Banking leverage with complete diversification.

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ABSTRACT:

The brutal adjustments of global banks' balance sheet regarding the economic activity have rekindled discussions about the procyclicality of the leverage. During an economic burst, the collateral value of banks decreases and their risk-taking capacity is reduced. Banks raise less funds and their leverage defined as total asset over equity goes down. The leverage is pro-cyclical. As the current literature focuses on framework with a single currency of denomination for assets and debts, I complete this gap by introducing a second currency of denomination in both sides of the balance sheet. This contribution is in line with the funding and investing strategies of European global banks which diversify part of their balance sheet with US dollar. Finally, diversification would alter the leverage's procyclicality depending on shocks.

JEL classification: F3, F4, G15

Keywords: liability, pro-cyclical leverage, global banks, diversification, currency mismatch.

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1 Introduction

The precrisis banking development followed by the crisis downturn have recently bring to researchers' attention the banking leverage adjustments. As developed by Adrian and Shin [2013], the leverage of banks which is defined as the ratio of total assets to equity is procyclical. This procyclicality comes from two specifications. First, banks' balance sheets are marked to market. Thus, an improvement of the economic activity increases their net worth. Second, banks are active in the management of their balance sheet. They reallocate the increase of their net worth in additional investment. The leverage increases. Figure 1 illustrates balance sheet's adjustment following an improvement of the economic activity. The procyclicality of the leverage is also related to less recent models of financial accelerators developed by Bernanke and Gertler [1989] and Kiyotaki and Moore [1997].

As banks are active in the management of their balance sheet, they behavior is compatible with a Value at Risk (VaR) rule. Adrian and Shin [2013] build a micro-founded model which links the leverage to the VaR. Banks adjust their balance sheet to maintain a given probability of failure.

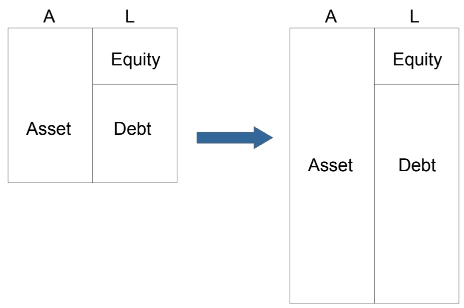

Figure 1: Pro-cyclical leverage - Adrian and Shin (2013)

Empirically, Adrian and Shin [2008], Gropp and Heider [2009], Kalemli-Ozcan et al.

[2011], Baglioni et al. [2013] study the determinants of the leverage and its supposed procyclicality. Only Gropp and Heider [2009] invalidate this relationship[2]. According to the other papers, the leverage is pro-cyclical especially for investment banks. Figure 2 supports these conclusion by ploting changes in assets with changes in debt for banks located in the euro area (left panel) and banks located in France (right panel). The slope is close to one for both geographic locations which translates the active management of banks.

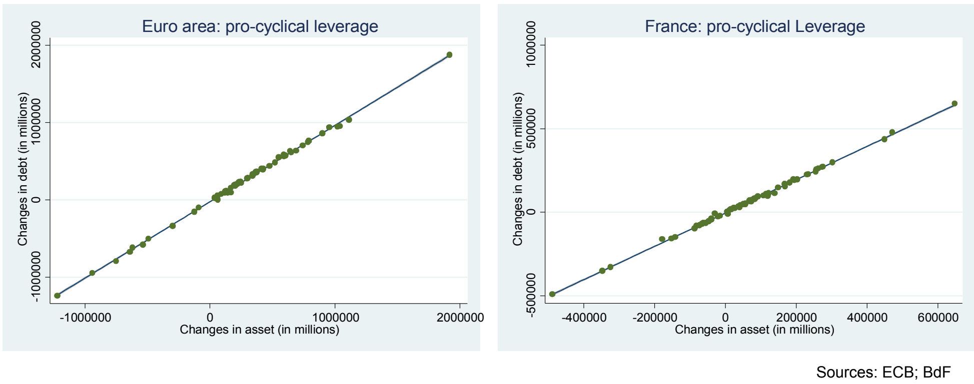

Figure 2: Pro-cyclical leverage - Adrian and Shin (2013)

Although the literature mostly concludes to a procyclical leverage, it also highlights the potential differences across geographic locations. According to Kalemli-Ozcan et al. [2011] the European investment banks may show a less procyclical leverage compared to the US investment banks. Looking at the differences in slope, Figure 2 also points out this heterogeneity. Banks located in France seem to have a less pronounced slope. The heterogeneity across banks may reflect the different composition of banks' balance sheet. Diversification in bank's balance associated with exchange rate fluctuations might change the banking expositions to specific economic condition. Unfortunately the cur-

[2]Banks' leverage converge to bank specific time invariant target.

rent empirical literature does not go further on this issue[3].

Theoretically, the micro-founded model from Adrian and Shin [2013] builds a framework with a single currency of denomination. There is no possible diversification in assets or liabilities. Bruno and Shin [2014] introduce a second currency in a cross-border network. However, as the local corporate completely bears the risk regarding exchange rate fluctuations, the model does not allow a diversification regarding the currency of denomination of each side of the bank's balance sheet. This gap in the literature is inconsistent with what we can observe on data.

Figure 3 breakdowns the currency of denomination of external banking positions from the BIS Data set. The share of international liabilities denominated in euro is higher than 20% over the period. The significance of the euro is even more pronounced looking at the international claims. It is clear from these stylized facts that banks have a diversified balance sheet. This diversification has to be included in the analysis.

Taking into accounts these banking specifications, the purpose of my paper is to bring a theoretical model which allows a diversification in both the assets and the liabilities of banks. As Adrian and Shin [2013] directly link the leverage to the economic activity, I choose to base my work on their model. I introduce a second currency of denomination in both side of the balance sheet. As illustrated in BIS [2010] global banks adopt global strategies regarding the composition of their aggregated balance sheet. Thus, my paper focuses on a aggregated framework where the complete diversification appears in their balance sheet. Contrary to Bruno and Shin [2014], the exchange rate is directly linked to the economic activity in each location. I would consider in my analysis two exchange rate regimes known a fixed regime and a floating regime.

[3] This might be explained by the scarcity of micro data on banking balance sheet.

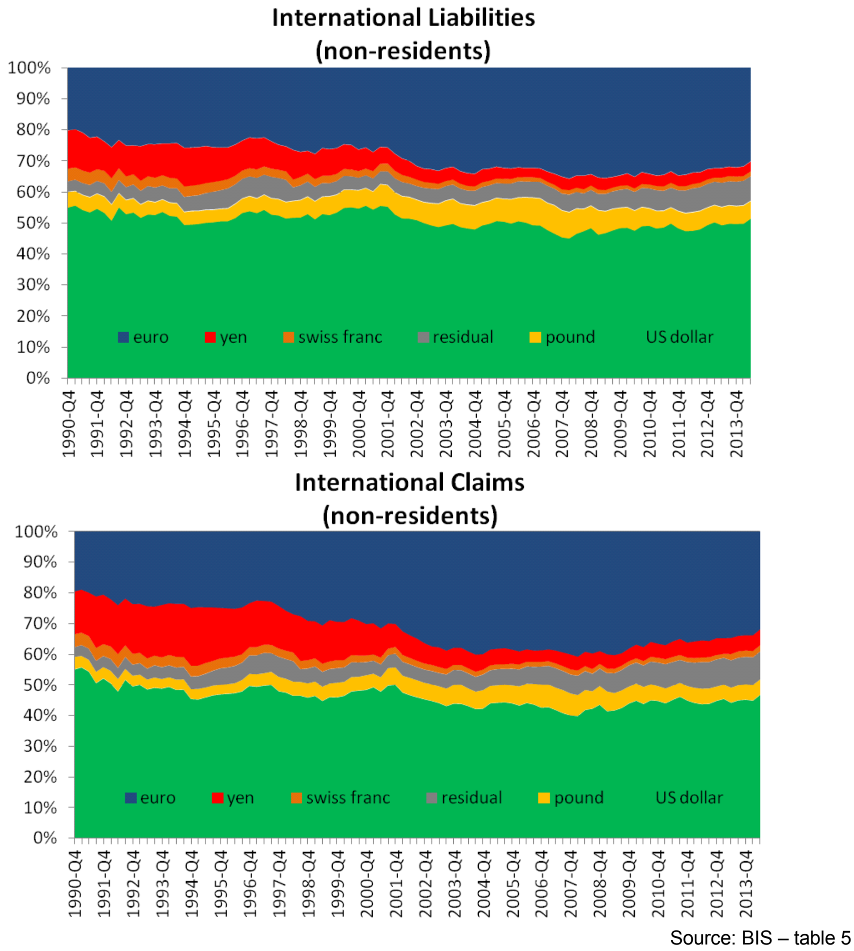

Figure 3: Currency breakdown of total reporting banks. International positions. BIS.

As banks follow a VaR rule, they adjust their balance sheet in order to maintain a constant probability of default. I study three specific types of shocks. A global positive shock impacts both economies related to each currency of denomination. In this circumstances, a diversification of the balance sheet does not change the procyclicality of the leverage for any exchange rate regimes. A positive shock would be characterized by a positive shock in the domestic economy and a less positive shock in the foreign economy. As long as the portfolio of the bank is diversified, the procyclicality of the leverage would be reduced in a fixed exchange. Having a floating exchange rate might bring more procyclicality because of its impact in the weighting of the portfolio. Finally, I introduce an asymmetric shock where the domestic economy is positively impacted and the foreign economy is negatively impacted. As the VaR rule forces the bank to be careful about its portfolio's risk, this last shock implies a counter-cyclical leverage. The diversification increases risks in the portfolio. This results is true even if the total expected return of the portfolio increases. However a floating exchange might reduce this counter-cyclicality.

The rest of the paper is organized as following. First, I develop a model with a complete diversification and a fixed exchange rate. This first section build the VaR rule from the maximization of the bank's and the creditors' utilities. The leverage is extracted to the VaR rule. In the second section I allow the exchange rate to fluctuate according to the two economic conditions.

2 Complete diversification with fixed exchange rate

The model is based on a representative bank's balance sheet. The bank invests in assets and raises funds from its creditors. There are two currencies of denomination for assets and debts. There is an economic condition specific to each currency. The economic conditions are known publicly and determine the distribution of assets' returns.

There are two periods T=0,1. Knowing the economic condition and the distribution of returns, the bank and the creditors agree on the amount potentially reimbursed at T=1 in order to satisfy the VaR rule. Then, this amount defined the level of debt the bank is able to raise at T=0. Finally, total debt determines the leverage.

2.1 Specifications of the agents

The first agent in this model is a representative bank which is similar to a European investment bank. It is domestic in the sense that its equity and its balance sheet are in domestic currency. The domestic currency is the first currency similar to the euro. The bank is risk neutral and equity E is exogenous[4]. The second type of agents refers to the creditor of the bank which can be seen as Money Market Funds or other investment banks. The creditor lend money to the bank in both currencies. The creditor is also risk neutral. The exchange rate S is defined as the number of domestic units per unit of foreign currency and it is fixed in this first development.

2.2 The accounting framework

There are two periods. At T=0, the bank raises funds backed by collateral in domestic and foreign currency with respectively A and $A^\star$. Total assets expressed in domestic currency equal $A + SA^\star$. Funds are in domestic and in foreign currency with D and $D^\star$ respectively. Thus, total funds from creditors expressed in domestic currency are equal to $D + SD^\star$. They are used by the bank to make an investment in asset. We note a the share of assets in domestic currency and $(1-a)$ the share of assets in foreign assets.

At T=1 the bank receives a total expected return from its investments $a(1+\bar{r})+(1-a)(1+\bar{r}^\star)$ where $\bar{r}$ and $\bar{r}^\star$ are respectively the expected returns from the domestic and the foreign asset. Returns depend on the economic condition specific to each currency area. Each currency is related to an economy with an economic condition θ and $\theta^\star$ for the domestic and the foreign economy respectively. θ and $\theta^\star$ define the distribution of returns received at T=1. They are known publicly since T=0 and they do not change between the two periods.

[4] An exogenous equity is in line with the theory of pro-cyclical leverage developed by Shin.

Also at T=1, the bank reimburses its domestic and foreign creditors with respectively $\bar{D}$ and $S\bar{D}^\star$. As θ and $\theta^\star$ are known for the two periods, there is no macroeconomics risk. However, the creditors of the banks are uninsured and they face a risk of default regarding the repayment at T=1. The risk of default would depend on the investment choice made by the bank. Two results emerge from this specification. First, it is assumed that $\bar{D} > D$ and $S\bar{D}^\star > SD^\star$ to remunerate the default risk. Second, creditor receive at T=0 a defaultable debt claim which would enters in its utility function.

From this first set of information I can develop the bank's balance sheets at each period:

T=0, at market value:

Assets	Liabilities
A	E
$SA^\star$	D
	$SD^\star$

T=1, at notional value:

Assets	Liabilities
$(1+\bar{r})A$	$\bar{E}$
$(1+\bar{r}^\star)SA^\star$	$\bar{D}$
	$S\bar{D}^\star$

Four debt ratios are defined relative to each currency of funding and each period. Regarding the market value of debt ratios, I define the debt ratios relative to domestic and foreign currency respectively as:

$$d = \frac{D}{(A + SA^\star)} \quad \text{and} \quad d^\star = \frac{SD^\star}{(A + SA^\star)}$$

Then, focusing on the notional value of debt ratios we have respectively for the domestic and the foreign currency:

$$\bar{d} = \frac{\bar{D}}{(A + SA^\star)} \quad \text{and} \quad \bar{d}^\star = \frac{S\bar{D}^\star}{(A + SA^\star)}$$

$\bar{E}$ is the equity at the notional value. It is equivalent to the equity at the market value augmented by interests. In other words, I assume that $E < \bar{E}$ and $a(1+\bar{r}) + (1-a)(1+\bar{r}^\star) > (\bar{d} + \bar{d}^\star)$.

I define the leverage λ as the ratio the total assets to equity. Since I am interested in the bank's global funding strategy (relative to its consolidate balance sheet), my leverage also takes into account the consolidate measure of the bank's balance sheet. Thus I have:

$$\lambda = \frac{(A + SA^{\star})}{E} = \frac{(A + SA^{\star})}{(A + SA^{\star}) - (D + SD^{\star})} = \frac{1}{1 - (d + d^{\star})}$$

As the leverage positively depends on the total debt ratio at T=0, its procyclicality would come from this component.

Looking at the investment choice, the bank makes an indivisible choice between two types of portfolios. Each portfolio is made of assets in domestic currency and assets in foreign currency. The weighting relative to each type of assets is introduced with a and $(1-a)$. Put differently, the portfolio's distribution comes from a mixture distribution of the two asset's return distribution. As each asset's return follows a GEV distribution, the portfolio's return is also defined by a GEV distribution. The first portfolio is a good portfolio with a total expected return of $[ar_H + (1-a)r_{H^{\star}}]$. The second portfolio is a less good portfolio compared to the latter. Its total expected return $[ar_L + (1-a)r_{L^{\star}}]$ is lower and it includes more volatility[5]. The following equations develop respectively and individually the cumulative distribution[6] of the good asset in domestic currency, the cumulative distribution of the good asset in foreign currency, the cumulative distribution of the bad asset in domestic currency, and the cumulative distribution of the bad asset

[5] The introduction of an investment choice enable a contract model between the creditor and the bank such as Holmström and Tirole [1997].

[6] Following the GEV(θ, σ, ξ) where θ, σ and ξ are respectively the location parameter, the scale parameter, and the shape parameter. z is the *iid* random variable

in foreign currency:

$$F_H(z) = exp\left\{-\left(1+\xi\left(\frac{z-\theta}{\sigma}\right)\right)^{-\frac{1}{\xi}}\right\}$$

$$F_{H^\star}(z) = exp\left\{-\left(1+\xi\left(\frac{z-\theta^\star}{\sigma}\right)\right)^{-\frac{1}{\xi}}\right\}$$

$$F_L(z) = exp\left\{-\left(1+\xi\left(\frac{z-(\theta-k)}{\sigma m}\right)\right)^{-\frac{1}{\xi}}\right\}$$

$$F_{L^\star}(z) = exp\left\{-\left(1+\xi\left(\frac{z-(\theta^\star-k)}{\sigma m}\right)\right)^{-\frac{1}{\xi}}\right\}$$

I assume that the returns for a similar asset only differ in the location parameter of the distribution θ and $\theta^\star$. Their scale or shape parameters are alike. As I am interesting in the European global banks' behavior between two developed currencies, it is consistent.

Using a mixture distribution, the cumulative distribution function of returns when the bank invests in the good portfolio is of the form[7]:

$$F_{H,H^\star}(z) = a\ exp\left\{-\left(1+\xi\left(\frac{z-\theta}{\sigma}\right)\right)^{-\frac{1}{\xi}}\right\} + (1-a)\ exp\left\{-\left(1+\xi\left(\frac{z-\theta^\star}{\sigma}\right)\right)^{-\frac{1}{\xi}}\right\}$$

In contrast, if the bank invests in the bad portfolio, the cumulative distribution function is defined by:

$$F_{L,L^\star}(z) = a\ exp\left\{-\left(1+\xi\left(\frac{z-(\theta-k)}{\sigma m}\right)\right)^{-\frac{1}{\xi}}\right\} + (1-a)\ exp\left\{-\left(1+\xi\left(\frac{z-(\theta^\star-k)}{\sigma m}\right)\right)^{-\frac{1}{\xi}}\right\}$$

Figure 4 illustrates different forms of the mixture distribution when the bank invests in the good portfolio. As the location parameters change the distribution of returns, the

[7]More details here: `http://en.wikipedia.org/wiki/Mixture_distribution`.
This new framework that uses a mixture distribution is still compatible with a Second Order Stochastic Dominance as in the referring model.

distribution is more or less concentrated on one side of the distribution depending on economic condition. The blue dashed line corresponds to $\theta < \theta^\star$ while the green dotted line represents the case where $\theta > \theta^\star$. Thanks to this representation, I can already anticipate that shocks in θ and $\theta^\star$ might change risks in the portfolio by deepening the tail of the distribution.

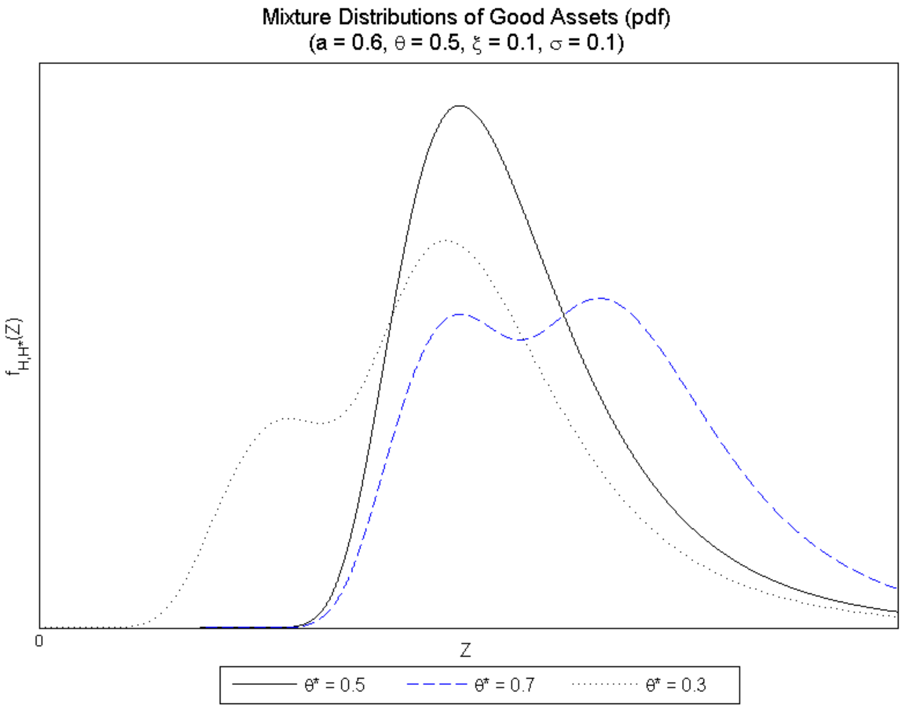

Figure 4: Mixture Distribution - Good Asset - PDF

The cumulative distribution function is also useful to define the probability of default α when the bank invests in the good portfolio. This risk of default appears if the realized total return is below a given level that might be equal to the total debt ratio at the notional value for instance. Thus, the probability of default α is defined by the cumulative distribution function such as:

$$\alpha = F_{H,H^\star}(z) = a\ exp\left\{-\left(1+\xi\left(\frac{z-\theta}{\sigma}\right)\right)^{-\frac{1}{\xi}}\right\}+(1-a)\ exp\left\{-\left(1+\xi\left(\frac{z-\theta^\star}{\sigma}\right)\right)^{-\frac{1}{\xi}}\right\}$$

The introduction of a second currency changes the value of α depending on the two economic conditions and on the degree of diversification. Figure 5 illustrates the different form of the CDF when the banks invests in the good portfolio. Depending on the locations parameter of the two economies the probability of default changes for a given value of Z. The dashed (*dotted*) line on the figure illustrates the situation where $\theta^\star > \theta$ ($\theta^\star < \theta$). The determination of α is linked to the VaR rule that I develop in the next sections.

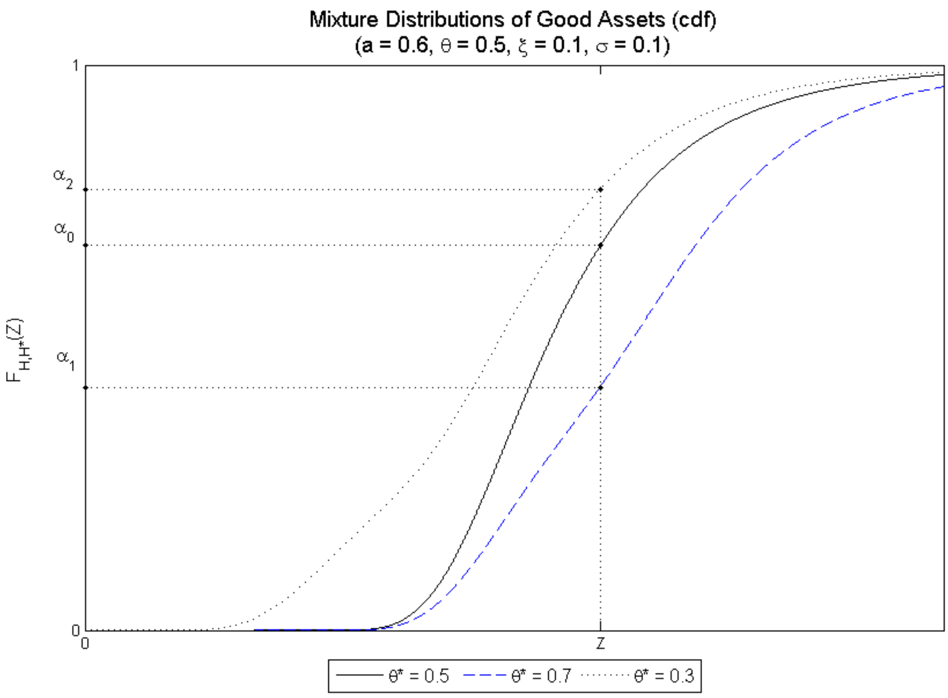

Figure 5: Mixture Distribution - Good Asset - CDF and the probability of default.

Since the creditors are uninsured, they hold a defaultable debt claim in returns to the funds they lent to the bank at T=0. This claim will enter in their utility because it is part of their wealth at T=0. Thanks to Merton [1974], I am able to divide the value of this defaultable debt claim into two components known as some cash $(\bar{D} + S\bar{D}^{\star})$ and a short position on a put option π. Since the risk differs between the two types of portfolios, I have to distinguish the put option relative to each investment choice.

If the bank invests in the good portfolio, I obtain the following put option price[8]:

$$\pi_{H,H^{\star}}(\bar{D}+S.\bar{D}^{\star}, A+SA^{\star}) \quad \leftrightarrow \quad (A+SA^{\star}).\pi_{H,H^{\star}}(\bar{d}+\bar{d}^{\star}, 1) \equiv (A+SA^{\star}).\pi_{H,H^{\star}}(\bar{d}+\bar{d}^{\star})$$

In contrast, if the bank invests in the bad portfolio:

$$\pi_{L,L^{\star}}(\bar{D}+S.\bar{D}^{\star}, A+SA^{\star}) \quad \leftrightarrow \quad (A+SA^{\star}).\pi_{L,L^{\star}}(\bar{d}+\bar{d}^{\star}, 1) \equiv (A+SA^{\star}).\pi_{L,L^{\star}}(\bar{d}+\bar{d}^{\star})$$

Now that I have defined all components of the model, I can focus on the utility of each agent's type.

2.3 Creditor's incentive constraint

The creditor of the bank is risk neutral. The maximization of its utility consists of maximizing its total net expected payoff.

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[8]The price of the put option depends on the total amount reimbursed at the end of the period - $\bar{D} + S.\bar{D}^{\star}$ - and on the total value of asset $A + SA^{\star}$.

If the bank invest in the good portfolio, I obtain the following net expected payoff:

$$
\begin{aligned}
U^{c+c^\star}_{H,H^\star}(A+SA^\star) &= \text{Defaultable debt claim } - \text{ Money lent at T} = 0 \\
&= (\bar{D}+S\bar{D}^\star) - (D+SD^\star) - (A+SA^\star)\pi_{H,H^\star}(\bar{d}+\bar{d}^\star) \\
&= (A+SA^\star)\left((\bar{d}+\bar{d}^\star) - (d+d^\star) - \pi_{H,H^\star}(\bar{d}+\bar{d}^\star)\right) \qquad (1)
\end{aligned}
$$

Assuming that the utility is equal or higher than 0 gives me the first participation constraint (IR):

$$
\begin{aligned}
0 &\leq (\bar{d}+\bar{d}^\star) - (d+d^\star) - \pi_{H,H^\star}(\bar{d}+\bar{d}^\star) \\
(d+d^\star) &= (\bar{d}+\bar{d}^\star) - \pi_{H,H^\star}(\bar{d}+\bar{d}^\star) \qquad \text{(IR)}
\end{aligned}
$$

And similarly for an investment in the less good portfolio:

$$
\begin{aligned}
U^{c+c^\star}_{L,L^\star}(A+SA^\star) &= (A+SA^\star)\left((\bar{d}+\bar{d}^\star) - (d+d^\star) - \pi_{L,L^\star}(\bar{d}+\bar{d}^\star)\right) \\
(d+d^\star) &= (\bar{d}+\bar{d}^\star) - \pi_{L,L^\star}(\bar{d}+\bar{d}^\star) \qquad (2)
\end{aligned}
$$

The IR constraints define the total debt ratio at the market value relative to the total debt ratio at the notional value. The latter should be large enough to make an incentive for the creditor. The more the bank offer large reimbursement, the more the creditor is tempted to lend money at T=0. Under this form, it does not depend directly on the portfolios' returns' specifications. The introduction of the diversified assets is going to change the definition of the total debt ratio at the notional value which comes from the utility of the bank.

2.4 Bank's incentive constraint:

As the bank is risk neutral, it also maximizes its total net expected payoff. The introduction of a second investment currency changes the composition of the bank's net expected payoff $U^E_{H,H^\star}$. In this new framework, returns come from both assets in domestic and in foreign currency. Thus the net expected payoff when the bank invests in the good portfolio is equal to:

$$\begin{aligned} U^E_{H,H^\star} &= A.r_H + SA^\star r_{H^\star} + (D + S.D^\star) - (\bar{D} + S.\bar{D}^\star) + (A + SA^\star).\pi_{H,H^\star}(\bar{d} + \bar{d}^\star) \\ &= (A + SA^\star)\left(a.r_H + (1-a)r_{H^\star} + (d + d^\star) - (\bar{d} + \bar{d}^\star) + \pi_{H,H^\star}(\bar{d} + \bar{d}^\star)\right) \end{aligned} \quad (3)$$

In contrast, when the bank invests in the bad portfolio the net expected payoff is equal to:

$$\begin{aligned} U^E_{L,L^\star} &= A.r_L + SA^\star r_{L^\star} + (D + S.D^\star) - (\bar{D} + S.\bar{D}^\star) + (A + SA^\star).\pi_{L,L^\star}(\bar{d} + \bar{d}^\star) \\ &= (A + SA^\star)\left(a.r_L + (1-a)r_{L^\star} + (d + d^\star) - (\bar{d} + \bar{d}^\star) + \pi_{L,L^\star}(\bar{d} + \bar{d}^\star)\right) \end{aligned} \quad (4)$$

Assuming that $U^E_{H,H^\star} \geq U^E_{L,L^\star}$ gives me the incentive constraint IC:

$$\begin{aligned} a(r_H - r_L) + (1-a)(r_{H^\star} - r_{L^\star}) &\geq \pi_{L,L^\star}(\bar{d} + \bar{d}^\star) - \pi_{H,H^\star}(\bar{d} + \bar{d}^\star) \\ \text{Where: } (r_H - r_L) &= (r_{H^\star} - r_{L^\star}) \\ r_H - r_L &\geq \Delta\pi(\bar{d} + \bar{d}^\star) \\ r_H - r_L &= \Delta\pi(\bar{d} + \bar{d}^\star) \end{aligned} \quad \text{(IC)}$$

Since assets differ only in their location parameters, the spreads in returns for each currency of denomination are similar. Thus, the lhs of the IC constraint can be simplified as if the bank only has assets in domestic currency[9]

Thus, the IC constraint stipulates that for any economic condition there is a solution $(\bar{d} + \bar{d}^\star)$ that satisfies this identity. The unique solution illustrated in figure 6 comes from the SOSD between the two mixture distributions as in the referring model.

[9]For more details, see the appendix.

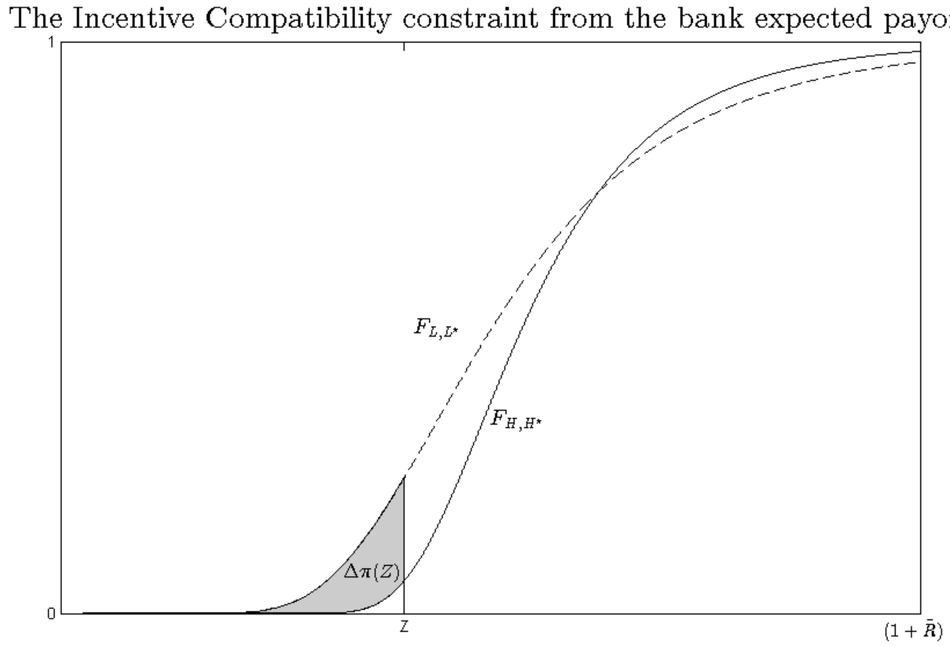

Figure 6: Private benefit from the moral hazard problem - IC constraint

As in Adrian and Shin [2013], the IC constraint also represents the moral hazard trade-off from Holmström and Tirole [1997]. The lhs represents the private benefit of investing in the good portfolio while the rhs is equal to private benefit of investing in the bad portfolio. Associated with the IR constraint from the creditor, bank is constrained to invest in the good portfolio.

However, under this form it is difficult to have a clear definition of $(\bar{d}+\bar{d}^\star)$. Additional assumptions are needed to have a closed form solution.

2.5 Value at Risk

As Adrian and Shin [2013], I suppose that $\xi = -1$ and that $m \mapsto 1$[10]. Thus, the cumulative distribution function of the mixture functions are of the form:

$$F_{H,H^\star}(z) = a\ exp\left\{\frac{z-\theta}{\sigma} - 1\right\} + (1-a)\ exp\left\{\frac{z-\theta^\star}{\sigma} - 1\right\}$$
$$F_{L,L^\star}(z) = a\ exp\left\{\frac{z-(\theta-k)}{\sigma} - 1\right\} + (1-a)\ exp\left\{\frac{z-(\theta^\star-k)}{\sigma} - 1\right\}$$

Where $F_{L,L^\star} = e^{\frac{k}{\sigma}} F_{H,H^\star}$

These assumptions allow me to simplify the rhs of IC as following[11]

$$(r_H - r_L) = \Delta\pi(\bar{d} + \bar{d}^\star) \tag{IC}$$
$$= (e^{\frac{k}{\sigma}} - 1)\sigma F_{H,H^\star}(\bar{d} + \bar{d}^\star) \tag{5}$$

Because $F_{H,H^\star}$ is the probability of default of the bank when it invests in the good portfolio, I can extract the following VaR:

$$\alpha = F_{H,H^\star}(\bar{d} + \bar{d}^\star) = \frac{(r_H - r_L)}{(e^{k/\sigma} - 1)\sigma} \tag{6}$$

As the rhs of (7) does not depend on θ or $\theta^\star$, the VaR holds and the probability of default α has to be maintained at the same level for any economic conditions and any level of diversification. The bank would adjust its total debt ratio at T=1 to satisfy this identity. It is important to notice that the VaR rule focuses on the tail of the distribution. If the tail is thickened by changes in economic conditions, the bank would have to decrease its total debt ratio to maintain α.

[10] $\xi = -1$ implies a bounded distribution function on the right side. As the VaR rule focus on the left side of the distribution, this assumption is not a problem.

[11] For more details, see the appendix.

As I am interested in this adjustment, I develop $F_{H,H^\star}(\bar{d}+\bar{d}^\star)$ in (7) to make $(\bar{d}+\bar{d}^\star)$ appear[12]:

$$\alpha = aF_{H^\star} + (1-a)F_{H^\star} = \frac{(r_H - r_L)}{(e^{k/\sigma}-1)\sigma}$$

$$\alpha = exp\left\{\frac{(\bar{d}+\bar{d}^\star)-\theta}{\sigma} - 1\right\}\left[a + (1-a)exp\left\{\frac{\theta-\theta^\star}{\sigma}\right\}\right] = \frac{(r_H - r_L)}{(e^{k/\sigma}-1)\sigma} \quad (7)$$

The VaR rule constrains the bank in its adjustment to the economic conditions. Thus, by extracting $(\bar{d}+\bar{d}^\star)$ from (7), I know the adjustment of the bank for each economic condition as:

$$(\bar{d}+\bar{d}^\star) = \theta + \sigma + \sigma ln\left(\frac{(r_H - r_L)}{(e^{k/\sigma}-1)\sigma}\right) - \sigma ln\left(a + (1-a)\ exp\left\{\frac{\theta-\theta^\star)}{\sigma}\right\}\right) \quad (8)$$

The definition of the total debt ratio at the notional value is given by (8). The procyclicity of the leverage is derived from the degree of total debt ratio adjustment regarding an economic shocks. If the two economies face a common global shock, the diversification of asset does not impact the pro-cyclicality of the leverage. The banking response or adjustment is a 1 for 1 as in Adrian and Shin [2013]. This first type of shocks is illustrated in Figure 7 with the two red curves. The adjustment goes from $(\bar{d}+\bar{d}^\star)_0$ to $(\bar{d}+\bar{d}^\star)_1$. The second shock I analyze is positive. The amplitude of the shock is lower in the foreign economy. The diversification of assets smooths the rise in total expected payoff. Compared to the previous situation, the procyclicality of the leverage is reduced as the risk in the portfolio is more important. The adjustment goes from $(\bar{d}+\bar{d}^\star)_0$ to $(\bar{d}+\bar{d}^\star)_2$. Finally, I study an asymmetric shock. It is characterized by a positive shock in the domestic economy and a negative shock in the foreign economy. This type of shock has important implications for the distribution of total return. As long as the bank di-

[12]I use the following arrangement: $F_{H^\star} = F_H\ .\ exp\left\{\frac{\theta-\theta^\star}{\sigma}\right\}$

versifies part of its assets, an asymmetric shock thickens the tail of the distribution and produces a counter-cyclical leverage. This situation is observed even if total expected return increases because the VaR rule focuses on the tail distribution. Reducing one of the assets' return increases risks. The adjustment goes from $(\bar{d}+\bar{d}^{\star})_0$ to $(\bar{d}+\bar{d}^{\star})_3$.

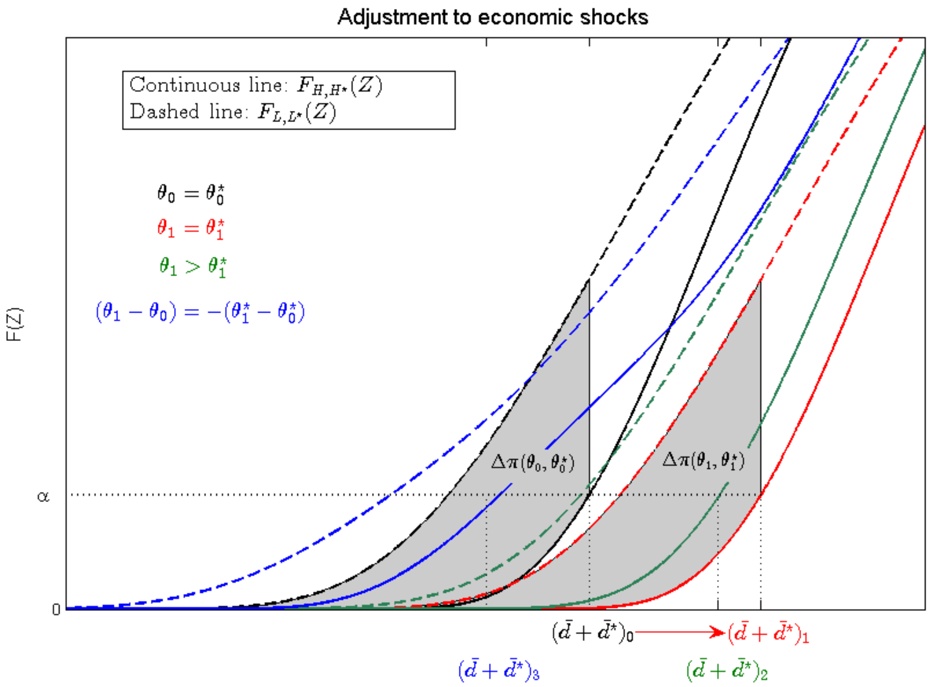

Figure 7: Procyclical leverage under diversification and fixed exchange rate.

In this first section I assume that the exchange rate is fixed. It allows me to have a first benchmark for the next extension where the exchange rate is floating relative to the economic conditions.

3 Complete diversification with floating exchange rate

3.1 Definition of the exchange rate:

As I am interested in the European global banks' implication in the US markets, I assume that the two economies in my model are developed and quiet similar. I also assume that there is a perfect capital mobility. Except their location parameters in the distributions, they are perfect substitute. Following this idea, I choose to defined the exchange rate S based on the Covered Interest Rate Parity.

S is defined as the number of domestic units per unit of foreign currency and I define it in the floating regime as:

$$S = 1 + \frac{r_{H^\star} - r_H}{1 + r_H} \tag{9}$$

Where :

$$r_{H^\star} = \theta^\star + \sigma H(\xi)$$

$$r_H = \theta + \sigma H(\xi)$$

As θ and $\theta^\star$ are known for the two periods, the exchange rate does not change between T=0 and T=1. Thus, the different constraints from the fixed exchange rate model hold. The introduction of the floating exchange rate is notable when the economic conditions change.

Starting from an initial situation where $\theta = \theta^\star$, a global shock on economies won't change the interest spread. The exchange rate is maintained at it initial value $S = 1$. On the contrary, if the positive shock is stronger in the domestic economy, the domestic currency appreciates and S decreases. Finally, an asymmetric shock such that $(\theta_1 - \theta_0) = -(\theta_0 -^\star \theta_1^\star)$ adds to the depreciation of the foreign currency. S becomes smaller.

3.2 The VaR Rule:

As S is fixed between the two periods, the different constraints from the fixed exchange rate model hold. The VaR rule is still the final constraint which defines the total debt ratio at T=1 and the leverage. I can directly go to the VaR rule:

$$\alpha = F_{H,H^\star}(\bar{d} + \bar{d}^\star) = \frac{(r_H - r_L)}{(e^{k/\sigma} - 1)\sigma} \tag{10}$$

As spreads in returns concern asset in the same currency, the introduction of a floating exchange rate does not impact the rhs of the VaR. The VaR still holds and the probability of default would be similar in both a fixed or a floating exchange rate regimes. As the bank is constrained by the VaR rule, it has to adjust to the economic shocks with its total debt ratios.

Proposition 1 *In both exchange rate regimes, the VaR rule holds and the probability of failure of the bank is constant for any economic condition. The bank adjusts its balance sheet to satisfy this constraint.*

The introduction of the floating exchange rate is notable with economic shocks. The adjustment adopted by the bank depends on the type of shocks.

3.3 The pro-cyclical leverage

A global shock:

A global shock is defined as a positive shock which impacts the two economies with the same amplitude. From the initial situation both location parameters increase. As illustrated in figure 8 the mixture distribution's pdf shift to the right, but its shape is unchanged.

As the shock is global, the weighting in the mixture distribution is constant. Diversification in assets does not change the procyclicality of the leverage as illustrated in figure

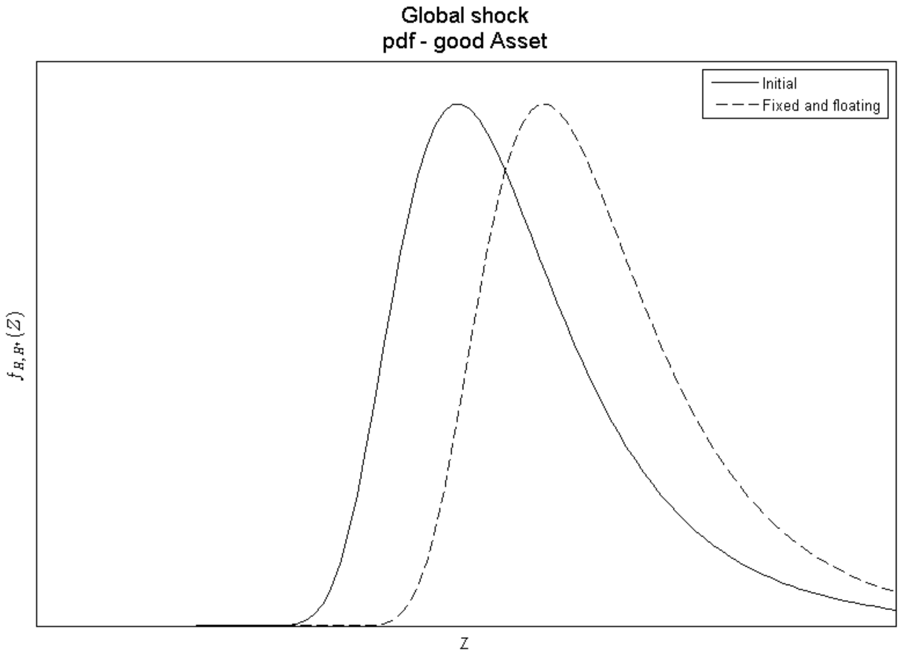

Figure 8: Pdf swift under global positive shock and floating exchange rate.

9. Total debt ratio at the notional value shifts from $(\bar{d}+\bar{d}^{\star})_0$ to $(\bar{d}+\bar{d}^{\star})_1$. At $(\bar{d}+\bar{d}^{\star})_1$, the probability of default is maintained at its initial level and the VaR rule is satisfied.

Proposition 2 *In both exchange rate regimes, diversification of banks' balance sheet does not change the procyclicality of leverage.*

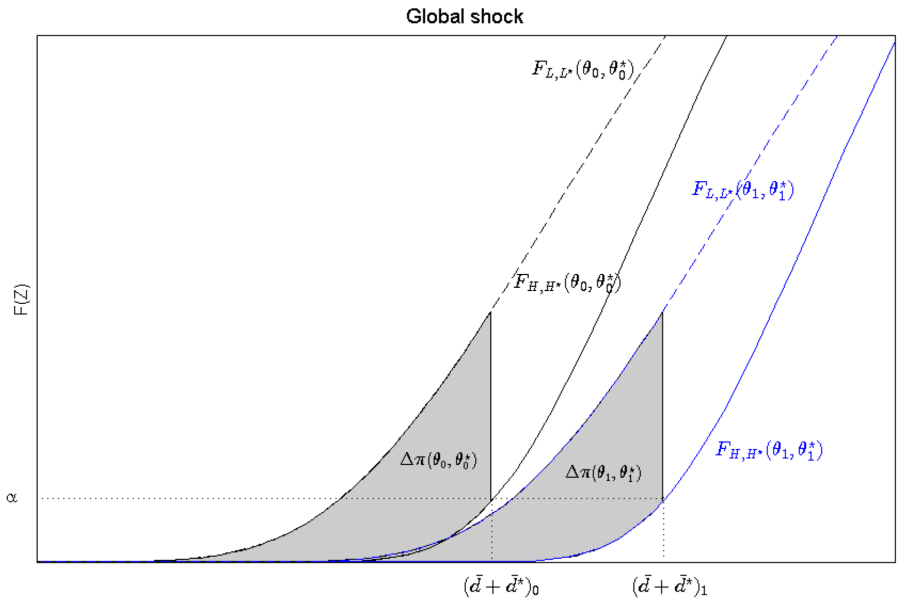

Figure 9: Procyclical adjustment under global positive shock and floating exchange rate.

A positive shock:

The introduction of a floating exchange rate does not impact each expected return r_H and $r_{H^\star}$. Both are referring to the expected return in the local currency. However, as the global bank invests in both economies, the total expected payoff converted in domestic currency would be impacted by the introduction of a floating exchange rate. Especially, it is the shape of the mixture distribution that would be impacted through the implication of S in the foreign asset's value and the weightings a and $(1-a)$.

A positive economic shock which is more pronounced at a domestic level depreciates the foreign currency. Compared to the value of the domestic asset, the foreign asset converted in domestic currency loses its importance. Consequently, an appreciation of the domestic currency is associated with an increase of the weighting a in the mixture distribution.

Graphically, figure 10 illustrates the swifts of the pdf under a positive shock and a floating exchange rate. As the domestic currency appreciates with the economic shock it increases the weighting of the good domestic asset in the portfolio. Consequently, the tail of the distribution which measures the probability of default would be reduced.

As the bank follows a VaR rule, the introduction of a floating exchange rate increases

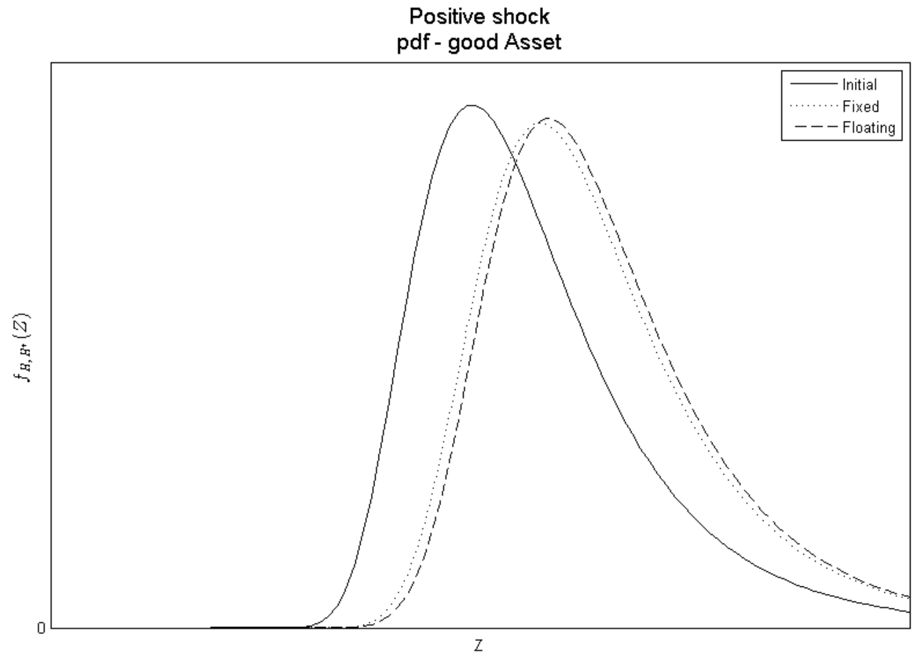

Figure 10: Pdf swift under a positive shock and a floating exchange rate.

its risk-taking capacity. For a given probability of default, the bank would be more procyclical under a positive shock and a floating exchange rate as illustrated in figure 11. Total debt ratio goes from $(\bar{d}+\bar{d}^{\star})_0$ to $(\bar{d}+\bar{d}^{\star})_1$.

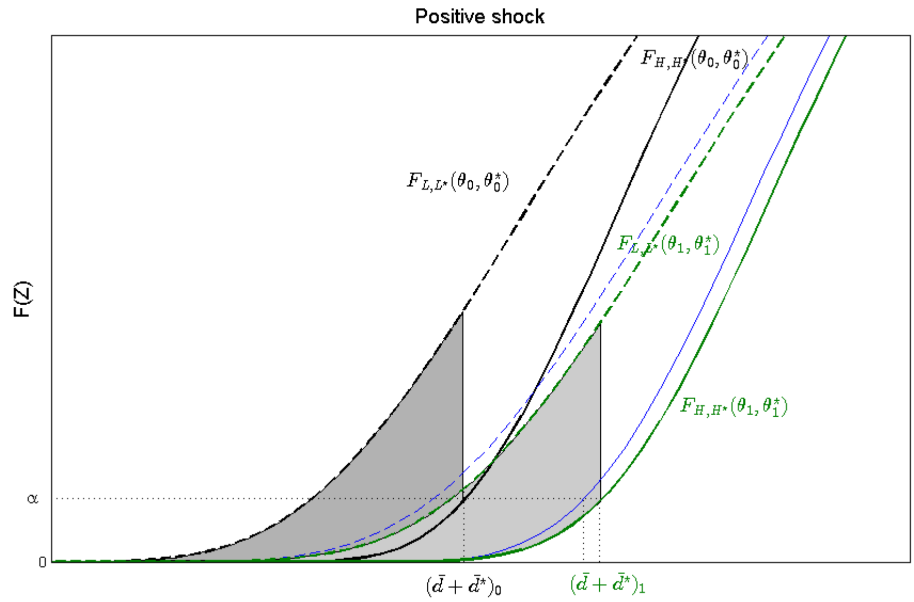

Figure 11: Procyclical adjustment under a positive shock and a floating exchange rate.

An asymmetric shock:

Finally, I introduce an asymmetric shock. The domestic economy is positively impacted while the foreign economy is negatively impacted. Under a floating exchange rate the domestic currency appreciates strongly. This appreciation increases the weighting of the domestic asset in the mixture distribution as illustrated in figure 12. As long as the assets of the banks are still diversified, an asymmetric shock still thickens the tail of the distribution. However, the floating exchange rate reduces the negative impact on the portfolio return though the change in weighting.

Compared to the fixed exchange rate situation, a floating exchange rate increases the risk-taking capacity of the bank. Thus, the counter-cyclicality of leverage is reduced under a floating exchange rate. In figure 13, the total debt ratio at T=1 goes from $(\bar{d}+\bar{d}^\star)_0$ to $(\bar{d}+\bar{d}^\star)_1$. The introduction of a floating exchange rate does avoid the

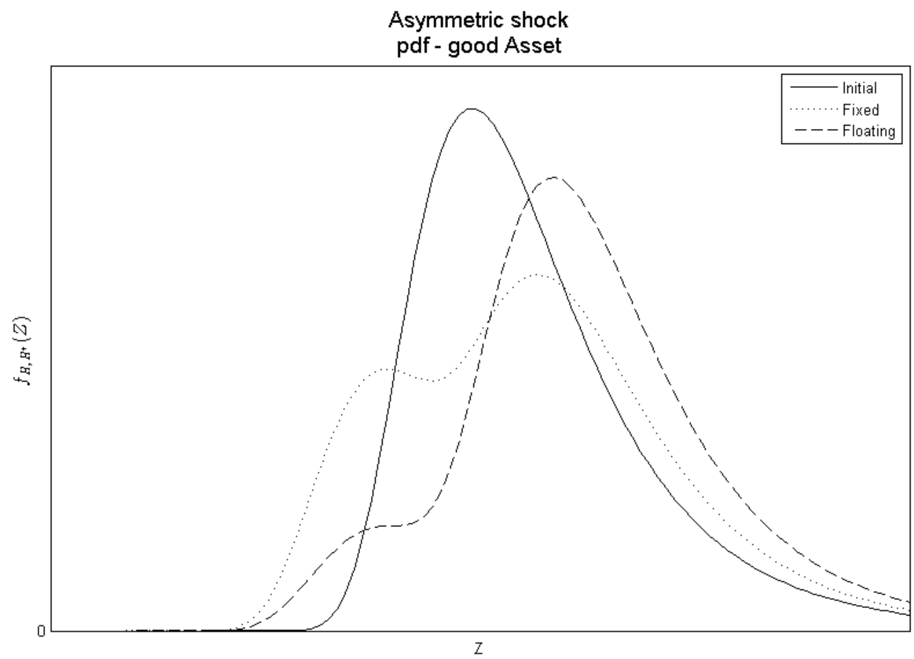

Figure 12: Pdf swift under asymmetric shock and floating exchange rate.

portfolio to become riskier.

The following propositions retake the previous results.

Proposition 3 *In both exchange rate regimes, an asymmetric shock leads to a counter-cyclical leverage when assets are diversified.*

Proposition 4 *The introduction of a floating exchange rate increases the risk-taking capacity of the banks.*

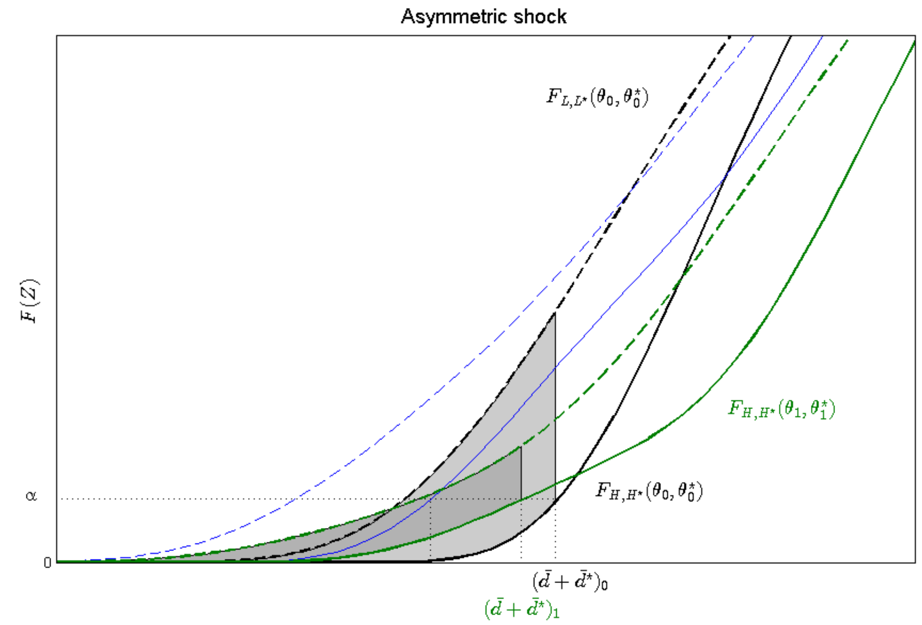

Figure 13: Procyclical adjustment under an asymmetric shock and floating exchange rate.

Conclusion

Global banks have global strategies regarding the composition of their assets and liabilities. A significant part of their balance sheet is diversified. According to the recent empirical literature, this diversification may have an impact on the leverage procyclicality. However, these results do not take into account the complete diversification of banks' balance sheet.

Theoretically, recent models succeed in reproducing the procyclicality of the leverage. However, most of them are based on a single currency framework. Put differently, global banks in these models do not diversify their balance sheet relative to different currencies.

My paper offers a first theoretical model which introduces a complete diversification in global banks' balance sheet. This contribution enables a more consistent approach regarding the current strategies of global banks.

Based on [Adrian and Shin, 2013], the model micro-founds the VaR rule and confirms the active behavior of banks in response to economic shocks. However depending on the type of shocks and on the exchange rate regime, the introduction of diversification changes the banking adjustment. Under a global shock the introduction of diversification does not modify the procyclicality of the leverage for both exchange rate regimes. By contrast, an asymmetric shock associated with diversification increases the portfolio's risk. As banks follow a VaR rule, an asymmetric shock implies a counter-cyclical leverage in both exchange rate regimes. Finally, a floating exchange rate increases the risk-taking capacity of the banks.

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4 Appendix

.1 Constant spreads:

As assets only differ in their location parameters, the spreads in interest rate are equal and constant relative to the economic condition.

$$\begin{aligned}
& a(r_H - r_L) + (1-a)(r_{H^\star} - r_{L^\star}) \\
& = a.\left(\theta + \sigma H(\xi) - (\theta - k) - m\sigma H(\xi)\right) + (1-a)\left(\theta^\star + \sigma H(\xi) - (\theta^\star - k) - m\sigma H(\xi)\right) \\
& = a.\left(k - \sigma(m-1)H(\xi)\right) + (1-a)\left(k - \sigma(m-1)H(\xi)\right) \\
& = \left(k - \sigma(m-1)H(\xi)\right) \\
& = Cst \qquad (11)
\end{aligned}$$

.2 IC development:

The simplifying assumptions gives the following IC constraint:

$$\begin{aligned}
(r_H - r_L) &= \Delta\pi(\bar{d} + \bar{d}^\star) \qquad \text{(IC)} \\
&= \int_0^{\bar{d}+\bar{d}^\star} F_{L,L^\star}\,\mathrm{d}z - \int_0^{\bar{d}+\bar{d}^\star} F_{H,H^\star}\,\mathrm{d}z \\
&= e^{\frac{k}{\sigma}} \int_0^{\bar{d}+\bar{d}^\star} F_{H,H^\star}\,\mathrm{d}z - \int_0^{\bar{d}+\bar{d}^\star} F_{H,H^\star}\,\mathrm{d}z \\
&= (e^{\frac{k}{\sigma}} - 1) \int_0^{\bar{d}+\bar{d}^\star} F_{H,H^\star}\,\mathrm{d}z \\
&= (e^{\frac{k}{\sigma}} - 1)\sigma F_{H,H^\star}(\bar{d} + \bar{d}^\star) \qquad (12)
\end{aligned}$$