# The Real Exchange Rate, Real Interest Rates, and the Risk Premium

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Define the excess return or "risk premium" on Foreign s.t. bonds:

$$\lambda_t \equiv i_t^* + E_t s_{t+1} - s_t - i_t = r_t^* + E_t q_{t+1} - q_t - r_t$$

The famous Fama regression demonstrates that as $r_t^* - r_t$ falls, $\lambda_t$ falls - I verify this for real rather than nominal interest rates

On the other hand, as $r_t^* - r_t$ falls (i.e., $r_t - r_t^*$ rises), Home currency appreciates "excessively" – more than can be explained by expectations of future interest rates under UIP

Are these two findings:

$$\text{cov}(-\lambda_t, r_t - r_t^*) < 0$$
$$\text{cov}(q_t - q_t^{IP}, r_t - r_t^*) < 0$$

arising from the same source?

No. They seem to say the opposite.

$\text{cov}(-\lambda_t, r_t - r_t^*) < 0$

means when home $r_t$ is high (relative to $r_t^*$, relative to average), home deposits are *riskier*.

$\text{cov}(q_t - q_t^{IP}, r_t - r_t^*) < 0$

means when home $r_t$ is high (relative to $r_t^*$, relative to average), the home currency is stronger than it would be under interest parity. Why? Because home deposits are *less risky*.

1. Empirical methodology
2. Empirical results
3. Why findings are a puzzle
   -- not readily explainable by complete-market risk-premium models
   -- not readily explainable by "delayed overshooting" models
4. Type of model that resolves puzzle.

Real interest rates and real exchange rates.

Rewrite: $q_t - E_t q_{t+1} = -(r_t - r_t^* - \bar{r}) - (\lambda_t - \bar{\lambda})$

Iterate forward to get:

$$q_t - \lim_{j\to\infty}\left(E_t q_{t+j}\right) = -R_t - \Lambda_t$$

where

$$R_t \equiv \sum_{j=0}^{\infty} E_t(r_{t+j} - r_{t+j}^* - \bar{r}) \qquad \Lambda_t \equiv \sum_{j=0}^{\infty} E_t(\lambda_{t+j} - \bar{\lambda})$$

$\Lambda_t$ - “level risk premium”

We find evidence for long run purchasing power parity: $\lim_{j\to\infty} E_t q_{t+j} = \bar{q}$

$q_t = q_t^{IP} - \Lambda_t$

## Data

U.S., Canada, France, Germany, Italy, Japan, U.K., and “G6”

G6 is like doing panel regressions

Exchange rates – last day of month (noon buy rates, NY)
Prices – consumer price indexes
Interest rates – 30-day Eurodeposit rates (last day of month)

Monthly, June 1979 – October 2009

VAR methodology
Two different VAR models:

Model 1: $\left[ q_t, i_t - i_t^*, i_{t-1} - \pi_t - (i_{t-1}^* - \pi_t^*) \right]$

Model 2: $\left[ q_t, i_t - i_t^*, \pi_t - \pi_t^* \right]$

(Extensions include long-term bond yields and stock returns.)

Estimate VAR with 3 lags. (Extension with 12 lags.)

Use standard projection measures to estimate

$r_t - r_t^* = i_t - i_t^* - (E_t \pi_{t+1} - E_t \pi_{t+1}^*)$, and $q_t^{IP} \equiv -\sum_{j=0}^{\infty} E_t (r_{t+j} - r_{t+j}^* - \overline{r}) + \overline{q}$

Then $\lambda_t$ is constructed as $\lambda_t \equiv r_t^* + E_t q_{t+1} - q_t - r_t$
$\Lambda_t$ estimate is constructed from $\Lambda_t = q_t^{IP} - q_t$

## Fama Regression in Real Terms: $q_{t+1} - q_t - \hat{r}_t^d = -\zeta_q - \beta_q \hat{r}_t^d + u_{q,t+1}$

| *Country* | $\hat{\beta}_1$ | *90% c.i.(* $\hat{\beta}_1$*)* |
|---|---|---|
| Canada | 0.862 | (-0.498,2.222) |
| | | (-0.632,2.908) |
| | | (-0.676,2.800) |
| France | 1.576 | (-0.117,3.269) |
| | | (0.281,3.240) |
| | | (-0.125,3.602) |
| Germany | 1.837 | (-0.015,3.689) |
| | | (0.687,4.458) |
| | | (0.589,4.419) |
| Italy | 0.360 | (-1.336,2.056) |
| | | (-1.087,2.136) |
| | | (-1.358,2.328) |
| Japan | 2.314 | (0.768,3.860) |
| | | (0.746,4.300) |
| | | (0.621,4.441) |
| United Kingdom | 2.448 | (0.854,4.042) |
| | | (0.873,4.614) |
| | | (1.039,4.846) |
| G6 | 1.933 | (0.318,4.548) |
| | | (0.510,3.932) |
| | | (0.473,4.005) |

## Regression of $q_t$ on $\hat{r}_t - \hat{r}_t^*$ : $q_t = \beta_0 + \beta_1(\hat{r}_t - \hat{r}_t^*) + u_{t+1}$

| *Country* | $\hat{\beta}_1$ | *90% c.i.( $\hat{\beta}_1$ )* |
|---|---|---|
| Canada | -48.517 | (-62.15,-34.88) |
| | | (-94.06,-31.41) |
| | | (-140.54,-27.34) |
| France | -20.632 | (-32.65,-8.62) |
| | | (-44.34,-1.27) |
| | | (-54.26,1.75) |
| Germany | -52.600 | (-67.02.-38.18) |
| | | (-85.97,-25.35) |
| | | (-105.29,-19.38) |
| Italy | -39.101 | (-51.92,-26.28) |
| | | (-67.63,-16.36) |
| | | (-90.01,-13.70) |
| Japan | -19.708 | (-29.69,-9.72) |
| | | (-42.01,-1.05) |
| | | (-46.53,-4.33) |
| United Kingdom | -18.955 | (-31.93,-5.98) |
| | | (-40.19,-3.08) |
| | | (-55.94,4.08) |
| G6 | -44.204 | (-55.60,-32.80) |
| | | (-73.17,-23.62) |
| | | (-82.87,-21.74) |

## Regression of $\hat{\Lambda}_t$ on $\hat{r}_t - \hat{r}_t^*$ : $\hat{\Lambda}_t = \beta_0 + \beta_1(\hat{r}_t - \hat{r}_t^*) + u_{t+1}$

| Country | $\hat{\beta}_1$ | *90% c.i.(* $\hat{\beta}_1$*)* |
|---|---|---|
| Canada | 23.610 | (15.12,32.10) |
| | | (12.62,51.96) |
| | | (11.96,63.71) |
| France | 13.387 | (1.06,25.72) |
| | | (-2.56,36.25) |
| | | (-6.98,42.40) |
| Germany | 34.722 | (19.66,49.78) |
| | | (9.34,57.59) |
| | | (3.68,69.36) |
| Italy | 27.528 | (17.58,37.48) |
| | | (14.98,48.32) |
| | | (12.51,58.54) |
| Japan | 15.210 | (4.76,25.66) |
| | | (-0.45,37.08) |
| | | (0.91,38.87) |
| United Kingdom | 14.093 | (0.33,27.86) |
| | | (0.39,34.46) |
| | | (-8.70,46.45) |
| G6 | 31.876 | (20.62,43.13) |
| | | (16.89,54.62) |
| | | (16.78,60.89) |

Implications:

$\text{cov}(\lambda_t, r_t - r_t^*) < 0$ (Fama regression in real terms)

$\text{cov}(\Lambda_t, r_t - r_t^*) = \text{cov}(\sum_{j=0}^{\infty} E_t \lambda_{t+j}, r_t - r_t^*) > 0$ (from VAR estimates)

$\rightarrow$ $\text{cov}(E_t \lambda_{t+j}, r_t - r_t^*) > 0$ for some *j* (as in previous figure)

Explaining $\text{cov}(\lambda_t, r_t - r_t^*) < 0$ and $\text{cov}(E_t \lambda_{t+j}, r_t - r_t^*) > 0$ is a challenge for risk premium models – when $r_t - r_t^*$ is high, the home currency is both riskier than average and expected to be less risky than average.

Notation: $d_{t+1} = q_{t+1} - q_t$

$$E_t \hat{\lambda}_{t+k} = \beta_{\lambda k}(\hat{r}_t - \hat{r}_t^*)$$

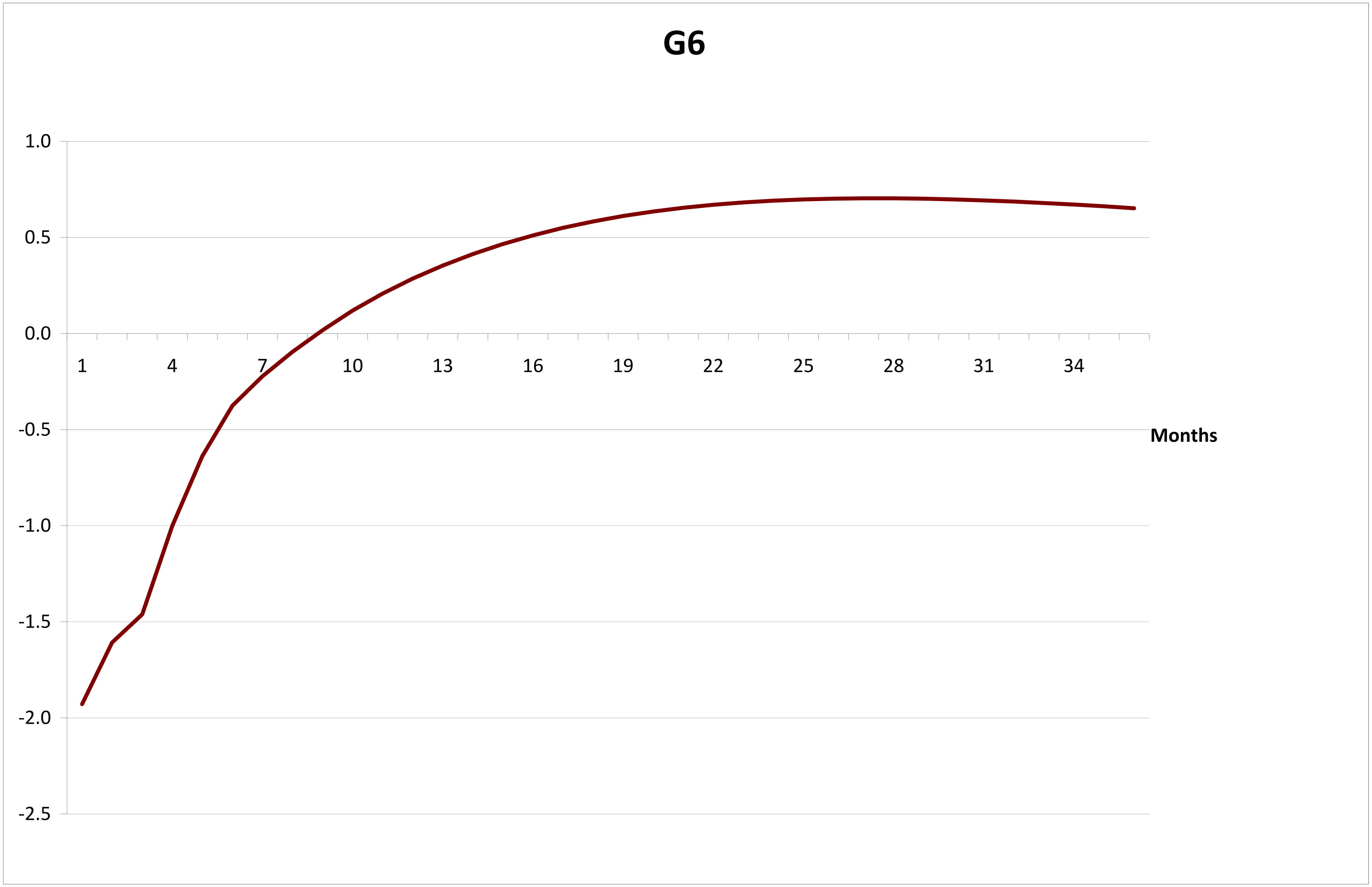


Figure 2 plots slope coefficients from the following regressions (Data are monthly, interest rates are 1-month, end-of-month. For this slide, U.S. relative to weighted average of rest of G7):

$$q_{t+k} = \alpha_{qk} + \beta_{qk}(r_t - r_t^*) \qquad \beta_{qk} = \text{cov}(q_{t+k}, r_t - r_t^*)/\text{var}(r_t - r_t^*)$$

$$q_{t+k}^{IP} = \alpha_{Rk} + \beta_{Rk}(r_t - r_t^*) \qquad \beta_{Rk} = \text{cov}(q_{t+k}^{IP}, r_t - r_t^*)/\text{var}(r_t - r_t^*)$$

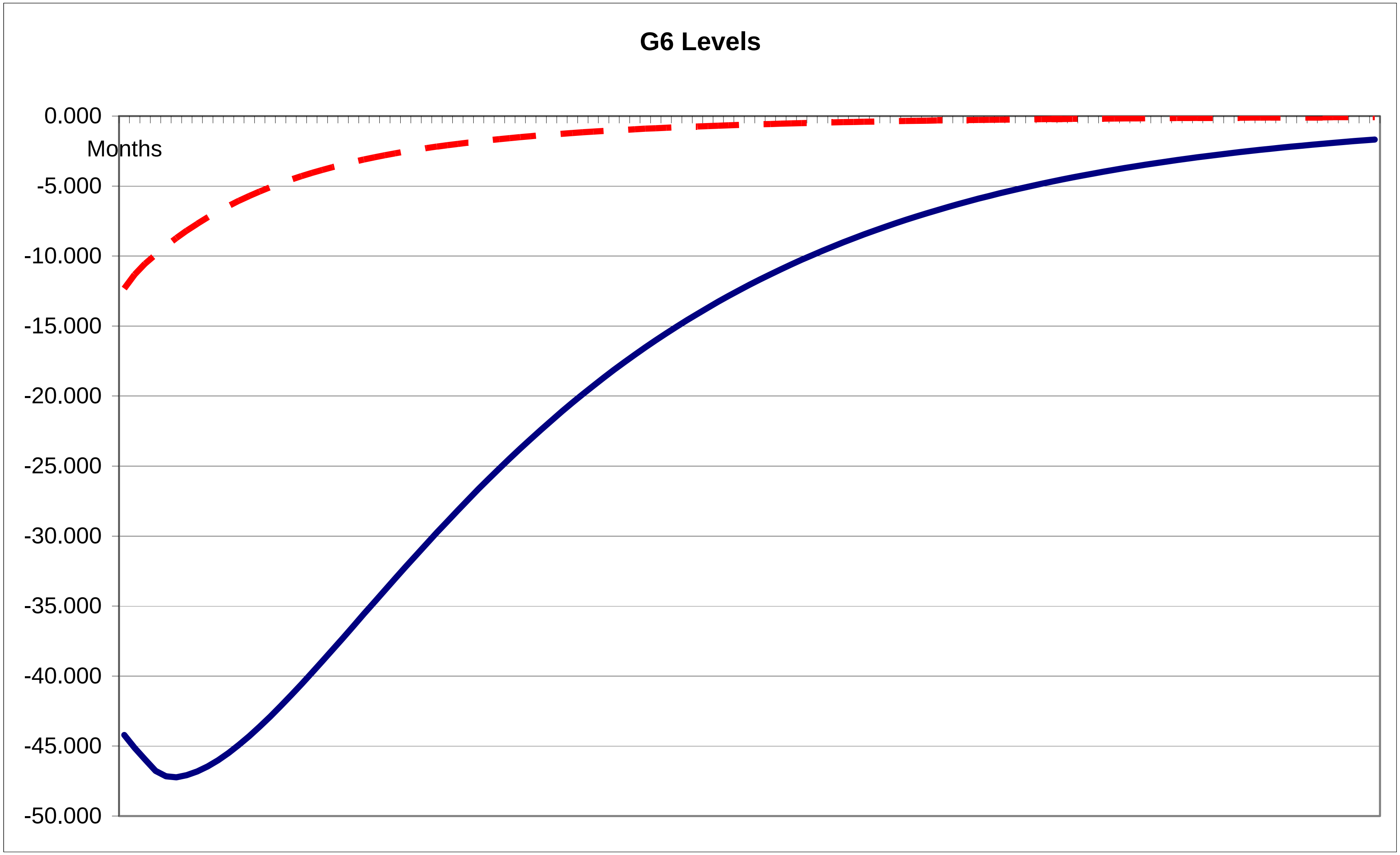
(Real interest rates themselves are estimates)

Difference between $q_{t+k}^{IP}$ and $q_{t+k}$ is $\Lambda_{t+k}$:

$$\Lambda_{t+k} = q_{t+k}^{IP} - q_{t+k}.$$

So difference in lines is $\beta_{\Lambda k} = \text{cov}(\Lambda_{t+k}, r_t - r_t^*)/\text{var}(r_t - r_t^*)$

$$q_{t+k} = \alpha_{qk} + \beta_{qk}(r_t - r_t^*),\ q_{t+k}^{IP} = \alpha_{Rk} + \beta_{Rk}(r_t - r_t^*)$$



Puzzle is $\text{cov}(\lambda_t, r_t^d) < 0$ but $\text{cov}(\Lambda_t, r_t^d) > 0$

Can complete markets risk premium models explain this?

$m_{t+1}, m_{t+1}^*$ are logs of home, foreign stochastic discount factors

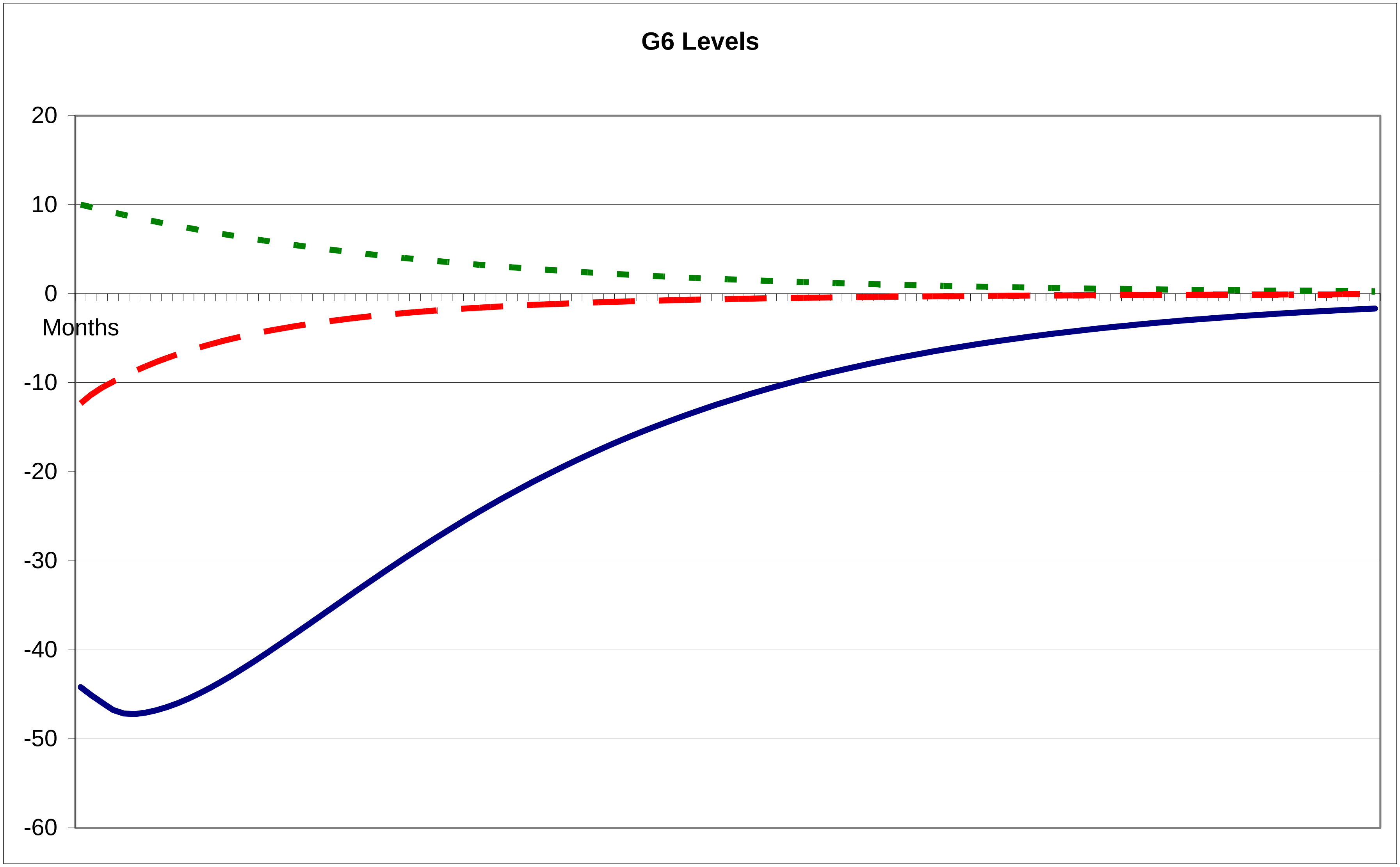
$$r_t^d = -E_t(m_{t+1} - m_{t+1}^*) - \tfrac{1}{2}(\text{var}_t\, m_{t+1} - \text{var}_t\, m_{t+1}^*)$$

$$\lambda_t = \tfrac{1}{2}(\text{var}_t\, m_{t+1} - \text{var}_t\, m_{t+1}^*)$$

Models are specified to account for UIP puzzle. When Home relative risk increases so $\lambda_t$ goes up, Home precautionary saving increases sufficiently that $r_t^d$ goes down.

These preference assumptions must imply $\text{cov}(\Lambda_t, r_t^d) < 0$.

## $q_{t+k} = \alpha_{qk} + \beta_{qk}(r_t - r_t^*)$, $q_{t+k}^{IP} = \alpha_{Rk} + \beta_{Rk}(r_t - r_t^*)$, and Model



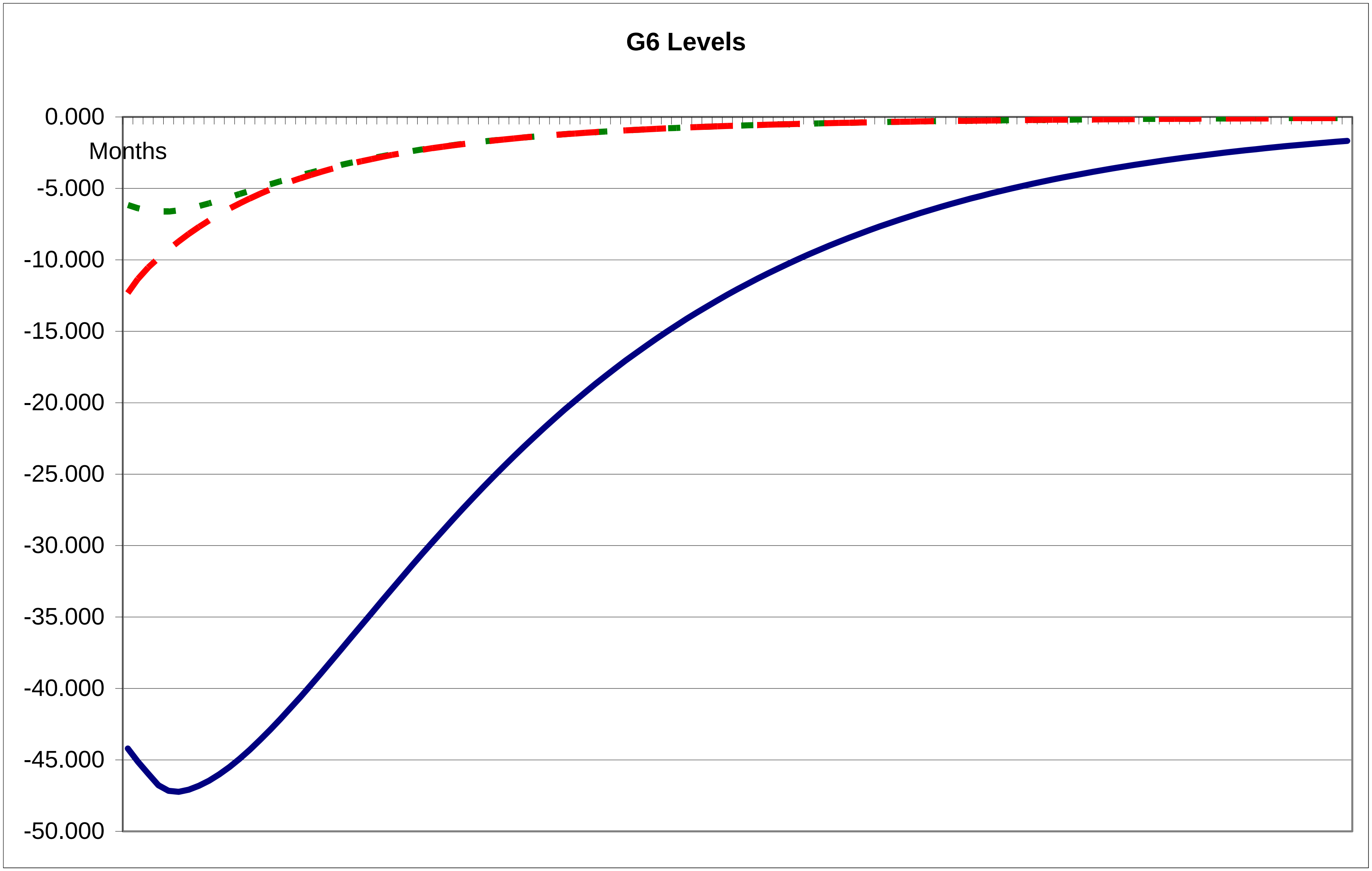
Delayed overshooting to monetary shocks has been explained in models of delayed reaction in the foreign exchange market
Froot and Thaler (1990), Bacchetta and van Wincoop (2010)

The impulse response function for $q_t$ starts off negative, declines for awhile, and then increases.

Model can give us $\text{cov}(\lambda_t, r_t^d) < 0$, and even $\text{cov}(d_{t+1}, r_t^d) < 0$,
but implies $\text{cov}(\Lambda_t, r_t^d) < 0$

The real exchange rate **underreacts** to the increase in real interest rates, rather than overreacting.

## $q_{t+k} = \alpha_{qk} + \beta_{qk}(r_t - r_t^*)$, $q_{t+k}^{IP} = \alpha_{Rk} + \beta_{Rk}(r_t - r_t^*)$, and Model



Models with a single economic variable driving $r_t^d$ and $\lambda_t$:

$$r_t^d = \sum_{j=0}^{\infty} a_j \varepsilon_{t-j} \qquad \lambda_t = \sum_{j=0}^{\infty} c_j \varepsilon_{t-j}$$

1. Single factor models: $r_t^d = k\lambda_t$

2. Unidirectional models: $a_j$ same sign $\forall j$, and $c_j$ same sign $\forall j$.

These are common assumptions. Sometimes both are made. Assumption 2, especially, seems sensible.

Theorem: we cannot get $\text{cov}(\lambda_t, r_t^d) < 0$ and $\text{cov}(\Lambda_t, r_t^d) > 0$

Need at least two shocks. One must matter in short run and deliver $\text{cov}(\lambda_t, r_t^d) < 0$. The other must be more persistent and have $\text{cov}(E_t\lambda_{t+j}, r_t^d) > 0$ in order to get $\text{cov}(\Lambda_t, r_t^d) > 0$.

An example of a model that would work:

Standard New Keynesian, except u.i.p. does not hold (good starting place because of implications under u.i.p.):

Open-economy Phillips curve:

$$\pi_t - \pi_t^* = \delta q_t + \beta E_t(\pi_{t+1} - \pi_{t+1}^*)$$

Taylor rule:

$$i_t - i_t^* = \sigma(\pi_t - \pi_t^*) + \varepsilon_t, \quad \varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \varsigma_t$$

"Liquidity" premium – short-term bonds have value as collateral

$$\lambda_t = \alpha\left[i_t - E_t\pi_{t+1} - (i_t^* - E_t\pi_{t+1}^*)\right] - \eta_t, \qquad \alpha > 0$$

$\eta_t$ -- exogenous increase in value of Foreign bonds

$\alpha\left[i_t - E_t\pi_{t+1} - (i_t^* - E_t\pi_{t+1}^*)\right]$ -- Home bonds are more valued as collateral during Home monetary policy contraction

This can account for $\text{cov}(E_t d_{t+1}, r_t^d) < 0$ and $\text{cov}(\Lambda_t, r_t^d) > 0$ when $\eta_t$ is more volatile but less persistent than $\alpha\varepsilon_t$.

$$\lambda_t = \alpha\left[i_t - E_t\pi_{t+1} - (i_t^* - E_t\pi_{t+1}^*)\right] - \eta_t$$

$\eta_t \uparrow \Rightarrow$ Foreign assets more valuable. Foreign currency appreciates, increasing inflationary pressure in Home. $\Rightarrow r_t^d \uparrow$.

This tends to give us $\text{cov}(\lambda_t, r_t^d) < 0$ and $\text{cov}(E_t d_{t+1}, r_t^d) < 0$ as in u.i.p. puzzle

$\varepsilon_t \uparrow \Rightarrow$ Home monetary contraction, $r_t^d \uparrow$. Relative liquidity value of Home assets rises, tends to make $\text{cov}(\lambda_t, r_t^d) > 0$.

If $\eta_t$ is more volatile, it dominates short run behavior of $\text{cov}(\lambda_t, r_t^d)$. If $\varepsilon_t$ is sufficiently persistent, it dominates long run behavior and determines $\text{cov}(\Lambda_t, r_t^d)$.

$$q_t = \frac{-(1+\alpha)(1-\rho_\varepsilon\beta)}{\delta(1+\alpha)(\sigma-\rho_\varepsilon)+(1-\rho_\varepsilon\beta)(1-\rho_\varepsilon)}\varepsilon_t + \frac{1}{1+\sigma\delta(1+\alpha)}\eta_t$$

$$\lambda_t = \frac{\alpha(1-\rho_\varepsilon\beta)(1-\rho_\varepsilon)}{\delta(1+\alpha)(\sigma-\rho_\varepsilon)+(1-\rho_\varepsilon\beta)(1-\rho_\varepsilon)}\varepsilon_t - \left(\frac{1+\sigma\delta}{1+\sigma\delta(1+\alpha)}\right)\eta_t$$

$$\Lambda_t = \frac{\alpha(1-\rho_\varepsilon\beta)}{\delta(1+\alpha)(\sigma-\rho_\varepsilon)+(1-\rho_\varepsilon\beta)(1-\rho_\varepsilon)}\varepsilon_t - \left(\frac{1+\sigma\delta}{1+\sigma\delta(1+\alpha)}\right)\eta_t$$

$$r_t^d = \frac{(1-\rho_\varepsilon\beta)(1-\rho_\varepsilon)}{\delta(1+\alpha)(\sigma-\rho_\varepsilon)+(1-\rho_\varepsilon\beta)(1-\rho_\varepsilon)}\varepsilon_t + \frac{\sigma\delta}{1+\sigma\delta(1+\alpha)}\eta_t$$

Conclusions:

A new puzzle. Our models for the UIP puzzle don't seem adequate.

Visually, the finding of $\text{cov}(\Lambda_t, r_t^d) > 0$ may be more important than the UIP puzzle, $\text{cov}(\lambda_t, r_t^d) < 0$.

Understanding this matters:

1. For understanding exchange rates
2. For understanding macroeconomics and finance.

$$q_{t+k} = \alpha_{qk} + \beta_{qk}(r_t - r_t^*),\ q_{t+k}^{IP} = \alpha_{Rk} + \beta_{Rk}(r_t - r_t^*)$$