Frictional and Keynsian unemployment in European economies

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May 7, 2015

Abstract

Knowledge of the unemployment structure (that consists of e.g. frictional and Keynesian unemployment) is necessary for the policy-makers to fight it effectively. The problem is that these components are not directly observable. This paper develops the unemployment decomposition method that is based on the DSGE model with two frictions (standard search frictions in the labor market and in the market for products) and price stickiness that allows for distinction between frictional and Keynesian unemployment. The model is used to study the structure of unemployment in four largest economies in the Eurozone: Germany, France, Italy and Spain.

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1 Introduction

It is well-understood that since the structure of unemployment is not homogenous, the policies that aim at decreasing unemployment should be adjusted to its specific heterogeneity. For the unemployment's components (like Keynesian or frictional unemployment) are not directly observable, there is a need for a theoretical method that decomposes the recorded time series of unemployment. We develop a framework that allows for such decomposition: we add two search frictions and price/wage rigidities into otherwise standard RBC model. We use this construction to analyze the unemployment structure in Germany, France, Italy and Spain.

Michaillat and Saez [10] have recently shown that models with frictions on both labor and goods markets can be used to decompose total unemployment into all three components: Keynesian, classical and frictional unemployment. They develop a theoretical, continuous-time model with search frictions both in the market for goods and the labor market, use their model to conduct a comparative-statics analysis and study the sources of labor market fluctuations in the US. They highlight the role of sticky wages and sticky prices in the propagation of shocks: with fixed prices, a drop in aggregate demand decreases product market tightness (the ratio of demand on products and manufacturer's capacity), which lowers sales made by producers and increases the idle time of hired employees. Since workers remain idle a larger proportion of the time, they become less profitable to employers, and the demand for labor decreases. The drop in labor demand raises unemployment. With flexible prices, a decrease in demand causes a decline of price level and hence it absorbed, so it does not affect either product market tightness or unemployment.

Our analysis can be seen as an attempt to incorporate the mechanism described by Michaillat and Saez into the RBC framework. There is however an important difference between our approach and Michaillat and Saez in nature of search costs in the market for goods: in our model, searching for consumer goods requires effort which is included in the utility function (like in Bai et al. [2]), whereas in their article agents have to purchase produced goods (which do not contribute to consumption) when they visit sellers. We have decided to introduce this change because the dynamics of the DSGE model in which search cost in the market for products was modelled as in Michaillat and Saez depended heavily on values of perfectly sticky prices. In particular, consumption increased in response to a positive demand shock when the level of demand (i.e. visits made by consumers to purchase products) was below its steady state value and it decreased when the level of demand was higher than the steady state level (these observations pertain to case when perfectly sticky prices are steady state values of flexible prices that decentralize a constrained-efficient allocation). In short, it was possible that consumption decreased in response to a positive demand shock.

Michaillat [9] has conducted a decomposition of unemployment for the US economy and has distinguished two main components: rationing unemployment and frictional unemployment. Rationing unemployment emerges in the Mortensen-Pissarides framework used by Michaillat when wages remain above marketclearing level and its source is the combination of diminishing marginal returns to labor and wage stickiness. Keynsian unemployment that is present in our analysis bears some conceptual similarities to those of rationing unemployment, but their source is different: we assume constant returns of scale and Keynsian unemployment arises as a result of three factors: price stickiness, wage stickiness and frictions in the market for goods. Our analysis is conducted in the standard DSGE framework (contrary to the Mortensen-Pissarides model of labor market used by Michaillat) and therefore allows for many potential extensions: e.g., studying fiscal and monetary policy.

Our work is related to Bai et al. [2], who show that demand shocks are responsible for the TFP volatility if the product market frictions are in place. However, they abstract from frictions in the labor market and from price rigidities which are present in our model and give rise to our decomposition method.

2 Model

The model is populated by identical, infinitely-lived households (workers) of meassure one. We assume existence of perfect insurance markets which implies that all households have identical income. Similarly to Bai et al. [2] agents in the model have to exert some effort to find goods they want to consume. This process is modeled in the following manner: households have to visit manufacturers if they want to buy goods. A single worker makes v_t visits (in period t) and gets one unit of consumption good per one visit with probability $q_G(\theta_{G,t})$, where by θ_G we denote the tightness in the market for goods (it is defined later). The number of visits v_t can be described as demand. It means that the total amount of consumption good purchased (when the number of visits and the probability $q_G(\theta_{G,t})$ are given) equals $c_t = q_G(\theta_{G,t})v_t$. It is assumed that search effort decreases utility which is captured by a convex function $G(v)$ that is included in our specification of preferences.

There are two stochastic, Markovian disturbances that affect economy in period t: the first one affects demand - $a_{d,t}$ and the other that has influence on the productivity level - $a_{z,t}$. By N_t we denote the fraction of employed workers at the begining of period t. Each worker derives utility $\exp(a_{d,t}) \cdot u(c_t)$ from goods consumed in current period and u is twice differentiable and strictly concave. Household uses its income (nominal labor income $w_t N_t$, where w_t is wage expressed in terms of price of shares and income from selling shares s_t together with dividends $\Pi_t s_t$) to purchase shares that can be sold in the following period - s_{t+1} and to buy consumption goods. It means that the dynamic problem of a worker can be described by the following Bellman equation:

$$W(s_t, N_t, a_{d,t}, a_{z,t}) = \max_{c_t, v_t, s_{t+1}} \{\exp(a_{d,t}) \cdot u(c_t) - G(v_t) \quad (1)$$

$$+\beta \mathbb{E}_t W(s_{t+1}, N_{t+1}, a_{d,t+1}, a_{z,t+1})\}$$

$$subject\ to:$$

$$c_t = q_G(\theta_{G,t}) v_t,$$

$$p_t c_t + s_{t+1} = s_t(1 + \Pi_t) + (1 - \sigma) w_t N_t.$$

where by p_t we denote the price of consumption goods, $\theta_{G,t}$, p_t, w_t are treated by workers as given and σ is exogeneous rate of destruction of a worker-employer relationship. Observe that we assume that job destruction takes place at the begining of period so that the number of hired workers who are paid wages is $(1-\sigma)N_t$. This is a convention used e.g. by den Haan et al. [5]. We use constraints to eliminate c_t and v_t, derive the FOC (with respect to s_{t+1}) and combine it with the envelope condition to get the Euler equation:

$$1 = \mathbb{E}_t \left(\left\{ \beta \frac{p_t}{p_{t+1}} \left[\frac{\exp(a_{d,t+1}) \cdot u'(c_{t+1}) - \frac{1}{q_G(\theta_{G,t+1})} G'(v_{t+1})}{\exp(a_{d,t}) \cdot u'(c_t) - \frac{1}{q_G(\theta_{G,t})} G'(v_t)} \right] \right\} (1 + \Pi_{t+1}) \right). \quad (2)$$

2.1 Firms

One can think of this sector as one big firm that has many "dormant" jobs (with opportunity cost equal zero). A dormant job can be activated by paying κ (so that a vacancy is posted) and finding a worker (it is a time-consuming process). Active job generates $\exp(a_{z,t})$ goods. Opportunity cost of a dormant job implies the following condition for opening a vacancy:

$$0 = -\exp(-\xi \cdot a_{z,t})\, \kappa + q_L(\theta_{L,t}) \mathbb{E}_t \left[\Delta_{t,t+1} (1 - \sigma) J_F(N_{t+1}, a_{d,t+1}, a_{z,t+1}) \right], \quad (3)$$

where $q_L(\theta_{L,t})$ is probability that an opened vacancy is filled, $\theta_{L,t}$ is the labor market tightness, , $\Delta_{t,t+1}$ is stochastic discount factor, $J_F(N_t, a_{d,t}, a_{z,t})$ is value ascribed by firm to filled vacancy in period t given employment and values of a_d, a_z and $\xi > 0$ is a parameter. The Bellman equation that characterizes J_F is:

$$J_F(N_t, a_{d,t}, a_{z,t}) = p_t f_G(\theta_{G,t}) \exp(a_{z,t}) - w_t \tag{4}$$

$$+\mathbb{E}_t \left[\Delta_{t,t+1}(1-\sigma) J_F(N_{t+1}, a_{d,t+1}, a_{z,t+1})\right],$$

where $f_G(\theta_{G,t})$ is probability that manufactured good is sold. Aggregate firm's profit in period t is given by:

$$\Pi_t = p_t f_G(\theta_{G,t}) \left(\exp(a_{z,t})(1-\sigma)N_t - \exp\left(-\xi \cdot a_{z,t}\right) \kappa v_{L,t}\right) - w_t(1-\sigma)N_t, \tag{5}$$

where $v_{L,t}$ is the number of vacancies posted by firm. Observe that output is linear with respect to number of workers - we would like to avoid job rationing described by Michaillat [9] so that Keynesian unemployment can be explained solely by the presence of frictions one the product market and price/wage stickiness. Notice, that productivity level $a_{z,t}$ appears in the expression for Π_t twice. One possible interpretation is that the total number of workers hired by the firm is delegated to complete two tasks: munufacturing and recruiting new workers. It implies that an increase in worker's productivity not only boosts the amount of goods produced by one worker but also decreases the number of workers which are needed to complete a certain amount of recruitment activities (and thereby avarage cost of posting one vacancy is reduced).

2.2 Law of motion in the labor market and consistency conditions

From now on, we use primes to denote forward lags of variables. As it has been already mentioned, it is assumed that the order (within the period) of job separations, hiring, posting vacancies and production is such as in den Haan et al. [5]. It means that the law of motion for employment is:

$$N' = (1-\sigma)N + M^L(1-(1-\sigma)N, v_L), \tag{6}$$

where M^L is Cobb-Douglas matching function and $U = 1-(1-\sigma)N$ denotes number of unemployed workers. A similar concept is present in the market for goods: there is a number of $M^G(v, \exp(a_z)(1-\sigma)N - \exp(-\xi \cdot a_z)\,\kappa v_L)$ successful trades given the number of visits v chosen by households and the total number of goods supplied by firms:

$$T = \exp(a_z)(1-\sigma)N - \exp(-\xi \cdot a_z)\,\kappa v_L. \tag{7}$$

The tightness in the labor market and the tightness in the market for goods are defined as follows:

$$\theta_L = \frac{1-(1-\sigma)N}{v_L}, \tag{8}$$

$$\theta_G = \frac{T}{v}. \tag{9}$$

Since both M^L and M^G are specified as constant returns to scale functions, then probabilities q_L, q_G, f_G can be expressed as functions of tightness that corresponds to a given market. In particular, we consider the following specifications of M^G and M^L:

$$M^G(v, T) = z_G v^{1-\alpha_G} T^{\alpha_G},$$

$$M^L\left(1-(1-\sigma)N,v_L\right)=z_L v_L^{1-\alpha_L}U^{\alpha_L},$$

where $0<\alpha_G<1$ and $0<\alpha_L<1$ and $z_L,\ z_G>0$, so that $q_L,\ q_G,f_G$ are:

$$q_L=\frac{M^L}{v_L}=z_L\theta_L^{\alpha_L},$$

$$q_G=\frac{M^G}{v}=z_G\theta_G^{\alpha_G},$$

$$f_G=\frac{M^G}{T}=z_G\theta_G^{\alpha_G-1},$$

arguments of functions were omitted to economize on notation. We assume the following form of stochastic discount factor:

$$\Delta_{t,t+1}=\beta\frac{p_t}{p_{t+1}}\left[\frac{\exp(a_{d,t+1})\cdot u'(c_{t+1})-\frac{1}{q_G(\theta_{G,t+1})}G'(v_{t+1})}{\exp(a_{d,t})\cdot u'(c_t)-\frac{1}{q_G(\theta_{G,t})}G'(v_t)}\right]. \qquad (10)$$

We impose the market clearing condition for the asset markets, i.e.:

$$\forall_t\ \ s_t=1. \qquad (11)$$

The resource constraint for the analyzed economy is:

$$c_t=f_G(\theta_{G,t})T_t. \qquad (12)$$

Stochastic disturbances are described by the following autoregressive processes:

$$a_{d,t+1}=\rho_D a_{d,t}+\epsilon_{d,t+1}, \qquad (13)$$

$$a_{z,t+1}=\rho_Z a_{z,t}+\epsilon_{z,t+1}, \qquad (14)$$

where $0 < \rho_Z, \rho_D < 1$ and $\begin{bmatrix} \epsilon_{d,t} \\ \epsilon_{z,t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \Sigma_{2\times2}\right)$, where $\Sigma_{2\times2}$ is variance-covariance matrix.

2.3 Equilibrium

We define equilibrium in a similar way to Michaillat and Saez [10].

Definition

A recursive competitive equilibrium (RCE) is a price function $p(N, a_d, a_z)$, a wage function $w(N, a_d, a_z)$, value functions $J_F(N, a_d, a_z)$ and $W(s, N, a_d, a_z)$, labor market tightness and product market tightness $\theta_L(N, a_d, a_z)$, $\theta_G(N, a_d, a_z)$, policy functions $c(N, a_d, a_z)$, $v(N, a_d, a_z)$, $s'(N, a_d, a_z)$, $v_L(N, a_d, a_z)$ and an employment function $N'(N, a_d, a_z)$ such that given processes $\{a_{Z,t}\}_{t=1}^{+\infty}$ and $\{a_{D,t}\}_{t=1}^{+\infty}$ and given N_0, $a_{Z,0}$, $a_{D,0}$, $s_0 = 1$:

1) Given prices, wages and product market tightness, $c(N, a_d, a_z)$, $v(N, a_d, a_z)$, $s'(N, a_d, a_z)$ solve worker's problem described by 1,

2) Given prices, wages, product market tightness and labor market tightness, 3 and 4 hold,

3) Worker's and firm's choices are consistent with $\theta_L(N, a_d, a_z)$ and $\theta_G(N, a_d, a_z)$, i.e.:

$$\theta_L(N, a_d, a_z) = \frac{1 - (1-\sigma)N(N_{-1}, a_{d,-1}, a_{z,-1})}{v_L(N, a_d, a_z)},$$

$$\theta_G(N, a_d, a_z) = \frac{\exp(a_z)(1-\sigma)N(N_{-1}, a_{d,-1}, a_{z,-1}) - \exp\left(-\xi \cdot a_z\right)\kappa v_L(N, a_d, a_z)}{v(N, a_d, a_z)},$$

4) Markets clear:

$$s'(N, a_d, a_z) = 1,$$

$$c(N, a_d, a_z) = f_G(\theta_G(N, a_d, a_z))$$

$$\cdot \left[\exp(a_z)(1-\sigma)N(N_{-1}, a_{d,-1}, a_{z,-1}) - \exp\left(-\xi \cdot a_z\right) \kappa v_L(N, a_d, a_z)\right],$$

5) Law of motion for employment holds:

$$N'(N, a_d, a_z) = (1-\sigma)N + M^L(1-(1-\sigma)N, v_L(N, a_d, a_z)).$$

2.4 Characterization of equilibrium

Equilibrium can be described by equations 2-14. It is a system of 13 equations that contains 15 variables. It means that values of prices and wages have to be pinned down by two additional conditions. Michaillat and Saez [10] consider three options: wages and prices determined by bargainig procedures, prices and wages that decentralize constrained-efficient outcome, perfectly sticky prices and wages (i.e. prices and wages become parameters). We adopt the last version of price and wage determination listed by Michaillat and Saez in our analysis. The logic behind our choice is supported by two arguments: firstly, wage rigidities can be an important source of propagation of productivity shocks (as noted by Hall [7]). Secondly, price stickiness plays a key role in amplification of demand shocks (as it is argued by Michaillat and Saez [10]).

The remaining issue is to choose the exact values for perfectly sticky prices and wages. We find that a natural choice is to set their values at levels that are consistent with steady state values of prices and wages that decentralize the constrained-efficient (or optimal given frictions) allocation (i.e. the solution to planner's problem that is constrained by frictions in the labor market and the market for products). In the next step we compute those values.

3 Constrained-efficient allocations

In this section we compute the planner's solution that corresponds to economy that is similar to the one presented above. Planner's allocations will play a significant role in the construction of our decomposition method. We use the notions: "constrained-efficient", "optimal given frictions" or simply "optimal" interchangeably.

3.1 Optimal allocation with two frictions

As we have mentioned, we decentralize the optimal allocation with two frictions to get some reasonable choice for perfectly rigid prices and wages which are then used in our empirical analysis. The social planner's problem that corresponds to the model presented earlier is:

$$V(N, a_d, a_z) = \max_{c, v_L, v, N'} \left\{ \exp(a_d) \cdot u(c) - G(v) + \beta \mathbb{E} V(N', a_d', a_z') \right\}$$

$$subject\ to:$$

$$c = M^G(v, T), \tag{15}$$

$$N' = (1-\sigma)N + M^L(1 - (1-\sigma)N, v_L), \tag{16}$$

$$T = \exp(a_z)(1-\sigma)N - \exp(-\xi \cdot a_z)\kappa v_L, \tag{17}$$

where V is the value function associated with planner's problem. We plug first constraint and third constraint into Bellman equation and we get:

$$V(N, a_d, a_z) = \max_{v, v_L} \{ \exp(a_d) \cdot u\left(M^G(v, \exp(a_z)(1-\sigma)N - \exp(-\xi \cdot a_z)\kappa v_L)\right)$$

$$-G(v) + \beta \mathbb{E} V(N', a_d', a_z')\},\ subject\ to:$$

$$N' = (1-\sigma)N + M^L\left(1-(1-\sigma)N, v_L\right).$$

We compute first order conditions:

$$\exp(a_d) \cdot u'(c) \cdot M^G_v = G'(v), \tag{18}$$

$$\beta \mathbb{E} V_N(Z') \cdot M^L_{v_L} = \exp(a_d) \cdot u'(c) M^G_T \kappa \exp\left(-\xi \cdot a_z\right), \tag{19}$$

where $Z = \{N, a_d, a_z\}$. The envelope condition is:

$$V_N(Z) = \exp(a_d) \cdot u'(c) \cdot M^G_T \cdot a_z(1-\sigma) + \beta \mathbb{E} V_N(Z')(1-\sigma)\left[1 - M^L_U\right]. \tag{20}$$

Equations 15-20 together with 8, 9, 13 and 14 characterize the planner's solution. The following proposition gives formulas for prices and wages which guarantee that the competitive equilibrium allocation replicates the optimal outcome.

Proposition

If the price function $p(Z)$ is given by

$$p(Z) = \frac{\alpha_G \exp(a_d) \cdot u'(c(Z))}{\beta \mathbb{E} W_s(1, Z)} \tag{21}$$

and wage function $w(Z)$ is characterized by the system:

$$\begin{cases} w(Z) = p(Z) \cdot f_G(\theta_G(Z)) \cdot a_Z + \mathbb{E}\left(\Delta(Z', Z)(1-\sigma) X(Z') V_N(Z') - X(Z) V_N(Z)\right), \\ X(Z') = \frac{(1-\alpha_L)\beta}{\exp(a_d) \cdot u'(c) M^G_T(Z) \cdot \Delta(Z', Z) \cdot (1-\sigma)}, \\ V_N(Z) = \exp(a_d) \cdot u'(c) \cdot M^G_T \cdot a_z(1-\sigma) + \mu(1-\sigma)\left[1 - M^L_U\right], \end{cases} \tag{22}$$

then the competitive allocation is optimal.

Proof

Our strategy is to show, that the allocation determined by equations 2-14, 21 and 22 satisfies conditions that characterize planner's solution. It is immeadiate that equations 15-17, 20, conditions that characterize θ_L, θ_G and shocks appear both in the system that characterizes competitive outcome and in the system that describes optimal allocation. It means that it remains to show that conditions that characterize the decentralized outcome imply 18 and 19.

Let us begin with equation 18. Observe that an alternative (to Euler equation 2) way to describe the maximizing behaviour of workers is equation[1] (we omit some arguments for notational convenience):

$$\exp(a_d) \cdot u'(c) \cdot q_G(\theta_G) - G'(v) = p \cdot q_G(\theta_G) \cdot \beta \mathbb{E} W_s(1, Z').$$

If we use the formula for price: $p(Z) = \frac{\alpha_G \exp(a_d) \cdot u'(c(Z))}{\beta \mathbb{E} W_s(1,Z)}$ then we get:

$$\exp(a_d) \cdot u'(c) \cdot (1 - \alpha_G) q_G(\theta_G) = G'(v),$$

from $M^L(1 - (1-\sigma)N, v_L) = z_L v_L^{1-\alpha_L} U^{\alpha_L}$ and 8 we obtain:

$$\exp(a_d) \cdot u'(c) \cdot M_v^G = G'(v),$$

which is identical to 18.

We derive 19 from conditions that describe the competitive allocation. The first equation that characterizes wages is:

$$w(Z) = p(Z) \cdot f_G(\theta_G(Z)) \cdot a_Z + \mathbb{E}\left(\Delta(Z', Z)(1-\sigma) X(Z') V_N(Z') - X(Z) V_N(Z)\right),$$

which together with 4 implies that:

$$J_F(Z) = X(Z) \cdot V_N(Z).$$

[1] Eliminate c and s' in 1 (using the household's constraints) and derive the FOC with respect to v.

We plug this formula into 3, use the second equation that describes wage formation: $X(Z') = \frac{(1-\alpha_L)\beta}{\exp(a_d)\cdot u'(c)M_T^G(Z)\cdot\Delta(Z',Z)\cdot(1-\sigma)}$ and the fact that $M_{v_l}^L = (1-\alpha_L)q_L$ to get:

$$\beta\mathbb{E}V_N(Z')\cdot M_{v_L}^L = \exp(a_d)\cdot u'(c)M_T^G\kappa\exp\left(-\xi\cdot a_z\right),$$

which is identical to 19. *Q.E.D.*

Values of p and w (defined in 21 and 22) computed for the stationary equilibrium (i.e. in which $a_{Z,t} = a_{D,t} = 0$ for all t) are denoted by $\bar{p}$ and $\bar{w}$.

Definition

Allocation and prices is said to be a *competitive equilibrium with perfectly sticky (rigid) prices and wages* if it is described with 2-14, $p(Z) = \bar{p}$ and $w(Z) = \bar{w}$.

Statistical characteristics of allocation that is associated with competitive equilibrium with perfectly sticky wages and prices will be compared with empirical data. The unemployment level for this allocation is denoted by U_s.

We conclude this section with two comments on derived results. The first one is an observation that optimal wage is equal to sum of two components. The first one (i.e. $p(Z)\cdot f_G(\theta_G(Z))\cdot a_Z$) is marginal product of a worker hired by firm. The second is $\mathbb{E}\left(\Delta(Z',Z)(1-\sigma)X(Z')V_N(Z') - X(Z)V_N(Z)\right)$ which can be interpreted as increase (if it is positive) or decrease (if negative) of worker's value for the firm. The second comment is the answer to the following question: is it possible to support formulas 21 and 22 with some price-setting protocols (e.g. Nash Bargaining)? We found it hard to support 22 with protocols that can be found in the literature. However, we have managed to justify 21 with the concept of competitive search equilibrium (analogous to the one presented by [11]) - which is shown in the Appendix.

3.2 Optimal allocation with one friction

In this part we describe the optimal allocation in economy with one friction (in the labor market). The relationship between this allocation and allocation generated by competitive equilibrium with perfectly sticky wages and prices is crucial for defining Keynesian and frictional unemployment in our work.

The social planner's problem in economy with one friction that corresponds to our model with two frictions is:

$$V_1(N, a_d, a_z) = \max_{c, v_L, N'} \{\exp(a_d) \cdot u(c) + \beta \mathbb{E} V_1(N', a_d', a_z')\}, \ \textit{subject to}: \quad (23)$$

$$c = \exp(a_z)(1-\sigma)N - \exp(-\xi \cdot a_z)\, \kappa v_L, \quad (24)$$

$$N' = (1-\sigma)N + M^L(1-(1-\sigma)N, v_L), \quad (25)$$

where V_1 is the value function associated with planner's problem. We plug constraints to Bellman equation and derive first order condition with respect to v_L:

$$\beta \mathbb{E} V_{1,N}(Z') \cdot M^L_{v_L} = \exp(a_d) \cdot u'(c) \cdot \kappa \exp(-\xi \cdot a_z), \quad (26)$$

where $Z = \{N, a_d, a_z\}$. The envelope condition is:

$$V_{1,N}(Z) = \exp(a_d) \cdot u'(c) \cdot a_z(1-\sigma) + \beta \mathbb{E} V_{1,N}(Z')(1-\sigma)\left[1 - M^L_U\right]. \quad (27)$$

By U_f we denote the level of unemployment for the allocation described in this section (it is called frictional unemployment from now on).

3.3 Frictional unemployment and Keynesian overemployment/underemployment

According to Keynesian tradition, the sources of periods characterized by long slumps and high unemployment are: imperfect adjustment of prices, wages and insufficient demand. Since wages do not fall during recessions then demand for labor remains insufficient for the employment level to recover. Symmetrically, these two elements are responsible for an amplification of increase in employment and output during economic booms: prices adjust upwards too slowly which in turn boosts the demand and production. Both of them appear in the model of competitive equilibrium with perfectly sticky wages and prices. Firstly, price stickiness is introduced by ascribing constant values to prices and wages. Secondly, we have an explicit formulation of demand in the model (i.e. the number of visits v_t) which can attain low levels if the demand shock $a_{d,t}$ decreases. Concluding, these considerations tell us (a similar conclusions were made by Hall [7]) that employment in economy with price rigidities and a productive role of aggregate demand is higher (lower) in booms (recessions) than in economy in which those features are absent.

All this means that if we want to isolate Keynesian "underemployment" or "overemployment" then we need to compare allocation generated by the model with perfectly sticky prices, wages and two frictions in place with the model with flexible prices and only one friction (in the labor market). Since in our analysis flexible prices are assumed to decentralize the optimal outcome, then to get an allocation generated by a model with flexible prices and one friction, we need to remove constraint 15 from the planner's problem with two frictions. The resulting dynamic problem is 23. The difference between the unemployment in model which describes the planner's problem with one friction - U_f and the unemployment rate in model with two frictions, perfectly rigid prices, perfectly sticky wages - U_s is called Keynesian overemployement (if the difference is positive) and Keynesian underemployment (if it is equal

0 or negative):

$$\begin{cases} Keynesian\,overemployment = U_f - U_s, & for\,U_f > U_s, \\ Keynesian\,underemployment = U_f - U_s, & for\,U_f \leq U_s. \end{cases}$$

4 Calibration and estimation

4.1 Missing specifications

We consider the following specification of the utiliy function:

$$\exp(a_d) \cdot u(c) - G(v) = \exp(a_d) \cdot \log(c) - \frac{\eta}{2}v^2.$$

4.2 Setting the parameter values

Parameters in our model can be divided into three groups:

- First group of parameters is calibrated to adjust some steady state proportions in the model to analogous proportions that can be computed from the data,
- Second group includes perfectly sticky prices.
- Third group contains parameters which are estimated by means of Bayesian methods (MCMC algorithm & Kalman filter).

4.2.1 Calibration

Calibrated parameters are: σ, β, α_L, z_L, κ and z_G. The value of σ for Germany, France, Italy and Spain is taken from Hobijn and Sahin [8]. We set the quarterly discount rate $\beta = 0.99$. We take $\alpha_L = 0.5$ as in Petrongolo and Pissarides [13] or Michaillat[9].We use the steady state version of system

Table 1: Targeted moments, calibration

	Germany		France		Italy		Spain	
	Data	Model	Data	Model	Data	Model	Data	Model
av. unemployment	9.1%	9.0%	8.9%	8.7%	8.9%	8.4%	15.3%	15.0%
av. cap. utilization	83.9%	83.9%	84.4%	84.4%	74.5%	74.5%	78.1%	78.1%
hiring probability	21%	32%	28%	36%	15%	23%	25%	33%

15-20, 8, 9[2] to find values of z_L, κ, z_G which imply that the moments generated by the model are close to their empirical equivalents. In particular, we take: unemployment rate $1-(1-\sigma)N_{ss}$, probability of getting a job by an unemployed person $\frac{M^L_{ss}}{U_{ss}}$ and capacity utilization $\frac{M^G_{ss}}{T_{ss}}$ as criterions for the comparison[3].

We think, that the choice of parameter z_G requires some broader comment. Firstly, it has been observed during our simulations of steady state values, that all moments (except for the capacity utilization) remain uneffected by choice of z_G, α_G and η. Since we have only one moment (capacity utilization) and three parameters to be pinned down, we do the following: we construct a function $z_G(\alpha_G, \eta)$ that for any values of α_G and η returns value of z_G for which the value of $\frac{M^G_{ss}}{T_{ss}}$ generated by the model is identical with the empirical value. Parameters α_G, η will be estimated by means of Bayesian methods.

[2]Note, that since we consider steady state version of the competitive allocation with perfectly sticky wages and prices and because we assume that sticky prices and wages are steady state values of prices and wages that decentralize the optimal solution, then stationary allocations 15-20, 8, 9 and 2-12 are identical so we can consider the planner's allocation which is more tractable.

[3]We use the data on the proportion of unemployed people who remain without a job less than one month to get the quarterly hiring rate $\frac{M^L_{ss}}{U_{ss}}$.

4.2.2 Solution method

We use the method suggested by P. Rendahl to solve the linearized version of the model, i.e. to obtain the following characterization of the dynamical system described by equations 2 to 14:

$$Y_{+1} = \Lambda^* \cdot Y,$$

where Y is a vector of steady state deviations (not necessarily in %) of all variables that appear in 2 to 14 and $\Lambda^*$is a transition matrix. The starting point of the algorithm is a linearized version of the model:

$$A \cdot Y_{+1} + B \cdot Y + CY_{-1} = \mathbb{O}. \tag{28}$$

We take initial guess of transition matrix Λ_0 and after making substitution $Y_{+1} = \Lambda_0 Y$ in 28 we get:

$$Y = -(A\Lambda_0 + B)^{-1} \cdot C \cdot Y_{-1}.$$

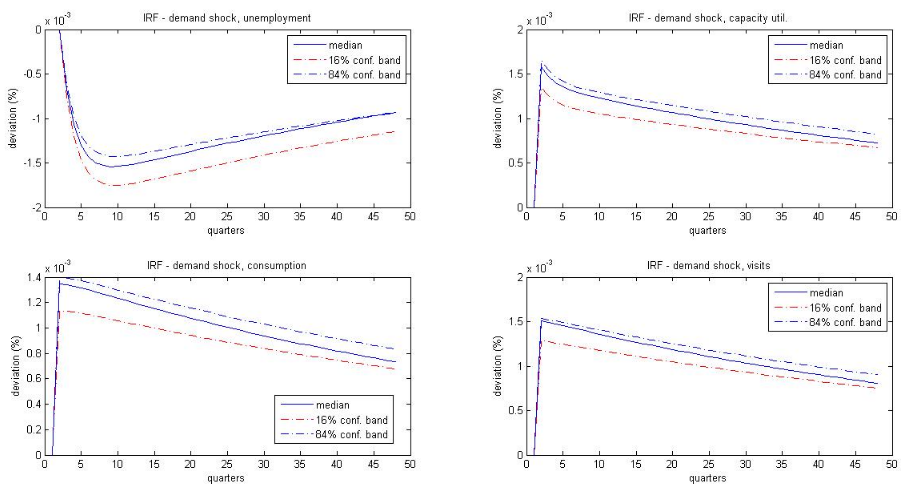
Matrix $-(A\Lambda_0 + B)^{-1} \cdot C$ becomes our next candidate for the transition matrix and we denote it by Λ_1. Then we substitute Λ_1 to 28 and obtain Λ_2. We repeat this procedure until convergence, i.e. until we find n that satisfies $\max_{i,j} \{|\Lambda_{i,j,n} - \Lambda_{i,j,n-1}|\} < \epsilon$, where ϵ is a small positive number.

4.2.3 Estimation

Bayesian methods are used for estimation of α_G, η, ξ and parameters that characterize stochastic processes: ρ_Z, ρ_D and $\Sigma_{2\times 2}$. It means that we have eight values to be estimated.

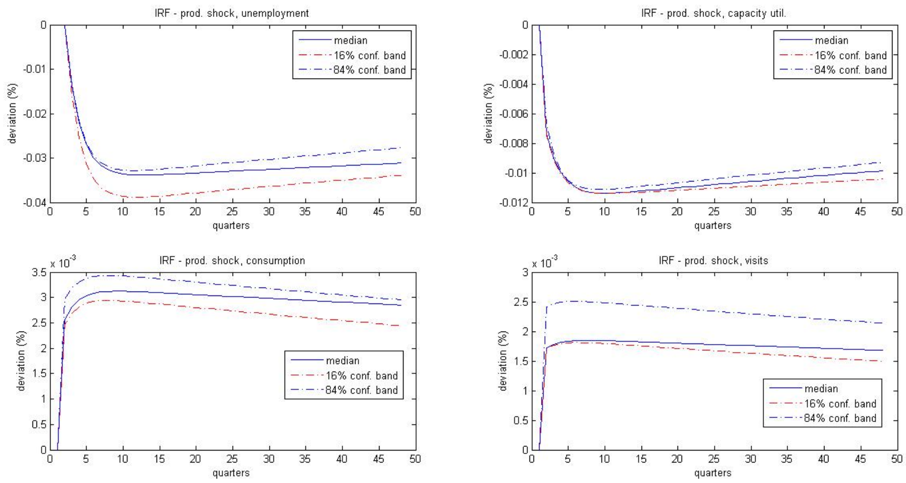
A standard MCMC algorithm is run: for every iteration it draws a set of parameters from the joint prior distribution and computes likelihood for the model specified for those values of parameters in each iteration. I use capacity utilization and unemployment as measured signals in the state space representation of the model. The remaining issue is whether we are able to identify shocks given these two time series. On the one hand impulse responses of unemployment are decreasing with respect to both shocks but on the other hand capacity utilization increases when economy is affected by a demand shock and decreases when the system is hit by a productivity shock (Figures 1 and 2[4]). It implies that shocks' impacts on the system are orthogonal and hence we can identify them. Estimated and calibrated parameter values are presented in the Appendix.

Figure 1: IRFs: demand shock, France



[4]We report the results for France in the main text, IRFs for other countries can be found in the Appendix.

Figure 2: IRFs: productivity shock, France



By means of the Kalman filter we compute conditional expectations of state variables given empirical data. Matrix Λ^* is used for generating the time series of capacity utilization and unemployment and compare it with empirical observations (see Figures 3 and 4[5]).

[5] We report the results for France in the main text, comparisons for other countries can be found in the Appendix.

Figure 3: Empirical data and simulated time series: unemployment, France

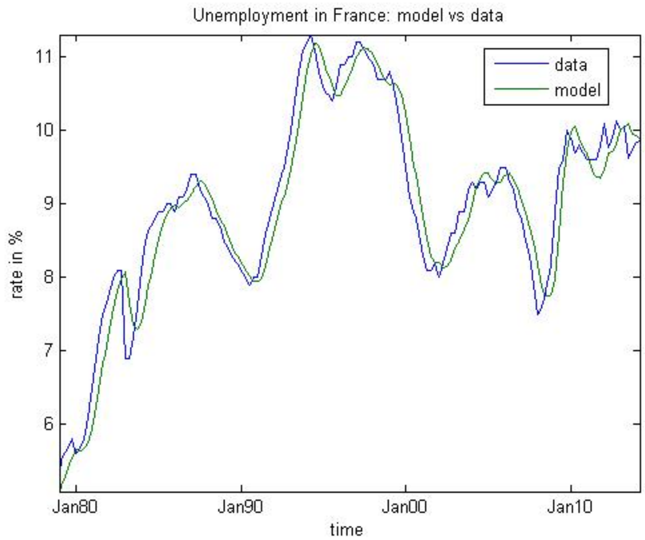
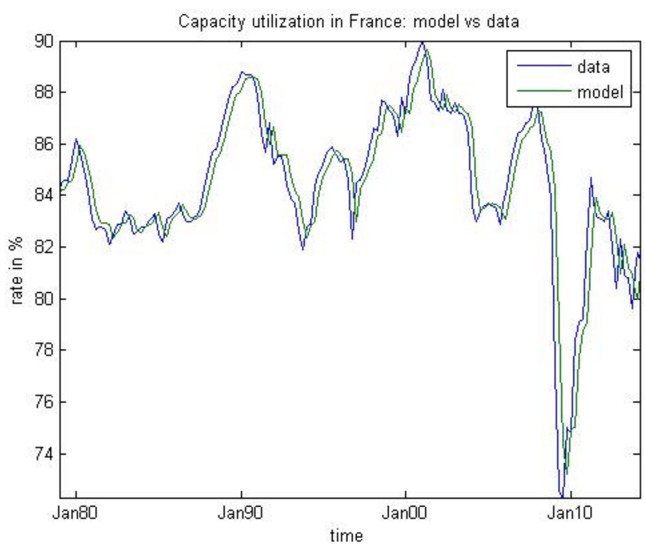


Figure 4: Empirical data and simulated time series: capacity utilization, France



5 Unemployment decomposition

We compute the steady state, simulate the model described with 24-27, 8-9, 13-14 and use the series of stochastic shocks that were calculated when the full model (two frictions, sticky wages and prices) was adjusted to data. The resulting path of unemployment is frictional unemployment U_f. In Figure 5 we compare it with the level of unemployment in the full model. The difference between two paths is Keynesian underemployment (overemployment) which is presented for the four analyzed economies in Figure 6.

Figure 5: Total unemployment and frictional unemployment, France

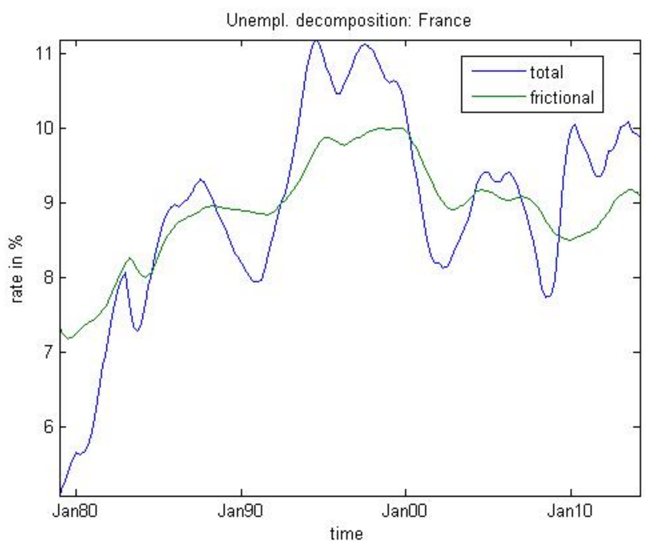
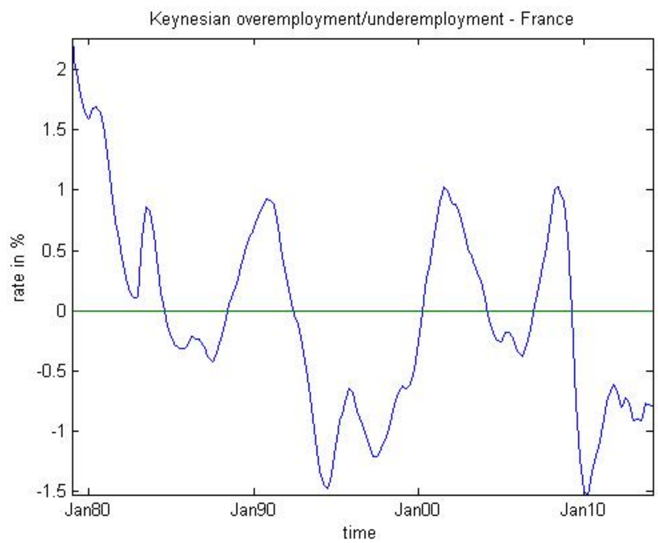


Table 2: Relative volatilities of the two components: frictional and Keynesian

	Germany	France	Italy	Spain
$\frac{\sigma_{U_f}}{\sigma_{U_s}}$	0.34	0.47	0.22	0.45
$\frac{\sigma_{U_f - U_s}}{\sigma_{U_s}}$	0.71	0.64	0.84	0.62

Figure 6: Keynesian overemployment/underemployment, France



We can observe, that frictional component plays a relatively less important role as a determinant of the total volatility of unemployment rate in Italy[6] than in the remaining economies: in Table 2 we report two indicators: the first one is the ratio of the standard deviation of frictional unemployment to the standard deviation of the total unemployment rate and the second is the ratio of the standard deviation of Keynesian overemployment (underemployment) to the standard deviation of the total unemployment rate.

Results presented in Table 2 suggest the direction of stabilization policies in the analyzed economies: measures that decrease price stickiness (e.g.,

[6]Again, figures that show the decomposition of unemployment for Germany, Italy and Spain can be found in the Appendix

liberalisation policies) or those which stimulate demand can be relatively more effective in Italy than in the remaining countries during recessions.

6 Concluding remarks

In this paper we have developed a method that allows for the decomposition of unemployment into two components: Keynesian and frictional. Since we conduct the analysis by means of the DSGE model, it is relatively easy to extend this framework to study various issues associated with effects of e.g. fiscal policy or labor market policies on unemployment.We have presented empirical results which indicate that the behavior of

7 Appendix

7.1 Microfoundations for the price setting formula

The concept which is similar to Moen's [11] competitive search equilibirum (and was described by Bai et al. [2]) is used to get an optimal price in the market for products. In particular it is assumed, that goods are sold in differentiated markets that are indexed by price and market tightness. Households choose one of these market when deciding where to make a visit. The situation is similar for the sector of firms: output generated by a single job-worker relationship (active job) is sent to the most atractive market (in terms of price and tightness).

Let ς denote the value of an outside option for active jobs that go to the most attractive market. To implement the analog of Moen's construction in this model, we proceed in the following way. It is assumed, that households maximize the profit from making a visit and choose the most convenient market (indexed by price and tightness) subject to profit maximization made by an active job-worker relationship, i.e.:

$$\max_{p,\theta_G} \left\{ \exp(a_d) \cdot u'(c) \cdot q_G(\theta_G) - G'(v) = p \cdot q_G(\theta_G) \cdot \beta \mathbb{E} W_s(1, Z') \right\}, \; subject\ to:$$

$$p f_G(\theta_G) \exp(a_z) - w + \mathbb{E}\left[\Delta(1-\sigma) J_F'\right] \geq \varsigma.$$

First order conditions associated with this maximization problem are:

$$\exp(a_d) q_G'(\theta_G) u'(c) - p q_G'(\theta_G) \beta \mathbb{E} W_s(1, Z') + \lambda p f_G'(\theta_G) a_z = 0,$$

$$q_G(\theta_G) \beta \mathbb{E} W_s(1, Z') = \lambda f_G(\theta_G) a_z,$$

where λ is Lagrange multiplier associated with the constraint. We can compute λ from the second FOC and plug it to the first one to get:

$$p(Z) = \frac{\alpha_G \exp(a_d) \cdot u'(c(Z))}{\beta \mathbb{E} W_s(1, Z)}, \tag{29}$$

which is identical to 21.

7.2 Calibrated and estimated parameter values

Table 3: Values of calibrated parameters

Parameter	Germany	France	Italy	Spain
z_L	0.41	0.45	0.31	0.43
σ	0.032	0.034	0.021	0.059
κ	0.71	0.73	0.70	0.69

Figure 7: IRFs: demand shock, Germany

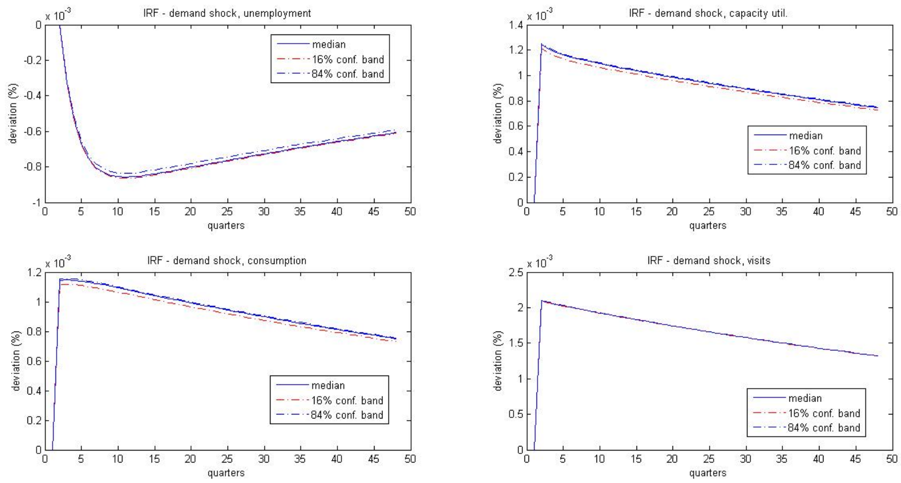


Table 4: Values of estimated parameters (means of posterior distribution)

Parameter	Germany	France	Italy	Spain
ξ	3.68	5.11	4.33	3.23
η	1.3	0.63	3.1	1.2
α_G	0.49	0.36	0.44	0.54
ρ_D	0.992	0.989	0.986	0.991
ρ_Z	0.988	0.992	0.995	0.984
σ_D	0.05	0.04	0.06	0.04
σ_Z	0.02	0.01	0.03	0.01
ρ_{corr}	0.27	0.35	0.21	0.23

7.3 Impulse response functions

Figure 8: IRFs: demand shock, Italy

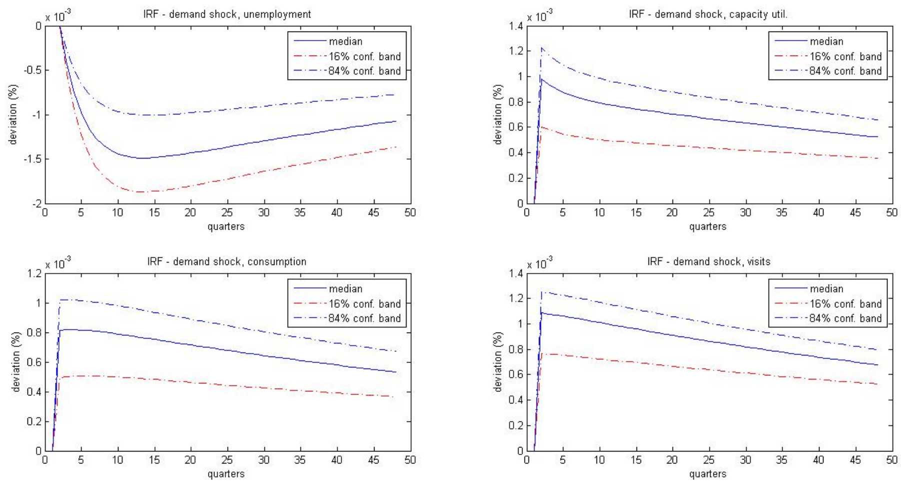


Figure 9: IRFs: demand shock, Spain

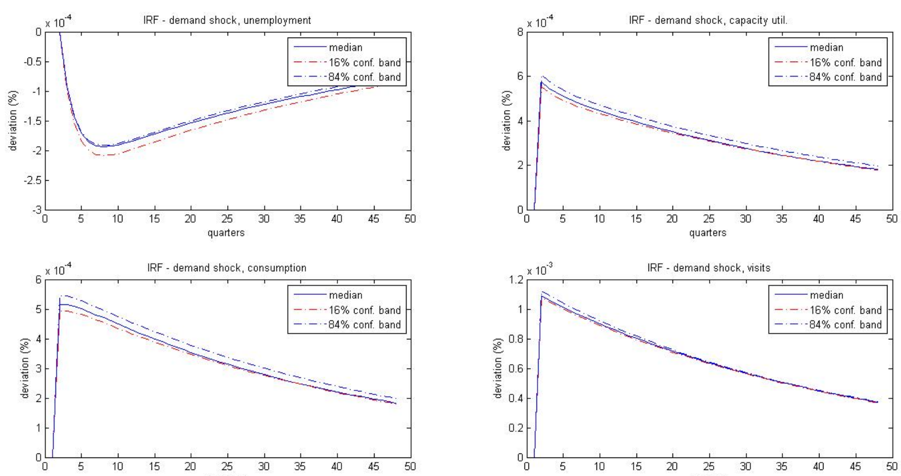


Figure 10: IRFs: productivity shock, Germany

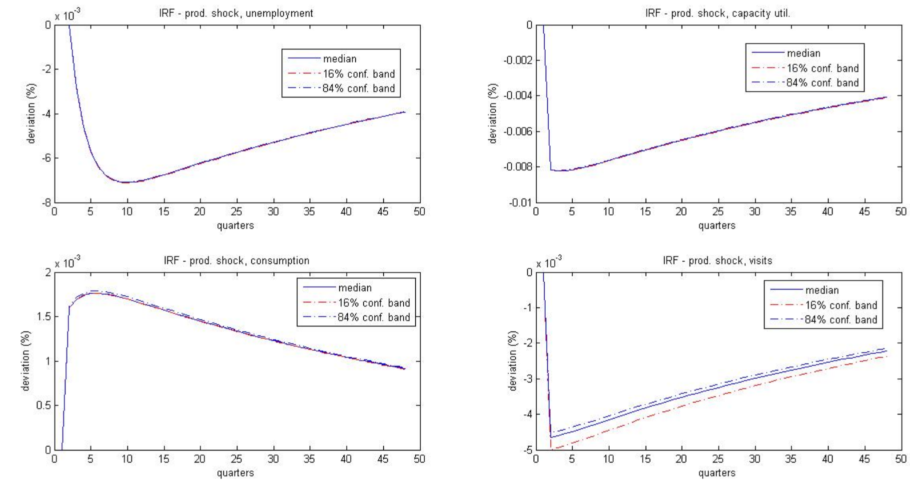


Figure 11: IRFs: productivity shock, Italy

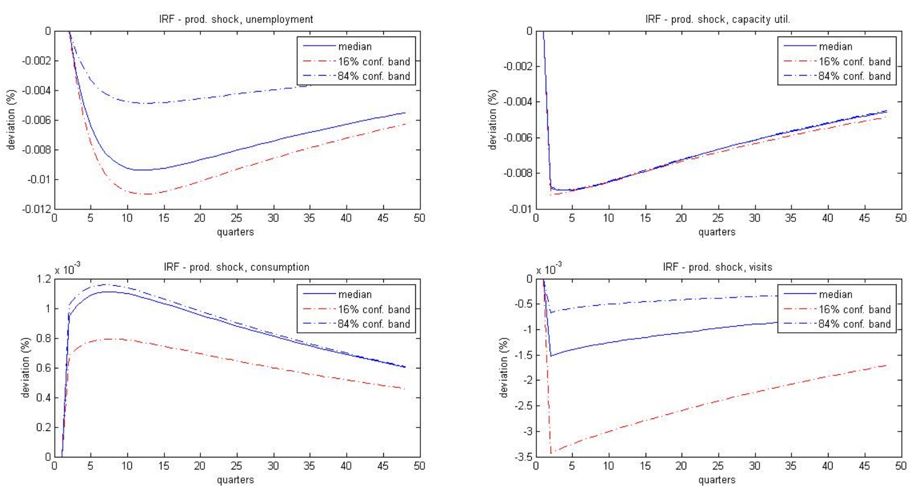


Figure 13: Empirical data and simulated time series: unemployment, Germany

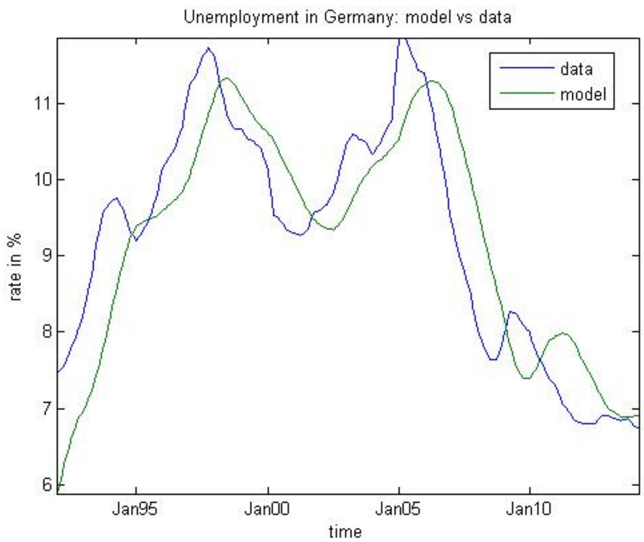
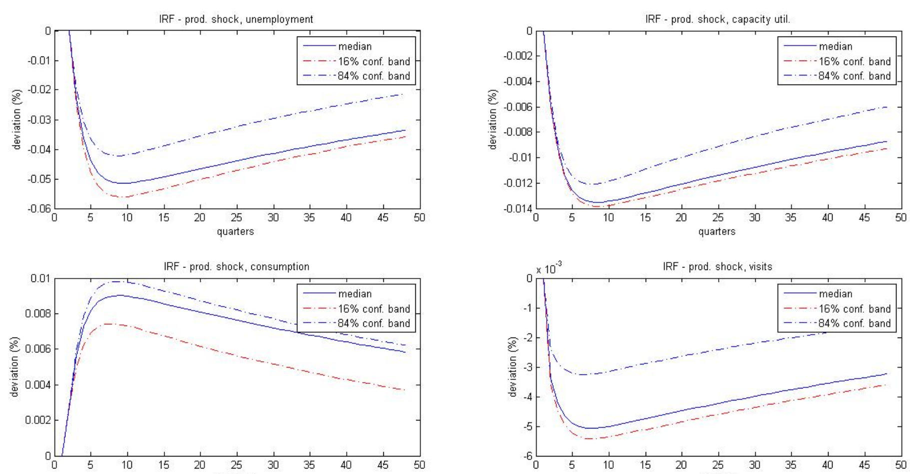


Figure 12: IRFs: productivity shock, Spain



7.4 Data fit

Figure 14: Empirical data and simulated time series: unemployment, Italy

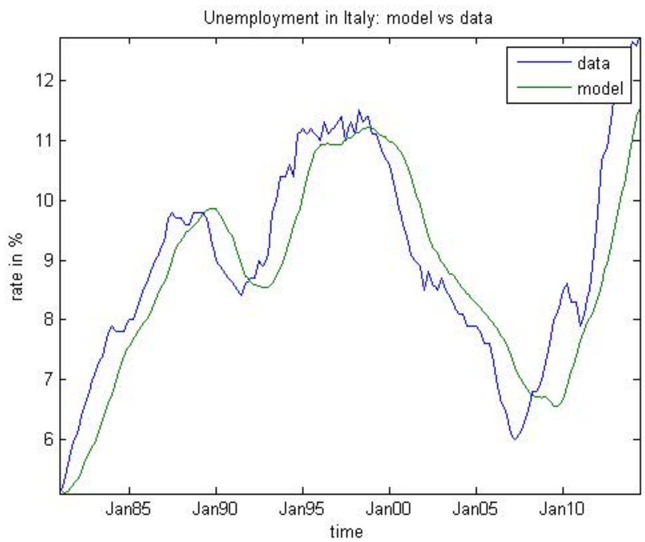


Figure 15: Empirical data and simulated time series: unemployment, Spain

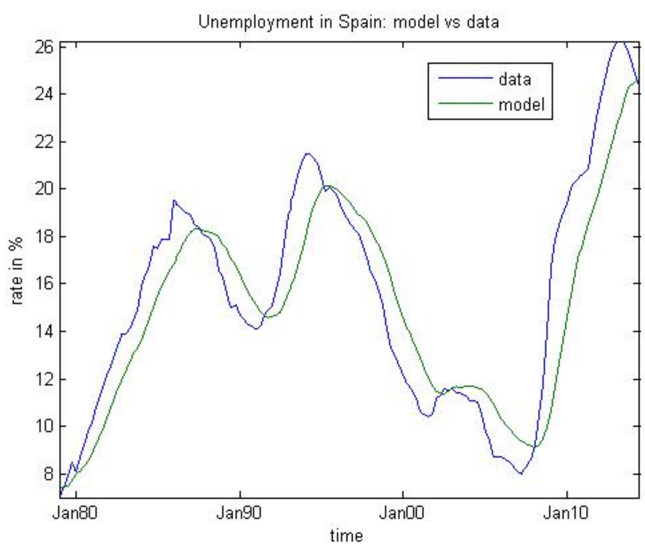


Figure 16: Empirical data and simulated time series: capacity utilization, Germany

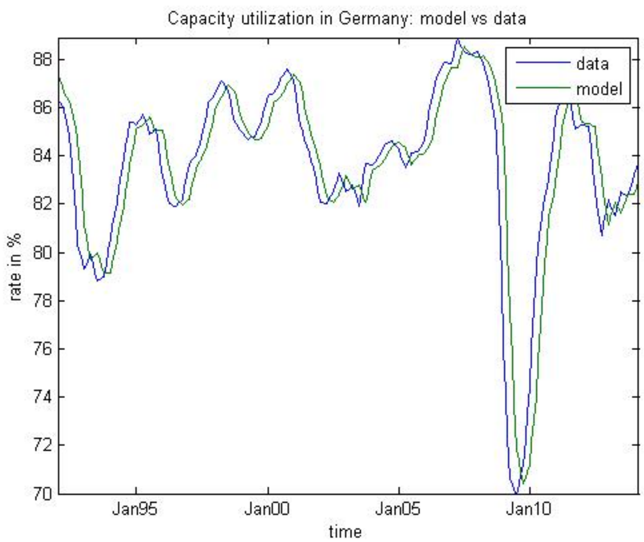


Figure 17: Empirical data and simulated time series: capacity utilization, Italy

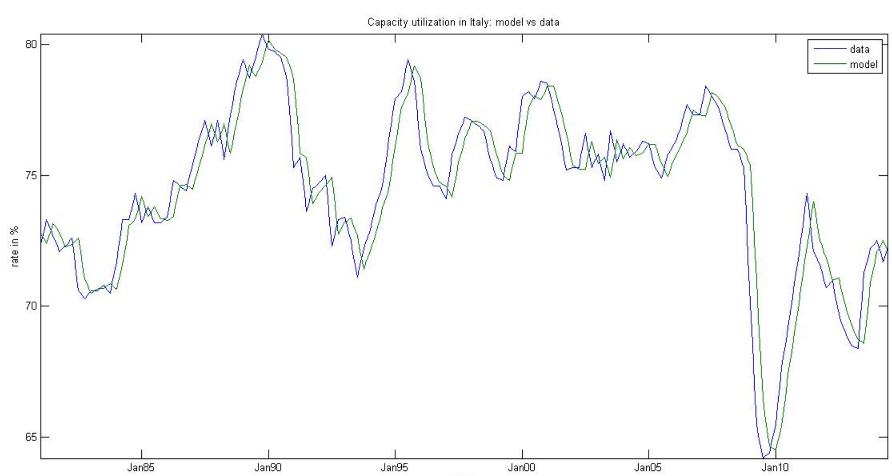


Figure 19: Total unemployment and frictional unemployment, Germany

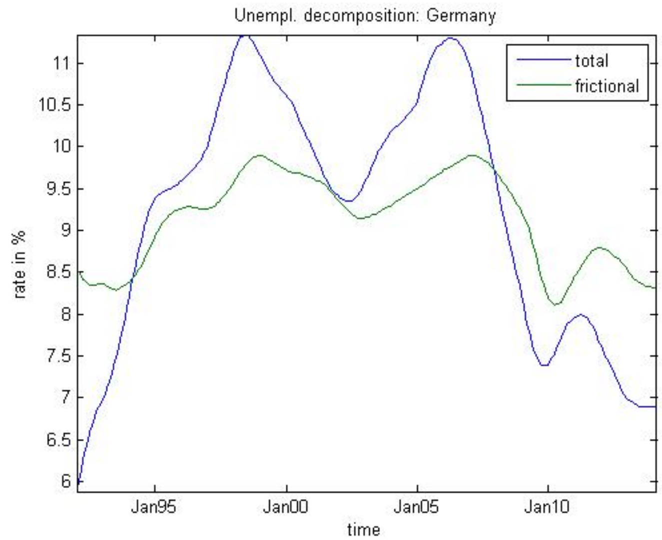
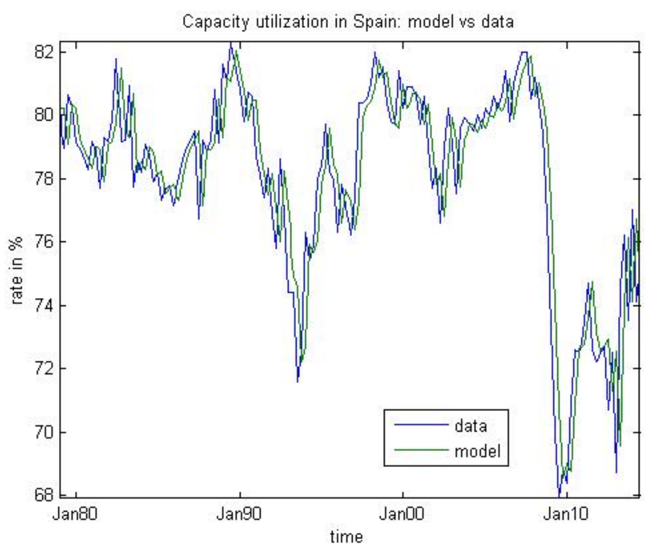


Figure 18: Empirical data and simulated time series: capacity utilization, Spain



7.5 Unemployment decomposition

Figure 20: Total unemployment and frictional unemployment, Italy

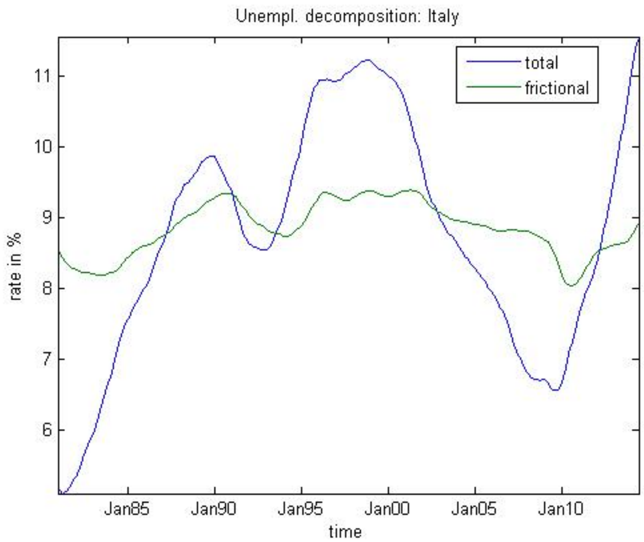


Figure 21: Total unemployment and frictional unemployment, Spain

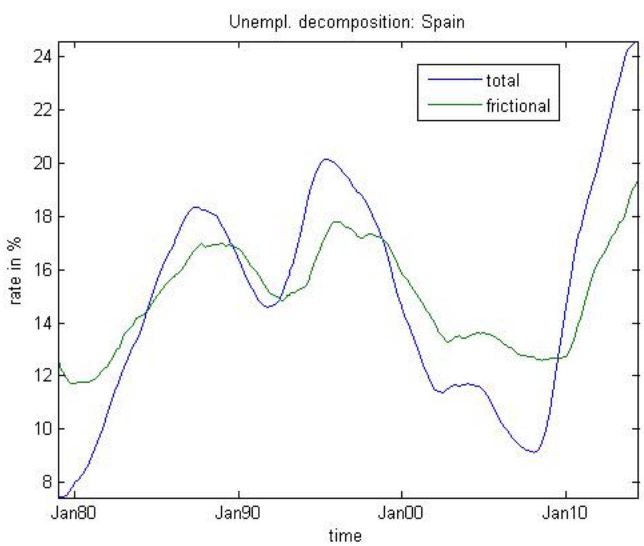


Figure 22: Keynesian overemployment/underemployment, Germany

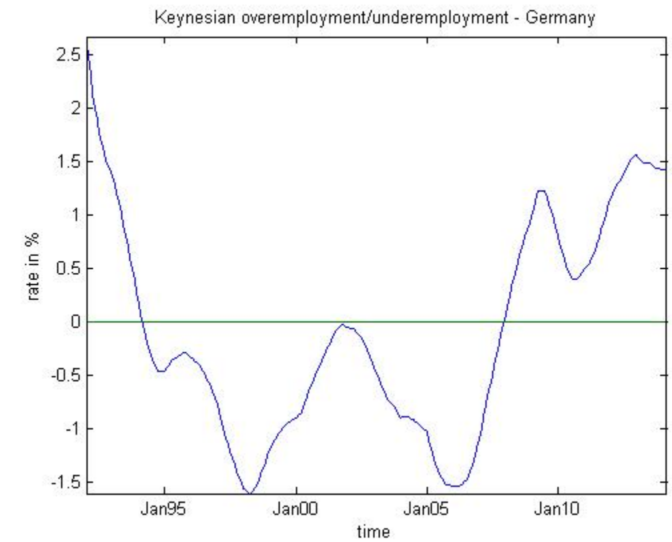


Figure 23: Keynesian overemployment/underemployment, Italy

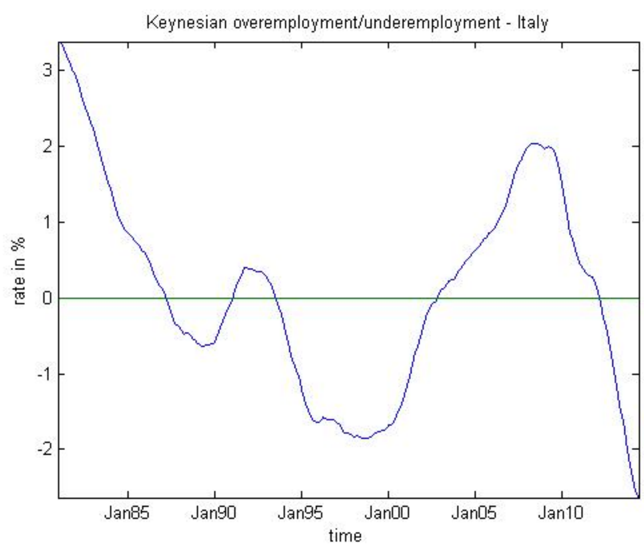
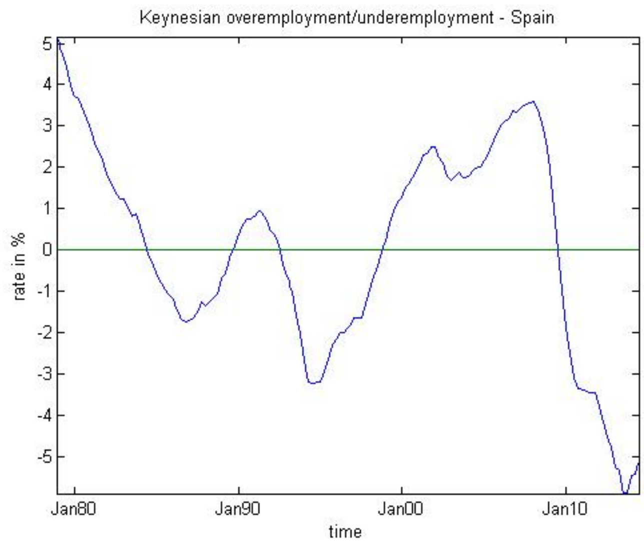


Figure 24: Keynesian overemployment/underemployment, Spain



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