRM world. This limits price declines and further ameliorates the impact of the housing crisis. Note that the effects are largely concentrated among first-time home buyers since existing homeowners are hedged against the downturn.

These effects are summarized in Figure 5, which plots the mass of homeowners defaulting and

Figure 6: FRM vs. ARM: Housing Market Outcomes


Note: The figure shows the impulse response to a simulated downturn preceded by an expansion with loose credit under FRM and ARM both ex ante and ex post. In the downturn, the maximum LTV falls from 95.0 percent to 80 percent and the economy falls into a deep downturn for an average of 5.6 years.
renters purchasing at each age in the period in which the crisis begins under FRMs and under the FRM $\rightarrow$ ARM counterfactual along with the difference. Because the pre-downturn distribution is the same, this only reflects differences in policy functions in the FRM and FRM $\rightarrow$ ARM worlds. Similar comparisons by age, income, and LTV reveal that the additional default under FRM comes from low income, low savings, and high LTV borrowers, while the additional demand comes from renters with moderate savings and income reflective of first-time homebuyers.

### 5.1.2 Ex-Ante Effect of Switching From FRM to ARM

Our second experiment considers the ex-ante effects of adjustable-rate mortgages, in addition to the ex-post effects. That is, relative to the baseline case in which all mortgages are fixed-rate, we now consider a world in which all mortgages are adjustable-rate, both pre- and post-crisis. We call this the ARM counterfactual. The results are presented in Figures 6, with the FRM world shown in blue and the ARM world shown in orange.

Perhaps surprisingly, the benefits of adjustable-rate mortgages are partially eliminated. Prices

Figure 7: FRM vs. ARM: Pre-Crisis LTV Distribution

fall by 3.4 percentage points, relative to 4.1 percentage points in the $\mathrm{FRM} \rightarrow \mathrm{ARM}$ counterfactual. More households are initially underwater and over eight years, there are 33.5 percent few defaults than under FRM, as opposed to 60.2 percent fewer defaults in the $\mathrm{FRM} \rightarrow \mathrm{ARM}$ counterfactual.

The crisis is worse in the ARM world relative to the FRM $\rightarrow$ ARM world because homeowners understand the hedging properties of the ARM mortgage and take on more risk by increasing their LTV. That is, homeowners understand that with an ARM mortgage, their mortgage payments will fall when the economy enters into a recession, when they are more likely to experience a drop in their labor income. These hedging properties encourage home purchasers to consume more when young relative to the FRM world by taking out loans with higher initial leverage or by cash-out refinancing. This creates macro fragility, which households do not internalize. These effects of switching to ARM ex ante on the LTV distribution is shown in Figure 7. In the ARM world in orange, there is more mass at the highest LTV ratios than the FRM world in blue. Consequently, while the ARM offers ex-post benefits by lowering mortgage payments and reducing default, given ex-ante decisions it leads to a crisis which is similar to the all FRM economy. The degree to which the ex-post benefits of ARMs are undone by the ex-ante buildup of fragility - at least for house prices and default - is a numerical result. This highlights the need for realistic quantitative models of the type we analyze.

### 5.1.3 Welfare Benefits of Switching From FRM to ARM

To further evaluate the benefits of switching from FRM to ARM, we calculate the consumptionequivalent welfare cost of the crisis under each mortgage for generations that are alive when the crisis hits. To do so, we calculate how much each agent would be willing to reduce their consumption per year of their remaining life to avoid a crisis. This is calculated as equivalent variation, as described in the Appendix, and we aggregate by the pre-crisis distribution of individuals across states. We report the difference between the FRM and ARM (both ex ante and ex post) economies.

Overall, households alive when the crisis hits would be willing to give up 1.27 percent more of their annual consumption for their remaining life to avoid the crisis in the FRM world than the ARM world. However, this aggregate calculation masks substantial heterogeneity across groups,

Figure 8: FRM vs. ARM: Welfare Loss By Age


Note: Each panel shows the percentage point difference in consumption equivalent welfare between FRM and ARM. Consumption equivalent welfare is calculated as the equivalent variation amount each agent would be willing to reduce their consumption per year of their remaining life to avoid a crisis for generations alive when the crisis hits. This is aggregated by age in panel A, by mortgage amount in panel B , by savings in panel C , and by idiosyncratic income in panel D using the pre-downturn distribution of agents across stage. The y axis in each panel is the difference between the FRM and ARM economies for this calculation, with a negative number indicating household are worse off under FRM. We repeat the calculations separately for renters and owners, as indicated in the legend.
as shown in Figure 8, which reports the welfare difference separately by age, LTV in the first period of the crisis, savings, and idiosyncratic income. The figure also splits out the relative welfare differentials between ARM and FRM by renters and owners. A negative value indicates a larger welfare loss under FRMs than in the ARM counterfactual, and all figures are in percentage points of consumption.

Figure 8 reveals that the welfare benefits of ARMs are concentrated on two groups. The first group is young, low savings, low income, and high LTV homeowners, who would be willing to give up as much as four percent more of their annual consumption to avoid the crisis in the FRM world relative to the ARM world. These households cannot refinance when the crisis hits due to the LTV constraint, are stuck at a high interest rate, and do not have much liquid savings to help cushion an income shock. The welfare effect is also large for this group because they have lower consumption and are borrowing constrained, expecting their income to rise later in life. Their baseline marginal utility of consumption is therefore high. Consequently, they are willing to pay a substantial amount of annual consumption to switch to an ARM, as the lower interest rate translates to a much smaller minimum payment and boosts consumption substantially.

Figure 9: FRM vs. ARM: The Role of Equilibrium Effects


Note: The figure shows the impulse response to a simulated downturn preceded by an expansion with loose credit under FRM and ARM both ex ante and ex post. In the downturn, the maximum LTV falls from 95.0 percent to 80 percent and the economy falls into a deep downturn for an average of 5.6 years. For the "no equilibrium effects" counterfactual, we take the price path and distribution of agents across states from the FRM model as given and report default and consumption using policy functions computed with ARMs with the FRM forecast rule.

The second group that benefits from the ARM is the very old, who know they will have to sell their house soon. In the FRM world, they do so at a much lower price.

There are, however, a few groups that are better off under FRM. In particular, renters are better off because they buy at a lower price. In particular, renters who are saving for retirement and have a lot of liquid assets receive a windfall when prices fall dramatically.

### 5.2 ARM vs. FRM: The Importance of Equilibrium Effects

An important feature of our analysis that differentiates it from the preceding literature is that house prices and mortgage spreads are determined in equilibrium. Figure 9 shows the impact of these equilibrium effects on consumption and default in a simulated downturn in our model. To calculate the impulse response in the ARM economy with no equilibrium effects, we take the price path and distribution of agents across states from the FRM model as given and report default and consumption using policy functions computed with ARMs with the FRM forecast rule. Figure 9 reveals that the equilibrium feedbacks account for one third of the difference in default and one quarter of the difference in consumption between the ARM and FRM counterfactuals.

### 5.3 ARM vs. FRM in Stochastic Simulations

To further elucidate the differences between the ARM and FRM worlds, we compare moments from stochastic simulations. Table 2 shows the standard deviation of price, default, and consumption for all ARMs relative to an all-FRM economy.

Although mean prices are the same in the ARM and FRM worlds, the insurance benefits of ARMs become evident when once examines the volatility of price, consumption, and particularly default.

Table 2: FRM vs. ARM: Moments From Stochastic Simulations

| Moment | ARM Rel to FRM |
| :---: | :---: |
| Std Dev of Price | $92.8 \%$ |
| St Dev of Default Rate | $67.6 \%$ |
| Std Dev of Consumption | $93.1 \%$ |

Notes: All series are percentages for the all-ARM economy relative to the same statistic for the all-FRM economy.

As we have seen, for particularly severe crises, ARMs can dramatically reduce housing market volatility. Together, then, these results suggest that indexing mortgages to aggregate conditions through ARMs provide important insurance.

## 6 Evaluating New Mortgage Designs

We have so far focused on comparing ARMs and FRMs to clearly elucidate the insurance benefits of indexation. However, ARMs do have some drawbacks. In particular, if the central bank raises interest rates to fight inflation, our assumption that in recession short rates fall may be violated. This was the case in the Volcker recessions in the early 1980s. In these cases, ARMs are worse from an insurance perspective, the reverse of our main results because the covariance of interest rates and income shocks switches, as we show in Section 7. Furthermore, by revealed preference most Americans prefer fixed rate mortgages, which on average account for roughly 80 percent of mortgage originations. ${ }^{22}$

Given these downsides, in this section we evaluate several mortgage designs that allow for state contingency in a crisis while preserving the benefits of FRMs in normal times. We also evaluate a contract that provides more insurance than a standard ARM. In the future, we plan to evaluate shared-appreciation mortgages. Our main finding is that mortgage designs that front-load payment relief in the crisis outperform mortgages that spread payment relief over the life of the mortgage.

### 6.1 Eberly-Krishnamurthy Convertible Mortgage

Eberly and Krishnamurthy (2014) propose a fixed rate mortgage that can at any time be converted to an adjustable rate mortgage, but not back. This is similar to a world in which one can choose between ARM and FRM with two important distinctions. First, homeowners who do not satisfy the LTV constraint can still switch to an ARM. Since in our simulated crisis 70 percent of homeowners have under 20 percent equity, this is likely to be significant. Second, homeowners who are so highly constrained that they cannot afford to pay the fixed costs of refinancing can convert to an ARM costlessly.

[^12]Figure 10: Eberly-Krishnamurthy Convertible Mortgage: Housing Market Outcomes


The figure shows the impulse response to a simulated downturn preceded by an expansion with loose credit under FRM and the Eberly-Krishnamurthy mortgage both ex ante and ex post. In the downturn, the maximum LTV falls from 95.0 percent to 80 percent and the economy falls into a deep downturn for an average of 5.6 years.

Table 3: FRM vs. ARM: Moments From Stochastic Simulations

| Moment | Eberly-Krishnamurthy | FRM Underwater Refi | Option ARM |
| :---: | :---: | :---: | :---: |
| Std Dev of Price | $96.9 \%$ | $100.6 \%$ | $98.2 \%$ |
| St Dev of Default | $73.5 \%$ | $83.7 \%$ | $78.6 \%$ |
| Std Dev of Consumption | $93.9 \%$ | $100.6 \%$ | $91.3 \%$ |

Notes: All series are percentages relative to the same statistic for the indicated FRM model.

This mortgage has three added benefits. First, introducing it in practice is likely to be less disruptive, as it can act exactly like a FRM and does not take away the FRM option. Second, in the event that the covariance of interest rates and monetary policy changes, it performs more like an FRM, as we show in Section 7. Third, to the extent that it cushions bank losses in a crisis, the spread banks charge will fall.

The convertible mortgage does, however, come at a cost as borrowers will convert to an ARM when it is beneficial to them and then refinance into a new loan that starts as an FRM when this is no longer the case. Because ARM rates are lower than FRM rates when rates fall, this behavior reduces the NPV of the mortgage to the lender through standard prepayment risk and results in a higher spread. This lower NPV effect is partially offset by the lower default losses under the ARM. On net, in our model the prepayment risk is larger the Eberly-Krishnamurthy mortgage has a 8.6 basis point higher mortgage rate relative to an FRM.

Figure 10 shows a simulated downturn under the Eberly-Krishnamurthy mortgage, both prior to and after the crisis (as will be the case for our other designs). The crisis looks roughly 75 percent of the way to the all-ARM world and realizes many of the consumption-smoothing and macroprudential benefits of ARMs. This is the case because on average the share of mortgages
that are functioning as ARMs rises from 15 percent pre crisis to over 85 percent during the crisis as households who need the payments relief the most convert. The convertible mortgage does not look fully like an ARM because older, richer household keep to their FRM and because of the higher spread. On net, the consumption equivalent welfare gain is 0.92 percent of consumption, compared to 1.27 percent for all-ARMs.

Table 3 compares the Eberly-Krishnamurthy mortgage with an FRM in stochastic simulations. Again, the convertible mortgage has most of the benefits of the all-ARM economy shown in Table , although quantitatively is not quite as beneficial as the ARM.

### 6.2 Fixed Rate Mortgage With Underwater Refinancing

The second mortgage that we consider is a fixed-rate mortgage that can be refinanced even when the LTV constraint at origination is not satisfied, which we call an FRM mortgage with underwater refinancing. This is similar to the Home Affordable Refinance Program (HARP) pursued in the recent downturn. Campbell (2013) has advocated FRMs with underwater refinancing, which are used in Denmark through a system of covered bonds that allow for refinancing when the LTV constraint at purchase is violated.We implement this in our model by allowing FRM households to refinance - but not originate a mortgage for anew purchase - at a new interest rate as long as they do not reduce their principal.

FRM with underwater refinancing acts like an FRM except it directly addresses the issue of not being able to refinance when one is underwater, undoing the pecuniary default externality in our model that operates through one person's default driving down prices and preventing others from refinancing. Because payments fall in bad states, it should help smooth consumption and reduce default losses for lenders. It will, however, increase lenders prepayment risk. On net, in our model, the FRM with underwater refinancing has a 1.6 basis point higher spread than an FRM.

Figure 11 shows the crisis under FRMs with underwater refinancing. The key to understanding the effects is to note that the FRM with underwater refinancing is priced off the long end of the yield curve, which falls less than the short end in a crisis. Because the insurance it provides is minimal, households do not take on much more risk, and consumption is roughly the same as FRMs in the crisis. The initial decline in prices is slightly shallower because there is initially less default as the initial relief kicks in, but more houses stay in a precarious position and are forced to default when an income shock occurs, so some of the initial benefit in terms of the default rate, consumption, and prices is undone as the crisis does not ameliorate as quickly. Overall, welfare is improved by only 0.13 percent of annual consumption. In normal times, the FRM with underwater refinancing behaves much like an FRM with slightly lower default volatility, as shown in Table 3. Overall, then, allowing for underwater refinancing provides some macroprudential benefits in a crisis but only limited consumption-smoothing benefits because the FRM is still priced off the long end of the yield curve. The comparison of the Eberly-Krishnamurthy mortgage and the FRM with underwater refinancing provides the clearest contrast for the central finding of the paper: that it is best to "front load" payment relief so that it is concentrated in a crisis. Both the Eberly-Krishnamurthy and FRM with underwater refinancing provide some insurance to underwater borrowers, however the Eberly-Krishnamurthy front loads the relief by delivering maximal relief in a recession while the

Figure 11: FRM With Underwater Refinancing: Housing Market Outcomes


The figure shows the impulse response to a simulated downturn preceded by an expansion with loose credit under FRM and the FRM with underwater refinancing both ex ante and ex post. In the downturn, the maximum LTV falls from 95.0 percent to 80 percent and the economy falls into a deep downturn for an average of 5.6 years.

FRM with underwater refinancing gives households a new fixed rate that lasts for longer, thereby spreading the payment relief over the length of the mortgage. From the perspective of a lender who can easily hedge interest rate risk, these two mortgages are quite similar. However, the EberlyKrishnamurthy mortgage helps constrained households in the crisis much more, which helps them smooth consumption, stems the tide of default and the price-default spiral. It also makes buying much more affordable for new homeowners, helping put a floor under house prices. The EberlyKrishnamurthy mortgage does hit household balance sheets more when rates rise, but by then the macroeconomy and housing market have stabilized, and house prices have recovered so that most households can refinance back into a new Eberly-Krishnamurthy that initially functions as an FRM if they so choose. Consequently, designs that front load the benefits of indexation so that they are concentrated in recessions does best in our quantitative analysis.

One aspect of refinancing that this analysis misses is the ability of underwater homeowners to extend their mortgage, as all mortgages amortize over the remaining lifetime of the borrower. For, example, refinancing a 10 year loan into a 30 year loan would substantially lower monthly payments, even if the loans are at the same interest rate and have the same principal (Lucas et al., 2011). Since the key issue during the crisis is liquidity, our analysis shows that such a reduction in payments could deliver significant benefits. However, there are a few important caveats. First, we have shown that the benefits are largest when payment reductions are front-loaded, and term extension reduces payments over the remaining life of the mortgage rather than front-loading them. Thus, if short rates fall by a significant amount during the crisis, underwater homeowners would still be better off with having an EK mortgage and taking the rate reduction, rather than having an FRMUR mortgage and doing a loan extension. Moreover, the benefits of a term extension depend on how many years the borrower has had her current mortgage. Crucially, we have shown
that the households most at risk of default and in need of liquidity are largely young households who have recently purchased a home and households who have experienced negative shocks and recently refinanced. For these households, the term extension would have a small impact as their pre-refinancing mortgage is already a relatively long-term mortgage. On the other hand, the EK mortgage can deliver substantial immediate relief if the monetary authority aggressively lowers short rates.

Finally, the FRM with underwater refinancing is very similar to ex post policies that refinance borrowers who are underwater, such as the Home Affordable Refinance Program pursued in the Great Recession. However, the HARP program occurred at least in part while the Federal Reserve pursued quantitative easing. We assess $\operatorname{HARP}$ and how it interacts with quantitative easing in Section 7.2.

### 6.3 Option ARM

Piskorski and Tchistyi (2010) use an optimal contracting model to argue that the optimal contract looks roughly like an option ARM. This is an ARM mortgage that became popular in the early 2000s boom that allows the borrower to make one of three payments: a fully amortizing payment, an interest only payment, and a potentially negatively amortizing payment equal to the minimum payment based on the interest rate at origination. The negative amortization is allowed up to a ceiling, and after several years the option ARM converts to a fully amortizing ARM. Piskorski and Tchistyi find this is roughly optimal because it provides borrowers with substantial insurance up until a ceiling, which is in place to protect the lender from default.

To mimic the option ARM in our model, we assume that for households in the last 25 years of their life the mortgage behaves like a normal ARM and amortizes. However, households in the first 20 years of life are able to choose a mortgage balance next period equal to the maximum of their current balance and the maximum balance under today's LTV constraint $\phi$. This allows for some negative amortization up to a ceiling defined by $\phi$, and makes it so that households are not margin called by the bank when the LTV constraint tightens at the beginning of the crisis.

The option ARM adds default risk, but the lender also gets increased interest payments in normal times if the loan does not default. In our model, the later slightly outweighs the former, and the spread is 7.1 basis points below the FRM spread.

Table 3 replicates Piskorski and Tchistyi's findings. In normal times, the option ARM provides higher average consumption and most of the reduced volatility that the ARM provides. This is in part because households can borrow more without refinancing, and thus have smoother consumption. However, Figure 12 shows that this comes at the cost of worse performance in the crisis. The option ARM provides more insurance, so households choose more leverage as with the ARM. Additionally, unlucky households who get negative income shocks choose the negative amortization option and increase their LTV further. The increased fragility stemming from more mass at the top of the LTV distribution leads to roughly the same amount of default under FRM. If the social costs of default are high, the option ARM, which performs well in normal times, may be imprudent from a macro perspective relative to other designs that introduce adjustable rates.

Figure 12: Option ARM: Housing Market Outcomes


The figure shows the impulse response to a simulated downturn preceded by an expansion with loose credit under FRM and the option ARM mortgage both ex ante and ex post. In the downturn, the maximum LTV falls from 95.0 percent to 80 percent and the economy falls into a deep downturn for an average of 5.6 years.

## 7 The Interaction of Mortgage Design and Monetary Policy

We now turn to examining how monetary policy interacts with mortgage design in a crisis. We begin by comparing the polar opposite case from our main analysis in which the real interest rate rises in a recession. We then turn to more aggressive monetary policy in a crisis, including quantitative easing. Throughout, we compare economies where all households have the same mortgage design both ex ante and ex post.

### 7.1 Fighting Inflation: Rising Real Rates in a Crisis

In this subsection we consider a housing-led recession with substantial inflation that causes the central bank to raise the real interest rate to contain inflationary pressures. One can think of this as a combination of the housing bust in the Great Recession and the inflation experienced in the 1981-2 recession. To consider this case, we keep the same calibration but increase the real short rate in a crisis to 5.6 percent, which is the average real rate in the 1981-2 recession instead of 0.26 percent. ${ }^{2324}$

Figure 13 shows how ARM, FRM, EK, and FRMUR in a simulated crisis. Because the covariance of the short rate and income has the opposite sign of before, an ARM makes the household's income stream net of mortgage payments more volatile and leads to higher price volatility and

[^13]Figure 13: Short Rate Rises in Crisis as in 1981-2 Recession


Note: The figure shows the outcomes in a simulated downturn in which the maximum LTV falls from 95.0 percent to 80 percent and there is a five year deep downturn under an FRM, ARM, EK, and FRMUR for the case where the central bank raises the short rate to 5.6 percent, the average real rate in the 1981-2 recession, in crisis states. The long rate adjusts according to the expectations hypothesis.
default and lower consumption. Quantitatively, prices fall by 5.9 percentage points more under ARM, and 11.8 percent of the housing stock defaults over eight years under ARM relative to 6.7 percent under FRM. On net, under ARM, welfare is lower by an equivalent of 1.43 percent of annual consumption.

The Eberly-Krishnamurthy convertible mortgage is in between FRM and ARM. Prices fall by 2.4 percentage points more than under FRM, and the cumulative default rate over eight years is 8.3 percent. Most households that take out a new mortgage or refinance do not convert their EK mortgage from fixed-rate to adjustable-rate, and the ARM share at the beginning of the crisis is 16.5 percent. However, a few households, who tend to be older and richer, converted their EK mortgage to adjustable-rate and did not refinance into a new EK mortgage that begins as a fixedrate prior to the crisis. Some of these households experience an adverse income shock in the crisis, are underwater, and end up defaulting, which is why default is higher and prices fall more by under FRM. That being said, the crisis is nowhere near as severe as under ARM because the youngest and most constrained households have an EK mortgage that is functioning as a fixed rate, which is why EK provides a nice balance of insuring against catastrophe in an inflationary environment while providing insurance when real rates fall in a crisis. Indeed, the consumption-equivalent welfare loss in the high inflation scenario under the EK mortgage is only 0.5 percent of annual consumption, while the gain in the baseline scenario is 0.92 percent of consumption for each year of the crisis.

### 7.2 Unconventional Monetary Policy Easing in a Crisis

We now consider alternative monetary policy in a crisis by studying three modifications to the central bank's behavior in the baseline scenario. In the first modification, we consider expansionary

Figure 14: Monetary Policy: Short Rate Falls By More in Crisis


Note: The figure shows in solid lines the outcomes in a simulated downturn in which the maximum LTV falls from 95.0 percent to 80 percent and there is a five year deep downturn under an FRM and ARM or the case where the central bank further reduces the short rate by 100 basis points but the long rate only responds modestly as it is set according to the expectations hypothesis. The baseline scenario without the additional 100 basis point reduction is shown in dashed lines for comparison.
monetary policy in crisis states whereby the central bank lowers the short rate and consequently ARM mortgage rates by an additional 100 basis points, but this change is only partially passed through to long interest rates, which adjust according to the expectations hypothesis. We think of this as corresponding to more aggressive traditional monetary policy. In the second modification, we assume that in addition to adjusting the short rate, in crisis states only the central bank takes actions that push the long rate down an additional 100 basis points beyond the expectations hypothesis response. We think of this as corresponding to unconventional policies that seek to affect the long end of the yield curve or long-term mortgage rates directly, such as quantitative easing. The third modification assumes that the central bank is doveish and pursues the second policy that combines short rates and QE not only in rare crises but also in more common regular recessions.

The results of the first modification, which reduces only short rates, are shown in Figure 14. Traditional monetary policy has very little impact on the severity of the crisis under FRM. While the return to saving in liquid assets changes, the FRM interest rate does not change appreciably since FRMs are priced off the long end of the yield curve. Since few households have liquid assets and most saving in the economy is for retirement, housing demand by young households and the behavior of high-LTV households that are primarily young is unchanged, and so the housing market equilibrium is not substantially affected.

On the other hand, more aggressive monetary policy that affects short rates is useful when homeowners have adjustable-rate mortgages. Price declines and default are lower and consumption is slightly higher in response to the more aggressive monetary response. This is, of course, not surprising. Lower short-rates leads to lower mortgage payments which leads to less default and a smaller price-default spiral.

Figure 15: Monetary Policy: Long Rate Falls More Than Expectations Hypothesis (QE)


Note: The figure shows the outcomes in a simulated downturn in which the maximum LTV falls from 95.0 percent to 80 percent and there is a five year deep downturn under an FRM and ARM, for the case where the central bank both further reduces the short rate by 100 basis points and pursues a policy that subsidizes the long rate by an additional 100 basis points in a crisis. The baseline scenario without the additional 100 basis point reduction is shown in dashed lines for comparison.

Figure 16: Monetary Policy: FRM With Underwater Refi With and Without QE


Note: The figure shows the outcomes in a simulated downturn in which the maximum LTV falls from 95.0 percent to 80 percent and there is a five year deep downturn under an FRM with underwater refinancing for the baseline monetary policy, the case where the central bank both further reduces the short rate by 100 basis points, and a policy where the central bank reduces the short rate by 100 basis point and subsidizes the long rate by an additional 100 basis points in a crisis.

Figure 17: Monetary Policy: Doveish Central Bank


Note: The figure shows the outcomes in a simulated downturn in which the maximum LTV falls from 95.0 percent to 80 percent and there is a five year deep downturn under an FRM and ARM for the case where the central bank both further reduces the short rate by 100 basis points and pursues a policy that subsidizes the long rate by an additional 100 basis points both in a crisis and in regular recessions. The baseline scenario without the additional 100 basis point reduction is shown in dashed lines for comparison.

When aggressive monetary policy in a crisis is passed through to long rates and FRM interest rates, there are smaller price declines in a world with all fixed-rate mortgages. We demonstrate this in Figure 15. However, the mechanism is quite different from the comparison of ARMs and FRMs. The payments of existing homeowners do not fall, and many remain liquidity constrained. Indeed, there is still significant default. Instead, new homeowners can now lock in cheap financing, which stimulates demand for housing and boosts house prices. Some exiting homeowners are no longer bound by the LTV constraint and are able to refinance due to the rise in prices, undoing some of the price-default spiral. In terms of welfare, households would be willing to pay an average of 0.94 percent more of their consumption to avoid a crisis under FRM relative to ARM, compared to 1.27 percent with the baseline monetary policy.

Our central finding about the added benefits of front loading payment relief suggests that ex post policies such as HARP, which setting aside implementation frictions is the same as FRM with underwater refinancing, are maximally effective only if paired with policies to push down long rates such as QE. To show this quantitatively, Figure 16 compares the FRM with underwater refinancing under three different monetary policies: the baseline policy, the more aggressive short rate policy, and the more aggressive short rate policy coupled with a policy that pushes down the long rate further such as quantitative easing. One can see that the benefits of the FRM with underwater refinancing are stronger for prices, default, and consumption when coupled with QE. Indeed, consumption equivalent welfare rises by an additional one percent of consumption per year of the crisis. HARP also works through a term extension effect that works through the same monthly payment channel we have analyzed that is often overlooked (Lucas et al., 2011).

Finally, Figure 17 shows the case where the central bank is doveish and pursues a more aggressive
monetary policy not only in rare crises but also in more frequent, garden-variety recessions. In this case, the crisis is much worse particularly for FRMs, with larger price declines, more default, and a greater consumption decline for homeowners. In terms of consumption equivalent welfare, the ARM world is now 1.49 percent better than the FRM world, compared to 1.27 percent with the baseline monetary policy. Households expect an activist central bank to stabilize the housing market, and they respond by taking on more debt, much like how with ARMs ex ante households take on more debt anticipating the hedging benefits of an ARM in a downturn. This highlights that aggressive monetary policy can backfire if it is anticipated and used too frequently.

## 8 Conclusion

We assess how can mortgages be redesigned or modified in a crisis to reduce housing market volatility, consumption volatility, and default and how mortgage design interacts with monetary policy. To do so, we construct a quantitative equilibrium life cycle model with aggregate shocks in which households have realistic long-term mortgages that are priced by risk-neutral and competitive lenders and household decisions aggregate up to determine house prices. We calibrate the model to match aggregate moments as well as quasi-experimental evidence on the effect of payment size and LTV on default so that our model is tailored to qualitatively assess the benefits of adding simple state contingency to mortgage contracts.

We use the model to assess the performance of various mortgage contracts in a realistic, recession-driven housing crisis. We compare simple adjustable-rate and fixed-rate mortgages to elucidate the economics of adding state contingency to mortgages. We find that adjustable-rate mortgages have substantial hedging benefits relative to fixed rate mortgages. If the central bank reduces interest rates in response to the crisis, mortgage payments automatically fall, which helps to smooth consumption, limits default by relaxing budget constraints in bad states, and stimulates housing demand by new homeowners. These hedging benefits are quite large for constrained, high LTV households who bear the brunt of the housing bust, making the overall welfare benefit to the economy of switching to ARMs large, approximately 3.1 percent of consumption each year of a housing bust. Crucially, these benefits depend on the extent to which the insurance provided by ARMs is anticipated by households, as households take on more debt when they expect their payments to fall in a crisis, leading to more macro fragility.

We then assess several proposed mortgage designs that add simple state contingency to standard mortgages. These designs are meant to improve on FRMs in the type of crisis that is our main focus while also performing well in normal times and in recessions where the central bank raises rates to fight inflation. We find an FRM with the option to costlessly convert to an ARM provides the best combination of insurance and macroprudential benefits. An FRM that can be refinanced underwater also provides macroprudential benefits but to a lesser degree since the benefits of spread out over time instead of front loaded, highlighting our central finding that it is best to concentrate payment relief in the crisis. Finally, option ARMs also can provide insurance but at the cost of significant macroprudential fragility.

Our analysis highlights the interaction between mortgage design and monetary policy. The
relative benefits of different mortgage designs depend on the conduct of monetary policy. The macroeconomic tradeoffs driving monetary policy depend on the mortgage designs adopted by agents. While we have not studied optimal monetary policy, our analysis, such as the quantitative easing experiment, elucidate this two-way interaction.

In a future draft we hope to improve the analysis along a number of dimensions. First, we plan to have lenders set spreads separately for each aggregate state rather than pooling risk across aggregate states. Second, we plan to consider shared appreciation mortgages. Third, we plan to do more to evaluate the interaction of mortgage design with monetary policy.

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[^1]:    ${ }^{1}$ For instance, Kaplan et al. (2016) assume that all mortgages have a single interest rate and that lenders break even by charging differential up front fees. By contrast, we maintain each borrower's interest rate and contract choice as a state variable.

[^2]:    ${ }^{2}$ The term $\alpha_{a}$ describes the utility from homeownership as a function of age. In our calibration, we will assume that $\alpha_{a}$ is decreasing in age so as to reflect the fact that at older ages the homeownership rate declines slightly.
    ${ }^{3}$ Including terminal wealth in the utility function is standard in OLG models of the housing market because otherwise households would consume their housing wealth before death. However, in the data the elderly have substantial housing wealth which they do not consume. The functional form for the utility derived from terminal wealth is standard.

[^3]:    ${ }^{4}$ We assume that houses are one size to maintain a computationally tractable state space in an environment with rich mortgage design. In practice, the average house size does grow over the life cycle with age (see e.g., Li and Yao, 2007) and house size grows with income. Assuming one house size leads richer agents in our economy to have more liquid assets than in the data and pay down their mortgage too quickly. This is not problematic for our calibration as the marginal agents for purchasing and default are poorer.
    ${ }^{5}$ We assume a fixed housing supply to keep the model tractable given the lags required to realistically model construction. Adding a construction response would dampen a boom but would not dramatically affect busts given the durability of housing (Glaeser and Gyourko, 2005).

[^4]:    ${ }^{6} i^{F R M}$ and $i^{A R M}$ represent the long and short mortgage rates, respectively, and different mortgage designs may have borrowers borrowing at $i^{F R M}$ and $i^{A R M}$ at different times.
    ${ }^{7}$ When we refer to an ARM in this paper, we refer to a fully-adjustable-rate mortgage that adjusts every year. In many countries, hybrid ARMs that have several years of a fixed interest rate and float thereafter are known as "adjustable rate." Aside from replicating the Fuster-Willen quasi-experiment in evaluating our calibration, we do not consider hybrid ARMs to maintain a tractable state space.
    ${ }^{8}$ The assumption that $d$ is drawn from a distribution rather than a single value helps smooth out the value functions in the numerical implementation, but is not crucial for our results. In practice, $d_{a}$ and $d_{b}$ are close and the model is essentially to a single default cost model.

[^5]:    ${ }^{9}$ We have found that a linear spline performs better than a linear relationship. The relationship is approximately linear in periods with no default and linear in periods with some default, although the line bends when default kicks in. A linear spline flexibly captures this relationship. We use the discretized price grid for our spline knot points.

[^6]:    ${ }^{10}$ We use a real model to focus the model on the benefits of interest-rate indexation in a scenario like the Great Recession. Indeed, our central points are fairly orthogonal to the literature on "mortgage tilt" and inflation, with the exception of the possibility that adjustable-rate borrowers may see their payments rise if inflation is high in a crisis and the central bank raises interest rates. We consider such a scenario in section 7 .
    ${ }^{11}$ Our model abstracts from regional heterogeneity in the strength of housing cycles and recessions. Given such heterogeneity, monetary policy - and mortgage design - is a somewhat blunt instrument because it does not treat different regions differently. See Beraja et al. (2017) for an analysis of regional heterogeneity and monetary policy in a model in which the mortgage market is important for the transmission of monetary policy.
    ${ }^{12}$ In practice, interest rates adjust gradually to the aggregate state of the economy. We assume immediate adjustment to keep the number of aggregate states tractable. With gradual adjustment, ARMs would provide less insurance. This is another plus for the Eberly-Krishnamurthy mortgage design that allows conversion between FRM and ARM, as agents may want to keep their FRM if ARM rates are not adjusting or are adjusting the wrong way.
    ${ }^{13}$ In practice, the 30 -year fixed rate mortgage is priced off of the 10 -year Treasury bond. We use the expectations hypothesis because we need to have counterfactual predictions of how changes in short rates are passed through into long rates and the literature on time-varying term premia remains unsettled. If term premia shocks are random, we can ignore them. There is some evidence that term premia are higher in recessions, which would mean long rates move less than in our calibration. By contrast, there is some evidence that term premia overreact to monetary policy at short horizons, which would mean that long rates move more than in our calibration.

[^7]:    ${ }^{14}$ Rather than including a deterministic income profile, we start households at lower incomes and let them stochastically gain income over time as the income distribution converges to its ergodic distribution. This does a good job of matching the age-income profile in the data.

[^8]:    ${ }^{15}$ Much of the literature calibrates to the "loss severity rate" defined as the fraction of the mortgage balance recovered by the lender. We calibrate to a fraction of the price because of a recent empirical literature that finds that in distressed markets, the loss recovery rate is much lower (e.g. Andersson and Mayock, 2014), which is consistent with a discount relative to price rather than a constant loss severity rate.
    ${ }^{16}$ We choose $d_{a}$ and $d_{b}$, the bounds of the uniform distribution from which $d$ is chosen, to add a small bit of mass around $\bar{d}$. In the calibration, $\bar{d}=40, d_{a}=35$, and $d_{b}=45$.

[^9]:    ${ }^{17}$ This figure is based on the default hazard in months 30 to 60 in Figure 1b. Fuster and Willen measure "default" as becoming 60 days delinquent rather than an actual foreclosure, so the actual default rate might be slightly lower.

[^10]:    ${ }^{18}$ In the SCF, we find that the ALT-A borrowers have low assets, are young, and have moderate-to-low income, as one would expect. We assume a uniform distribution between the 25 th and 75 th percentiles of age and assets in the SCF data and assume that individuals in the Fuster-Willen experiment have moderate to low income.
    ${ }^{19}$ Fuster and Willen find a substantial default hazard below 100 percent LTV for three likely reasons. First, the combined LTV that Fuster and Willen use is likely measured with error both due to missing liens in the data and due to error in the automatic valuation model. Second, Fuster and Willen measure default as 60 day delinquency and not a final foreclosure. In areas with substantial foreclosure backlogs, borrowers who are above water can become delinquent before they sell. Finally, there are some frictions in terms of time to sell and the fixed costs of sale that may cause above-water households to default. In our model, there is minimal default for above water households because some households get a moving shock with very low equity and decide to default.

[^11]:    ${ }^{20}$ Our model does not match the time series of house price indices, because prices fall to their lowest level on the impact of the shock, while in the data they decline gradually. This is the case because our Walrasian model does not have any house price momentum. See Guren (2017) for a summary of the literature on momentum.
    ${ }^{21}$ With a 2.5 percent annual trend, real personal consumption expenditures fell 9.4 percent relative to trend peak to trough in the recession. Our model may have a larger decline than in the data because we do not capture the upper tail of the income distribution well and because we do not model lenders.

[^12]:    ${ }^{22}$ If we allow for both FRMs and ARMs in our model and do not price either mortgage, about half of mortgage holders choose an ARM. However, with a small 20 basis point subsidy for the FRM we obtain a realistic ARM share. This subsidy may come from two sources. First, ARM borrowers are riskier, and in a future draft, we plan to price the ARM relative to the FRM and expect the ARM to have an endogenously higher spread. Second, the FRM is institutionalized by Fannie Mae and Freddie Mac, which provide a small subsidy for conforming (predominantly FRM) loans.

[^13]:    ${ }^{23}$ Combining the $2006-8$ credit crunch with the $1981-2$ recession results in rather extreme event, especially given that the Federal Reserve may not have raised rates as much in the $1981-2$ recession in an all-ARM world. We nonetheless evaluate this extreme event in order to stress-test our mortgage designs.
    ${ }^{24}$ Many ARMs have a ceiling on the amount that the interest rate can rise over the course of the loan. In this counterfactual, we assume that there are no such ceilings. If there were, the increase in the ARM interest rate would be smaller, but knowing this banks would increase the ex ante mortgage spread.

