

# Endogenous Life-Cycle Housing Investment and Portfolio Allocation

Cengiz Tunc<sup>1</sup> Denis Pelletier<sup>2</sup>

<sup>1</sup>Central Bank of the Republic of Turkey

<sup>2</sup>North Carolina State University

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# Motivation

- Two Empirical Observations
  - Low stock market participation rate
  - Moderate equity holding for the participants
  - 2007 SCF shows that only 55% of US households have direct or indirect holdings of risky asset
  - 1968-2007 PSID data shows median household's risky asset holding is zero
- Theoretical models with the prevailing equity premium assumption predict almost 100% of households should hold risky assets as the bulk of their financial portfolio.

# Motivation

- An important reason for the gap between the models and the empirical fact is housing investment as about 2/3 of the US households are homeowner.
- A typical household has a higher priority to be homeowner than investing in the stock market
- Dual benefits of housing
  - Durable consumption good from which owners derive utility
  - Investment tool that enables owners to hold home equity

# My Research

- Incorporate a comprehensive housing investment into a life-cycle asset allocation model to address these empirical observations.
- By using Epstein-Zin recursive preferences we estimate relative risk aversion and elasticity of intertemporal substitution parameters separately.
- Compare the portfolio allocation profiles under both the Epstein-Zin recursive preferences and CRRA form

# Literature

- Growing literature on housing-included portfolio allocation models : Grossman and Laroque (1990), Flavin and Nakagawa (2008), Longstaff (2009), Piazzesi et. all. (2007).
- Some papers closer to our research
  - (Hu, 2005), (Cocco, 2004), and (Yao and Zhang, 2005)
- Our contribution
  - Introducing a fairly comprehensive housing feature,
  - Estimating RRA and EIS by using Epstein-Zin preferences
  - Matching the empirical observations fairly well.

# Model

- Discrete time, each period is one year
- Households enter the market at age of 20 and live for up to 80 years

$$V_t = \left\{ u(C_t, H_t)^{\frac{1-\gamma}{\theta}} + \beta \left( E_t \left[ q_{t+1} V_{t+1}^{1-\gamma} + (1 - q_{t+1}) W_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \quad (1)$$

$$\theta = \frac{1 - \gamma}{1 - 1/\psi} \quad (2)$$

$$u(C_t, H_t) = [C_t^\psi + H_t^\psi]^{\frac{1}{\psi}}, \quad (3)$$

$$y_{it} = F_t + u_{it}^l, \quad (4)$$

$$F_t = \beta_0 + \beta_1 \text{age} + \beta_2 \text{age}^2 + \beta_3 \text{gender} + \beta_4 \text{marital\_status} + \beta_5 \text{educ}, \quad (5)$$

$$\eta_t^l = \rho \eta_{t-1}^l + w_t^l, \quad (6)$$

# Model

- All households are initially renter, homeownership decision is endogenously made in each period starting from the second period..
- Homeowners pay down payment, annual mortgage payments (prin. and int.), and M&D. expenses.
- Renters only pay annual rent.
- Two types of financial assets
  - Riskless asset with annual constant gross return  $R^b$
  - Risky asset with annual stochastic return
 
$$R_{t+1}^s - R^b = \mu_s + \varepsilon_{t+1}^s \text{ with } \varepsilon_{t+1}^s \text{ iid } (0, \sigma_{\varepsilon^s}^2).$$
- Per unit price of housing is denoted by  $P_t^h$  such that a house of size  $H_j$  has price  $P_t^h H_j$
- $\Delta p_t^h = \mu_h + \varepsilon_t^h, \varepsilon_t^h \text{ iid } (0, \sigma_{\varepsilon^h}^2).$
- Households may borrow up to the house value minus the down payment:  $RM_t \leq (1 - d)P_t^h H_j.$

# Budget Constraints

## ● Renters

$$X_t = R^b B_{t-1} + R_t^s S_{t-1} + Y_t \quad (7)$$

$$X_t = C_t + S_t + B_t + \text{FIX}_t \alpha_F Y_t + (1 - \text{HR}_t) \left[ \alpha_R P_t^h H_t \right] + \quad (8)$$

$$\text{HR}_t \left[ M_t + dP_t^h H_t \right] \quad (9)$$

## ● Homeowners

$$X_t = R^b B_{t-1} + R_t^s S_{t-1} + Y_t \quad (10)$$

$$X_t = C_t + S_t + B_t + \text{FIX}_t \alpha_F Y_t + \text{HR}_t (1 - \text{MS}_t) \left[ M_t + \delta P_t^h H_{t-1} \right] + \\ \text{HR}_t \text{MS}_t \left[ dP_t^h H_t + M_t + \delta P_t^h H_{t-1} - \left( (1 - \kappa) P_t^h H_{t-1} - \text{RM}_t \right) \right] + \\ (1 - \text{HR}_t) \text{MS}_t \left[ \alpha_R P_t^h H_t - \left( (1 - \kappa) P_t^h H_{t-1} - \text{RM}_t \right) \right], \quad (11)$$



# Optimization Problem

- The state variables are denoted by  $\Omega$ , where  $\Omega_t = \{t, LW_t, IFIX_t, O_{t-1}, H_{t-1}, RM_t\}$ .
- The control variables are denoted by  $\Psi_t = \{C_t, O_t, H_t, FIX_t, s_t\}$
- Household's optimization problem is then

$$V_t(\Omega_t) = \max_{\Psi_t} \left\{ u(C_t, H_t)^{\frac{1-\gamma}{\theta}} + \beta \left( E_t \left[ q_t V_{t+1}(\Omega_{t+1})^{1-\gamma} + (1 - q_t) W_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \quad (12)$$

subject to dynamics, restrictions and budget constraints mentioned above.

# Parameters

**Table:** Baseline Parameter Values

Description	Parameter	Value
Time discount factor	$\beta$	0.95
Gross return on the riskless asset	$R^b$	1.03
Equity premium	$\mu_s$	0.06
Liquidation cost	$\kappa$	0.10
Intratemporal elasticity of substitution	$\nu$	0.33
Rental rate	$\alpha_R$	0.06
Mortgage rate	$r^m$	0.03
Fixed entry cost	$\alpha_F$	0.05
Down payment	$d$	0.20
Depreciation and maintenance	$\delta$	0.01
Average growth rate on housing prices	$\mu_h$	0.01
Std. of persistent shock to labor income	$\sigma_{l_2}$	0.1632
Std. of temporary shock to labor income	$\sigma_{h_1}$	0.3272
Std. of shocks to return on housing inv.	$\sigma_h$	0.057
Std. of shocks to return on risky asset inv.	$\sigma_s$	0.20
Retirement income factor	$\xi$	0.66
Persistence parameter of labor income shocks	$\phi$	0.82
Correlations between shocks		
- return on housing and return on risky asset inv.	$\rho_{sh}$	0.20
- return on housing investment and labor income	$\rho_{hl}$	0.075
- return on risky asset inv. and labor income	$\rho_{sl}$	0.10

# Solution Technique

- Use numerical approximation method
- Backward induction due to the finite nature of the problem
- At time  $T + 1$ , value function coincides with the bequest function
- Obtain utility function for all combinations of relevant control variables
- Value function is the sum of the utility function plus the discounted expected continuation function

# Solution Technique

- Cubic spline interpolation if the continuation value doesn't lie on the state space grid
- Choose the maximum value function among all control variables
- The optimum policy rules for consumption, housing, and financial asset investments correspond to these chosen value functions
- Risky assets market decision: compare VF conditional on having paid the fixed cost vs not having paid the fixed cost
- Homeownership decision: compare VF conditional on being a homeowner vs. on being a renter
- Simulate life-cycle consumption, housing investment, and portfolio allocation profiles from these rules

# Estimation

- Conflicting evidence on the size of the RRA and the EIS:
- (Vissing-Jorgensen, JPE 2002), (Guvenen, JME 2003), (Hall, JPE 1988), (Vissing-Jorgensen and Attanasio, AER-PP 2003), and (Gomes and Michaelides, JF 2005).
- Use the minimum distance method for the estimation

# Estimation

Let us denote by  $w_{i,t}$  the value taken by variable  $i$  at age  $t$  over the life-cycle. Let us also denote by  $g_{i,t}(\gamma, \psi)$  the predicted value generated by the model for variable  $i$  at age  $t$  for a given value of the parameters  $\gamma$  and  $\psi$ .<sup>1</sup> A consistent estimator of the RRA and EIS parameters is then given by

$$(\hat{\gamma}, \hat{\psi}) = \arg \min_{\gamma, \psi} \sum_{i=1}^N \sum_{t=1}^K (w_{i,t} - g_{i,t}(\gamma, \psi))^2, \quad (13)$$

where  $N$  is the number of variables used in the minimum distance estimation.

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<sup>1</sup>We simulate a very large number of life-cycle paths (50,000) and use the average as the predicted value.

# Estimation

- Risky asset as a share of total financial assets, risky asset market participation rate, and homeownership status
- We use SCF 2007 for the data because SCF is probably the most comprehensive survey on US households financial assets.
- We define total investment in risky assets as the sum of stock mutual funds, bond mutual funds, mortgage-backed bonds, corporate bonds, publicly traded stocks and foreign bonds.
- On the other hand, riskless asset are composed of checking accounts, certificate of deposits, government-backed bond mutual funds, US Government bonds, saving bonds, municipal bonds, and cash and call money accounts.
- For some assets, households are asked how they are invested: either in risky, or riskless, or split between them. Depending on the answers we classify IRA-Keogh accounts, pensions and saving-money market accounts either risky, riskless, or split between both.

# Estimation

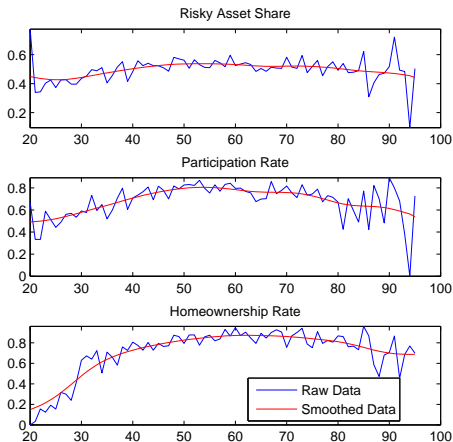


Figure: Raw and Smoothed Data for the Estimation



# Results

Estimated RRA and EIS are 3.78 and 0.34, respectively.

**Table:** Life-Cycle Profiles - Baseline Model

Age	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Risky Asset Share (Uncond.)	0.15	0.24	0.41	0.49	0.47	0.37	0.31	0.3
Risky Asset Share (Cond.)	0.33	0.47	0.63	0.66	0.7	0.72	0.65	0.62
Participation Rate	0.39	0.51	0.64	0.75	0.68	0.52	0.48	0.49
Homeownership Rate	0.32	0.7	0.73	0.74	0.74	0.78	0.86	0.94

# Results

**Table:** Life-Cycle Profiles - No Housing Investment

Age	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Risky Asset Share (Uncond.)	0.75	0.96	0.98	0.98	0.96	0.83	0.67	0.53
Risky Asset Share (Cond.)	0.77	0.97	0.98	0.98	0.96	0.88	0.78	0.68
Participation Rate	0.88	1	1	1	1	0.94	0.86	0.78

**Table:** Life-Cycle Profiles - Baseline Model

Age	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Risky Asset Share (Uncond.)	0.15	0.24	0.41	0.49	0.47	0.37	0.31	0.3
Risky Asset Share (Cond.)	0.33	0.47	0.63	0.66	0.7	0.72	0.65	0.62
Participation Rate	0.39	0.51	0.64	0.75	0.68	0.52	0.48	0.49
Homeownership Rate	0.32	0.7	0.73	0.74	0.74	0.78	0.86	0.94

# Results

Table: Life-Cycle Profiles - SCF Data

Age	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Risky Asset Share (Uncond)	0.32	0.38	0.46	0.49	0.46	0.43	0.4	0.38
Risky Asset Share (Cond)	0.43	0.47	0.52	0.54	0.52	0.52	0.49	0.46
Participation Rate	0.52	0.64	0.75	0.8	0.77	0.73	0.64	0.58
Homeownership Rate	0.27	0.61	0.78	0.85	0.87	0.84	0.76	0.69

Table: Life-Cycle Profiles - Baseline Model

Age	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Risky Asset Share (Uncond.)	0.15	0.24	0.41	0.49	0.47	0.37	0.31	0.3
Risky Asset Share (Cond.)	0.33	0.47	0.63	0.66	0.7	0.72	0.65	0.62
Participation Rate	0.39	0.51	0.64	0.75	0.68	0.52	0.48	0.49
Homeownership Rate	0.32	0.7	0.73	0.74	0.74	0.78	0.86	0.94

# Results

**Table:** Life-Cycle Profiles - House Price Risk

Age	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
<b>Panel A: Risky Asset Share (Cond.)</b>								
Std. of House Price								
0.20	0.44	0.7	0.84	0.91	0.92	0.91	0.82	0.72
0.16	0.43	0.56	0.76	0.85	0.87	0.88	0.79	0.7
0.12	0.59	0.51	0.7	0.77	0.79	0.81	0.73	0.63
0.08	0.33	0.44	0.64	0.69	0.78	0.88	0.8	0.67
0.04	0.48	0.56	0.66	0.71	0.79	0.92	0.86	0.65
0.00	0.32	0.5	0.63	0.56	0.7	0.91	0.85	0.61
<b>Panel B: Participation Rate</b>								
Std. of House Price								
0.20	0.36	0.68	0.82	0.91	0.9	0.8	0.71	0.63
0.16	0.38	0.61	0.76	0.87	0.84	0.74	0.66	0.61
0.12	0.42	0.53	0.69	0.78	0.74	0.58	0.51	0.5
0.08	0.39	0.51	0.6	0.65	0.57	0.39	0.36	0.44
0.04	0.43	0.52	0.58	0.6	0.52	0.35	0.31	0.42
0.00	0.41	0.49	0.56	0.77	0.6	0.28	0.25	0.38
<b>Panel C: Homeownership Rate</b>								
Std. of House Price								
0.20	0.49	0.69	0.46	0.3	0.37	0.52	0.69	0.88
0.16	0.47	0.72	0.56	0.4	0.44	0.56	0.72	0.89
0.12	0.36	0.73	0.64	0.59	0.64	0.77	0.85	0.93
0.08	0.32	0.7	0.73	0.73	0.74	0.79	0.87	0.95
0.04	0.26	0.66	0.73	0.74	0.76	0.82	0.89	0.96
0.00	0.25	0.64	0.77	0.80	0.80	0.85	0.90	0.96

# Results

Table: Life-Cycle Profiles - Entry Cost

Age	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
<b>Fixed Cost</b>	<b>Panel A: Risky Asset Share (Cond.)</b>							
0	0.65	0.57	0.6	0.65	0.74	0.87	0.81	0.63
0.02	0.64	0.57	0.6	0.65	0.74	0.87	0.8	0.63
0.04	0.64	0.56	0.6	0.65	0.74	0.87	0.8	0.63
0.06	0.65	0.57	0.61	0.66	0.75	0.87	0.81	0.63
0.08	0.65	0.57	0.65	0.7	0.78	0.89	0.82	0.65
0.10	0.65	0.57	0.66	0.7	0.78	0.89	0.82	0.65
<b>Fixed Cost</b>	<b>Panel B: Participation Rate</b>							
0	0.44	0.52	0.57	0.61	0.51	0.33	0.3	0.42
0.02	0.43	0.52	0.58	0.61	0.52	0.33	0.3	0.42
0.04	0.43	0.52	0.58	0.61	0.52	0.33	0.3	0.42
0.06	0.43	0.53	0.58	0.62	0.52	0.34	0.3	0.43
0.08	0.43	0.53	0.61	0.64	0.56	0.39	0.35	0.45
0.10	0.42	0.53	0.61	0.65	0.56	0.39	0.35	0.45
<b>Fixed Cost</b>	<b>Panel C: Homeownership Rate</b>							
0	0.32	0.72	0.77	0.78	0.79	0.84	0.9	0.96
0.02	0.32	0.71	0.77	0.78	0.79	0.84	0.89	0.96
0.04	0.32	0.71	0.77	0.78	0.79	0.84	0.9	0.96
0.06	0.31	0.7	0.77	0.77	0.78	0.83	0.89	0.96
0.08	0.32	0.71	0.72	0.72	0.74	0.8	0.87	0.95
0.10	0.31	0.7	0.71	0.72	0.74	0.8	0.87	0.95

# Results

Table: Homeowners and Renters

Age	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
<b>Homeowners</b>								
Risky Share(Uncond.)	0.12	0.15	0.21	0.31	0.28	0.21	0.22	0.28
Risky Share(Cond.)	0.37	0.33	0.4	0.47	0.5	0.54	0.56	0.61
Participation Rate	0.31	0.45	0.52	0.65	0.57	0.38	0.39	0.46
<b>Renters</b>								
Risky Asset Share (Uncond.)	0.18	0.47	0.91	1	0.99	0.97	0.85	0.85
Risky Asset Share (Cond.)	0.37	0.7	0.97	1	0.99	0.98	0.85	0.85
Participation Rate	0.49	0.66	0.94	1	1	1	1	1

# Results

Estimated RRA and EIS parameters are 3.14 and 0.31, respectively.

**Table:** Life-Cycle Profiles - CRRA

Age	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Risky Asset Share (Uncond)	0.16	0.36	0.54	0.65	0.59	0.44	0.47	0.51
Risky Asset Share (Cond)	0.35	0.61	0.72	0.75	0.75	0.75	0.8	0.86
Participation Rate	0.4	0.58	0.74	0.87	0.78	0.59	0.58	0.6
Homeownership Rate	0.28	0.6	0.75	0.72	0.7	0.76	0.77	0.93

**Table:** Life-Cycle Profiles - Baseline Model (RRA=3.78, EIS=0.34)

Age	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Risky Asset Share (Uncond.)	0.15	0.24	0.41	0.49	0.47	0.37	0.31	0.3
Risky Asset Share (Cond.)	0.33	0.47	0.63	0.66	0.7	0.72	0.65	0.62
Participation Rate	0.39	0.51	0.64	0.75	0.68	0.52	0.48	0.49
Homeownership Rate	0.32	0.7	0.73	0.74	0.74	0.78	0.86	0.94

# Conclusion

- First, we show that housing investment has strong crowding out effect on investment in risky assets and this effect is observable throughout the life-cycle.
- The unconditional and conditional risky shares and the participation rate over the life-cycle under the EZ preferences matches the empirical data better than the same share under the CRRA utility form while we observe similar patterns for the homeownership rate for these two cases.
- Third, the effect of the presence of housing investment on households portfolio allocation is larger than the effect of disentangling the relative risk aversion from the intertemporal elasticity of substitution through using EZ recursive preferences.
- We further find that the size of the fixed entry cost on risky asset investment has limited impact on portfolio allocation.