

Macroeconomic Dynamics Near the ZLB: A Tale of Two Countries

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- ZLB since late 1990s in Japan, since 2009 in U.S.
- Deflation around -1% in Japan since 1995.
- Prominent explanation for ZLB combined with deflation: economy moved towards an unintended (undesirable) steady state.
- Bullard (2010) [Seven Faces of “The Peril”] :
 - U.S. = Japan?
 - Promising zero interest rates for an extended period:
 - 1 stimulative policy that can generate inflation and lead the economy back to the desired steady state;
 - 2 inflation expectations might fall and the economy might move to the undesired steady state.

What We Do

- Take the idea of multiple equilibria and “sunspots” seriously in the context of New Keynesian models with ZLB.
- Provide first formal econometric analysis of U.S. / Japan shifting to a regime near deflationary (unintended) steady state.
- Construct a Markov switching sunspot equilibrium for an estimated small-scale New-Keynesian model with ZLB and two regimes.
 - Japan experienced a change in sunspot to a “deflationary regime”.
 - U.S. remained in the targeted-inflation regime.
 - Provide some informal interpretation linking to stance against deflation.
- Conduct policy experiments near the ZLB to demonstrate how same policies lead to radically different outcomes.
- First paper to use global approximation methods to compute a sunspot equilibrium for a DSGE model with a full set of stochastic shocks and ZLB.

- **Sunspots**

- Cass and Shell (1983): Extrinsic uncertainty; random phenomena that don't affect tastes, endowments or production possibilities.
- Lubik and Schorfheide (2004)

- **Multiple Equilibria in the presence of ZLB**

- Benhabib, Schmitt-Grohe and Uribe (2001a,b)
- Mertens and Ravn (2014)
- Schmitt-Grohe and Uribe (2013)
- Cochrane (2013)

- **Numerical solution of the targeted-inflation equilibrium**

- Judd, Maliar, and Maliar (2010)
- Fernandez-Villaverde et al. (2012)
- Gust, Lopez-Salido and Smith (2012)

- **Fiscal multipliers in the presence of ZLB**

- Eggertsson (2009)
- Christiano, Eichenbaum and Rebelo (2011)

- Multiple equilibria and sunspots in a two-equation model
- A New Keynesian DSGE model with ZLB constraint
 - Nonlinear solution
 - Properties of sunspot equilibrium
- Quantitative Analysis
 - Estimation of model parameters
 - Model dynamics
 - Extract model state variables from U.S. and Japanese data
 - Interpretation of the main result
 - Policy experiments

Two-Equation Model

Adapted from Benhabib, Schmitt-Grohe, and Uribe (2001) and Hursey and Wolman (2010).

- Fisher equation:

$$R_t = r\mathbb{E}_t[\pi_{t+1}]$$

- Monetary policy rule

$$R_t = \max \left\{ 1, r\pi_* \left(\frac{\pi_t}{\pi_*} \right)^\psi \exp[\sigma\epsilon_t] \right\}, \quad \epsilon_t \sim iidN(0, 1), \quad \psi > 1$$

- Combine:

$$\mathbb{E}_t[\pi_{t+1}] = \max \left\{ \frac{1}{r}, \pi_* \left(\frac{\pi_t}{\pi_*} \right)^\psi \exp[\sigma\epsilon_t] \right\}$$

- Model has two steady states. ($\sigma = 0$)
 - Targeted-inflation steady state (π_* , $R_* = r\pi_*$)
 - Deflation steady state ($\pi_D = 1/r$, $R_D = 1$)

Some solutions to the difference equation: ▶ Decision Rules

$$\mathbb{E}_t[\pi_{t+1}] = \max \left\{ \frac{1}{r}, \pi_* \left(\frac{\pi_t}{\pi_*} \right)^\psi \exp[\sigma\epsilon_t] \right\}$$

- Mimicking the (unique) local dynamics around the **targeted-inflation steady state**

$$\pi_t = \pi_* \gamma_* \exp \left[-\frac{1}{\psi} \sigma \epsilon_t \right].$$

- Mimicking similar dynamics around the **deflation steady state**

$$\pi_t = \pi_* \gamma_D \exp \left[-\frac{1}{\psi} \sigma \epsilon_t \right].$$

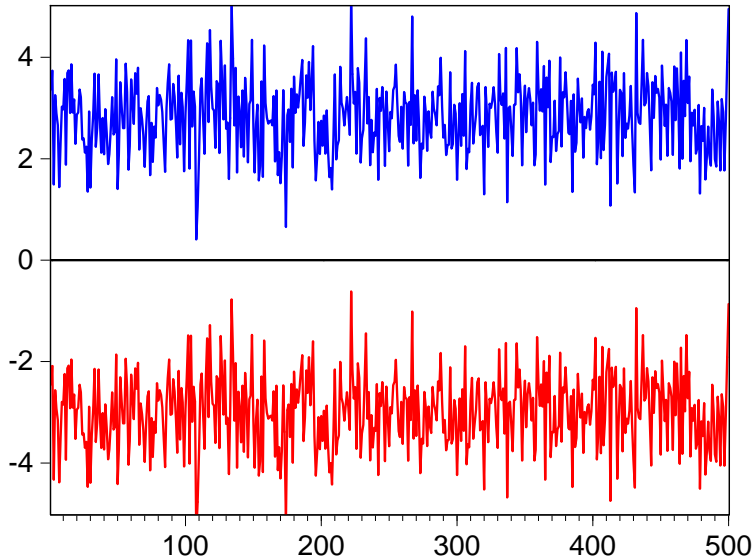
- Sunspot equilibrium that alternates between **targeted-inflation** and **deflation** regimes:

$$\pi_t = \pi_* \gamma(s_t) \exp \left[-\frac{1}{\psi} \sigma \epsilon_t \right]$$

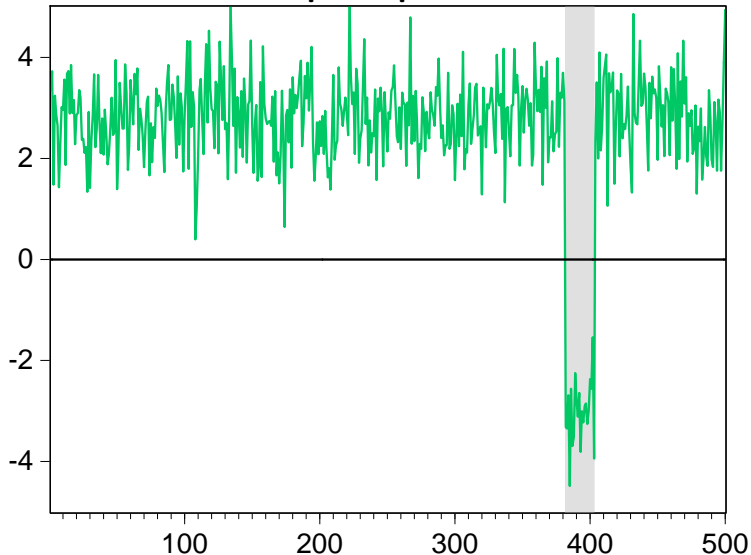
where $s_t \in \{0, 1\}$ follows a Markov-switching process.

Inflation Dynamics in Simple Model

Targeted Inflation Equilibrium and Deflation Equilibrium



Sunspot Equilibrium



We now consider a small-scale New Keynesian model...

- and compute three equilibria:
 - a targeted-inflation equilibrium,
 - a deflation equilibrium,
 - a Markov-switching sunspot equilibrium
- We solve for “minimal-state-variable” equilibria by
 - postulating flexible functional forms for agents' decision rules;
 - parameterizing these functions such that the equilibrium conditions are satisfied.

“Standard” Small-Scale New Keynesian DSGE Model

- **No discount factor shock.** Households maximize

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s \left(\frac{(C_{t+s}/A_{t+s})^{1-\tau} - 1}{1-\tau} - \chi_H \frac{H_{t+s}^{1+1/\eta}}{1+1/\eta} + \chi_M V \left(\frac{M_{t+s}}{P_{t+s} A_{t+s}} \right) \right) \right].$$

- Intermediate good j is produced by a monopolist with technology:

$$Y_t(j) = A_t H_t(j), \text{ where } \ln A_t = \ln \gamma + \ln A_{t-1} + \ln z_t$$

- Intermediate goods producers face **quadratic price adjustment costs**:

$$AC_t(j) = \frac{\phi}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - \bar{\pi} \right)^2 Y_t(j),$$

- Monetary policy rule **with ZLB enforced**:

$$R_t = \max \left\{ 1, \left[r \pi_* \left(\frac{\pi_t}{\pi_*} \right)^{\psi_1} \left(\frac{Y_t}{\gamma Y_{t-1}} \right)^{\psi_2} \right]^{1-\rho_R} R_{t-1}^{\rho_R} e^{\sigma_R \epsilon_{R,t}} \right\}.$$

- Resource constraint (g_t is a generic demand shock):

$$C_t + AC_t + G_t = Y_t \text{ where } G_t = \left(1 - \frac{1}{g_t} \right) Y_t$$

► Equilibrium Conditions

Equilibria of the Model

- Model has **two steady states** and **many stochastic equilibria**.
- We first solve for two minimum-state-variable (MSV) equilibria, with $R_{t-1}, y_{t-1}, g_t, z_t, \epsilon_{R,t}$ as states:
 - **Targeted-Inflation equilibrium**
 - **Deflation equilibrium**
- We then construct a sunspot equilibrium by adding an exogenous shock s_t following a Markov process.
 - $s_t = 1$: **Targeted-Inflation regime**
 - $s_t = 0$: **Deflation regime**
 - $\mathbb{P}(s_t = 1 | s_{t-1} = 1) = p_{**}, \mathbb{P}(s_t = 0 | s_{t-1} = 0) = p_{DD}$
- Empirical analysis is based on **Markov switching sunspot equilibrium**.

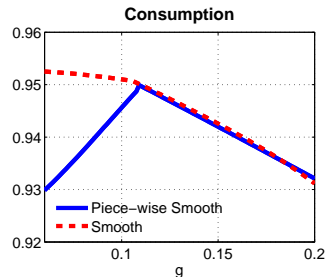
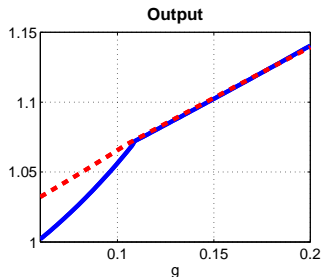
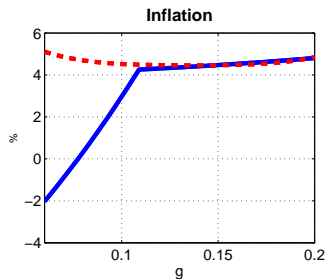
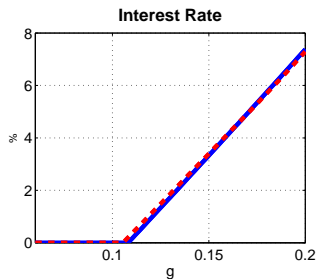
Sketch of Solution Method

- Consider decision rule $\pi(\mathcal{S}_t; \Theta)$.
- “Stitch” four functions for each decision rule:

$$\pi(\mathcal{S}_t; \Theta) = \begin{cases} f_{\pi}^1(\mathcal{S}_t; \Theta) & \text{if } s_t = 1 \text{ and } R(\mathcal{S}_t; \Theta) > 1 \\ f_{\pi}^2(\mathcal{S}_t; \Theta) & \text{if } s_t = 1 \text{ and } R(\mathcal{S}_t; \Theta) = 1 \\ f_{\pi}^3(\mathcal{S}_t; \Theta) & \text{if } s_t = 0 \text{ and } R(\mathcal{S}_t; \Theta) > 1 \\ f_{\pi}^4(\mathcal{S}_t; \Theta) & \text{if } s_t = 0 \text{ and } R(\mathcal{S}_t; \Theta) = 1 \end{cases}$$

- f_j^i are linear combinations of a complete set Chebyshev polynomials up to 4th order, with weights Θ .
- The “seam” is endogenous.
- Choose Θ to minimize sum squared residuals from the Euler Equations over a grid of points.
- Solve model over a grid of points representative of the ergodic distribution and the set of states needed to fit the data.

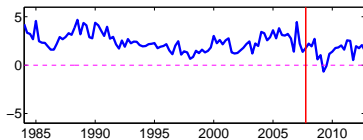
Sample Decision Rules - U.S. $s = 1$, fixed R_{-1} , y_{-1} , z , ϵ_R



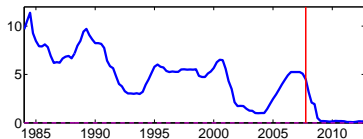
- Estimate model parameters using Bayesian methods (second-order approximation and particle filter) over non-ZLB periods.
- Data: Per capita output growth, inflation, interest rates
 - U.S. (1984Q1-2007Q4)
 - Real GDP growth
 - GDP deflator inflation
 - Federal funds rate
 - Japan (1981Q1-1994Q4)
 - Real GDP growth
 - GDP deflator inflation
 - Uncollateralized call rate

U.S.

Inflation (%)

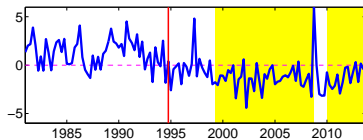


Nominal Interest Rate (%)

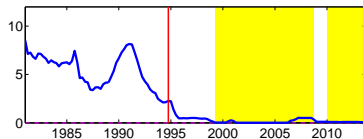


Japan

Inflation (%)

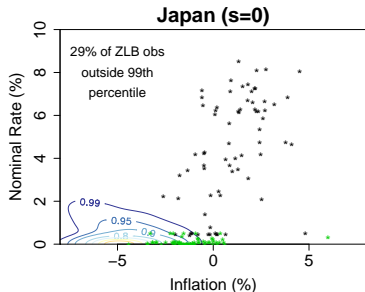
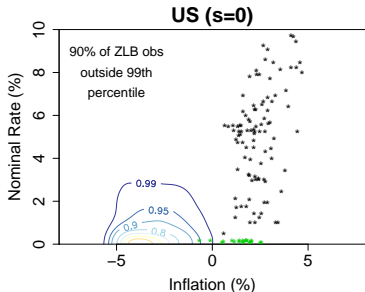
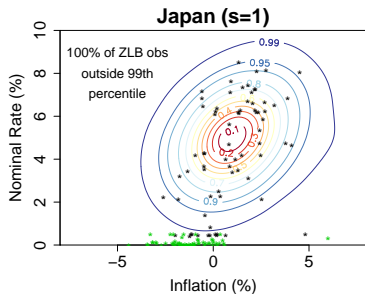
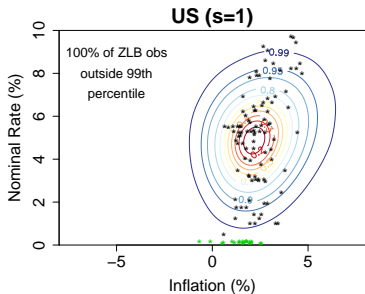


Nominal Interest Rate (%)



The red vertical line denotes the end of the estimation sample.

Equilibrium Dynamics - Model vs. Data



Some Moments

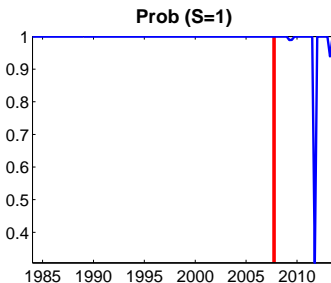
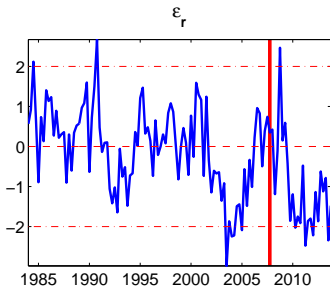
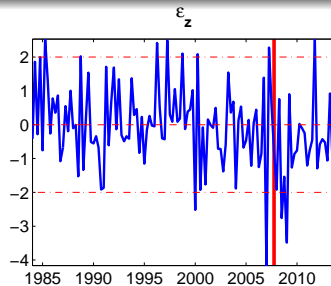
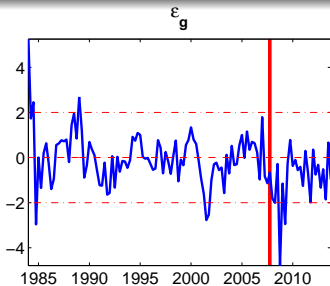
	U.S.			Japan		
	Mean	Stdev	Corr w/ y	Mean	Stdev	Corr w/ y
	s = 1 (82.6%)					
π	2.33	1.12	0.83	1.08	1.46	0.73
ZLB		0.0%			0.0%	
Deflation		1.09%			22.86%	
	s = 0 (17.4%)					
π	-3.35	0.92	-0.96	-4.70	1.08	-0.92
ZLB		84.2%			86.4%	
Deflation		99.95%			99.96%	

- Correlation of π and y similar to the findings in Eggertsson (2009) and Mertens and Ravn (2014).
 - Aggregate Demand (or MR's EE curve) becomes upward sloping near the ZLB.

Extracting Historical Shocks

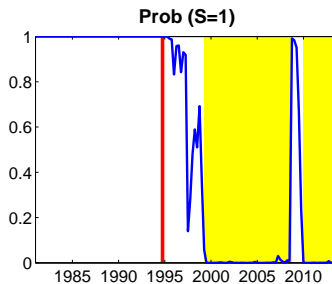
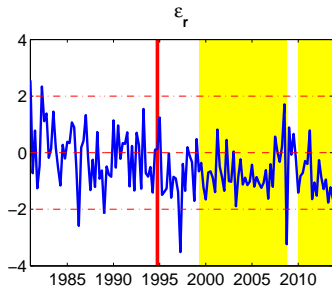
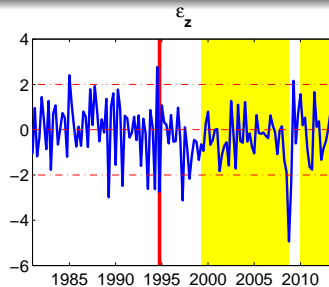
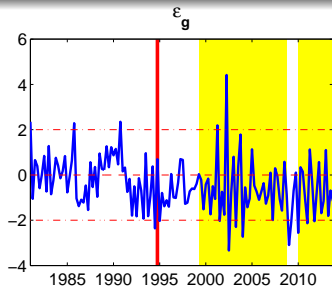
- We now use a particle filter to extract the latent states for our model.
- We use the post-estimation period which contains extended periods of ZLB.
 - U.S. at ZLB since 2009Q1.
 - Japan at ZLB since 1999Q2 except for 2007Q2-2008Q3 (policy rate at 0.5%).

Filtered Shocks - U.S.



The red vertical line denotes the end of the estimation sample.

Filtered Shocks - Japan

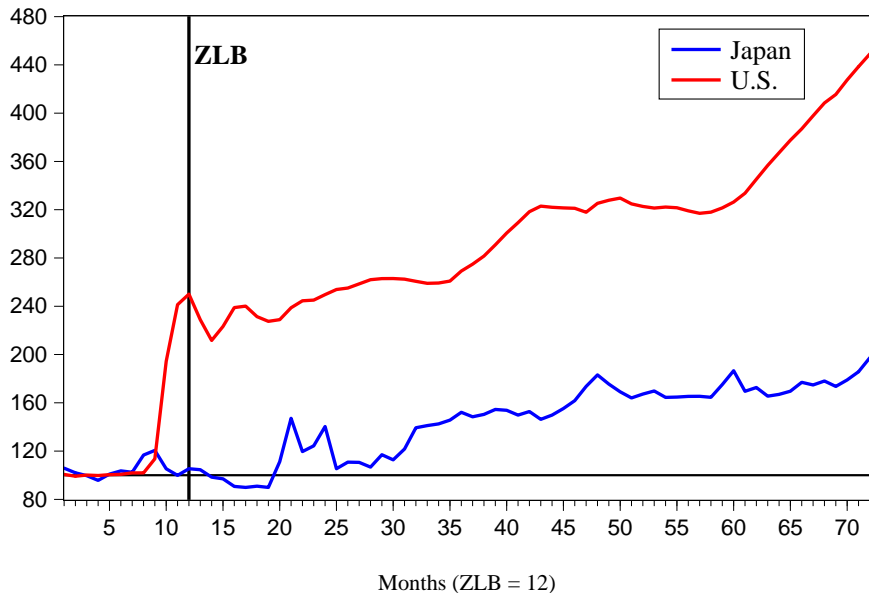


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Discussion of the Main Result

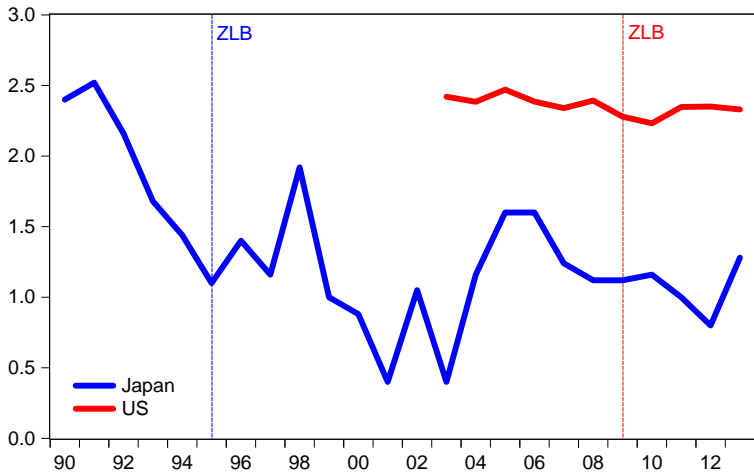
- Both countries have been subject to exogenous shocks prior to their ZLB episodes that manifest themselves as negative ε_g shocks.
 - **U.S.:** Financial crisis of 2007-2008
 - **Japan:** Burst of housing bubble (1992), East-Asian / Korean crisis (1997), Russian crisis (1998)
- In targeted-inflation regime, policy rates approached / reached ZLB.
- Very different monetary policy stance against deflation between two countries. (more next)
- Agents in Japan used the deflation stance of Bank of Japan as a coordination device (a sunspot) and started expecting deflation. (sunspot variable switched to $s = 0$)
- This didn't happen in the U.S. due to the stance of the Fed.

Discussion: Balance Sheets of the Fed and Bank of Japan



Discussion: Inflation Expectations

Inflation Expectations - 10 Year Ahead



Comparison of the Deflation Stance of the Central Banks

Japan:

- After ZLB in 1999, any further action (committing to an inflation target or QE) was expressly ruled out.
- When QE was implemented in 2001, it wasn't explained clearly nor previous claims refuted.
- BoJ had a credibility problem in which the markets and the public did not expect the BoJ to pursue expansionary monetary policy in the future, which would ensure that deflation would end. (Ito and Mishkin, 2006)

U.S.:

- Very forceful reaction to the financial crisis.
- Use of unconventional tools (QE) early on.
- Adoption of a formal inflation target
- Forward Guidance

- During their ZLB episodes both Japan and U.S. engaged in unprecedented fiscal and monetary policies.
 - Move the economy away from deflation (or prevent it)
 - Stimulate real activity.
- **U.S.:**
 - American Recovery and Reinvestment Act (ARRA) in February 2009 (5% of GDP awarded in 2009, 1.5% of GDP received)
 - Forward guidance by the Fed
- **Japan:**
 - Various fiscal programs after 1999 (up to 3% of GDP)
 - ▶ [Fiscal Programs Japan](#)
 - Policy rate kept at zero

Policy Experiments

- Distinguish two situations:
 - “Normal” times (non-ZLB periods) [1984Q1-2005Q2 for U.S., 1981Q2-1991Q2 for Japan]
 - ZLB episode [2009Q1-2011Q1 for U.S., 1999Q2-2005Q2 for Japan]
- Conduct the same policy, taking the conditions (filtered states) in a period as given.
 - Baseline path will be identical to data.
 - Integrate out the conditions that take the economy to ZLB
- Three experiments:
 - **Fiscal-only (Marginal)**: A *very small* σ_g shock to g such that ZLB still binds.
 - **Fiscal-only (Large)**: A $1.5\sigma_g$ shock to g .
 - **Joint fiscal (Large) and monetary**: Large fiscal shock low interest rate.

- Multiplier :
$$\mu_H = \frac{\sum_{h=0}^H (Y_{t+h}^I - Y_{t+h})}{\sum_{h=0}^H (G_{t+h}^I - G_{t+h})}$$

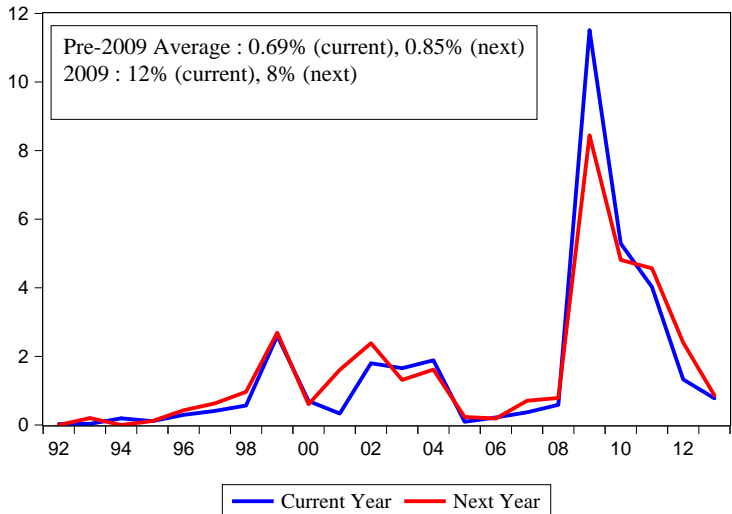
Government Spending Multipliers

Policy	U.S.				Japan			
	1Q	4Q	8Q	12Q	1Q	4Q	8Q	12Q
"Normal" Times								
Fiscal (Large)	0.62	0.67	0.68	0.70	0.58	0.56	0.56	0.56
ZLB Episode								
Fiscal (Marginal)	0.87	0.91	0.83	0.81	0.47	0.46	0.46	0.46
Fiscal (Large)	0.62	0.67	0.69	0.70	0.47	0.46	0.46	0.46
Fiscal (Large) & Monetary	1.16	1.23	1.25	1.24	0.47	0.46	0.46	0.46

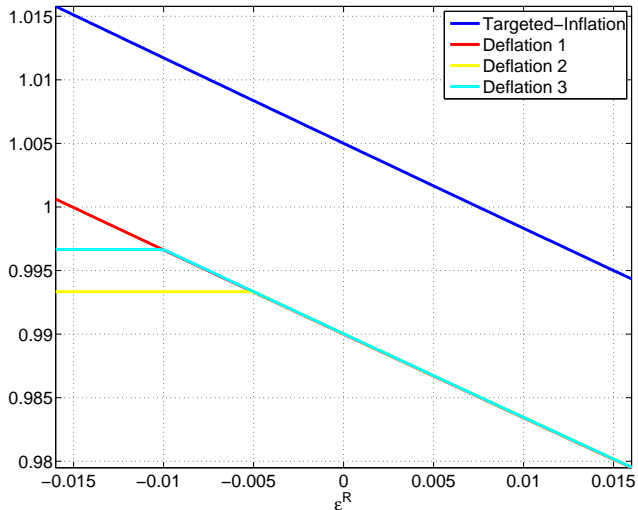
Conclusions

- First quantitatively-serious analysis of sunspots and multiple equilibria in the context of New-Keynesian models at the ZLB.
- Empirically compare the experiences of Japan and U.S. at the ZLB
- Very different sources of ZLB in two countries
 - Japan switched to **deflationary regime** in 1999 and remained there until end of sample.
 - U.S. experienced adverse demand shocks starting in 2008 and monetary policy intervention since then, but remained in **targeted-inflation regime**.
 - (Lack of) commitment of central banks to fighting deflation is key for differences.
- Policy effectiveness at the ZLB depends crucially on source of ZLB
 - If at ZLB when $s = 0$, marginal multiplier about half relative to when $s = 1$.

Probability that Annual Inflation will be Less Than 0% (SPF)



Decision Rules in Simple Model



▶ Return

Equilibrium Conditions

Equilibrium is $\{c_t, \pi_t, y_t, R_t\}$ (in terms of detrended variables, i.e., $c_t = C_t/A_t$ and $y_t = Y_t/A_t$)

$$1 = \beta \mathbb{E}_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\tau} \frac{1}{\gamma z_{t+1}} \frac{R_t}{\pi_{t+1}} \right]$$

$$1 = \frac{1}{\nu} \left(1 - \chi_H c_t^\tau y_t^{1/\eta} \right) + \phi (\pi_t - \bar{\pi}) \left[\left(1 - \frac{1}{2\nu} \right) \pi_t + \frac{\bar{\pi}}{2\nu} \right] \\ - \phi \beta \mathbb{E}_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\tau} \frac{y_{t+1}}{y_t} (\pi_{t+1} - \bar{\pi}) \pi_{t+1} \right]$$

$$c_t = \left[\frac{1}{g_t} - \frac{\phi}{2} (\pi_t - \bar{\pi})^2 \right] y_t$$

$$R_t = \max \left\{ 1, \left[r \pi_* \left(\frac{\pi_t}{\pi_*} \right)^{\psi_1} \left(\frac{y_t}{y_{t-1}} z_t \right)^{\psi_2} \right]^{1-\rho R} R_{t-1}^{\rho R} e^{\sigma_{R,t}} \right\}$$

and the laws of motion for g_t , z_t and $\epsilon_{R,t}$. [Return](#)

Approximate Dynamics In Targeted-Inflation Equilibrium

- Let $\tau = 1$, $\eta = \infty$, $\chi_h = 1$, $\gamma = 1$, $\bar{\pi} = \pi_*$, $\psi_1 = \psi$, $\psi_2 = 0$, and $\rho_R = \rho_g = \rho_z = 0$.
- Linearizing around targeted-inflation steady state yields the system

$$\hat{R}_t = \max \left\{ -\ln(r\pi_*), \psi \hat{\pi}_t + \sigma_R \epsilon_{R,t} \right\}$$

$$\hat{c}_t = \mathbb{E}_t[\hat{c}_{t+1}] - (\hat{R}_t - \mathbb{E}_t[\hat{\pi}_{t+1}])$$

$$\hat{\pi}_t = \beta \mathbb{E}_t[\hat{\pi}_{t+1}] + \kappa_* \hat{c}_t,$$

- Then solution is piece-wise linear

$$\hat{R}_t(\epsilon_{R,t}) = \max \left\{ -\ln(r\pi_*), \frac{1}{1 + \kappa\psi} \left[\psi(\kappa + \beta)\mu_\pi^* + \kappa\psi\mu_c^* + \sigma_R \epsilon_{R,t} \right] \right\}$$

$$\hat{c}_t(\epsilon_{R,t}) = \begin{cases} \frac{1}{1 + \kappa\psi} \left[(1 - \psi\beta)\mu_\pi^* + \mu_c^* - \sigma_R \epsilon_{R,t} \right] & \text{if } \hat{R}_t \geq -\ln(r\pi_*) \\ \ln(r\pi_*) + \mu_c^* + \mu_\pi^* & \text{otherwise} \end{cases}$$

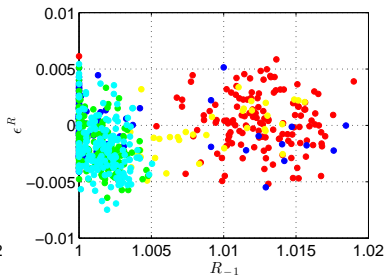
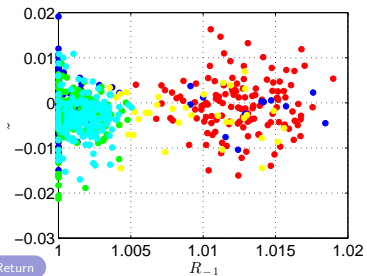
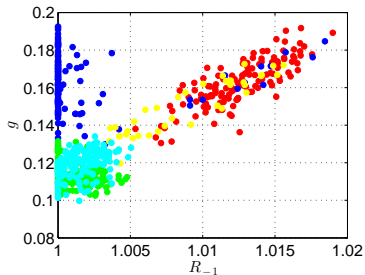
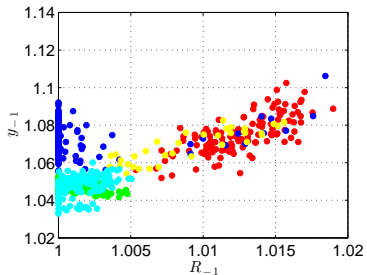
$$\hat{\pi}_t(\epsilon_{R,t}) = \begin{cases} \frac{1}{1 + \kappa\psi} \left[(\kappa + \beta)\mu_\pi^* + \kappa\mu_c^* - \kappa\sigma_R \epsilon_{R,t} \right] & \text{if } \hat{R}_t \geq -\ln(r\pi_*) \\ \kappa \ln(r\pi_*) + (\kappa + \beta)\mu_\pi^* + \kappa\mu_c^* & \text{otherwise} \end{cases}$$

Fiscal Programs in Japan

Date	Total % GDP	Central Gov't % GDP	Description	Real	ER	Defl
Apr-1998	3.12	0.9	Accelerate recovery	x		
Nov-1998	4.68	1.48	Recovery package	x		
Nov-1999	3.57	1.29	Economic stimulus	x		
Oct-2000	2.16	0.76	Economic recovery	x		
Dec-2001	0.81	0.51	Fear of global recession	x		
Dec-2002	2.97	0.6	Accelerate economic reform	x		
Aug-2008	2.33	0.36	Reduce uncertainty	x		
Oct-2008	5.37	0.96	Global recession	x		
Apr-2009	12.06	3.27	Ext. sector and fin. system	x		
Dec-2009	5.18	1.53	Recovery and fight deflation	x	x	x
Sep-2010	2.03	0.19	Overcome deflation	x	x	x
Oct-2010	4.37	1.06	Appreciation and deflation	x	x	x
Jan-2013	4.17	2.7	Post earthquake stimulus	x	x	x
Dec-2013	3.84	1.13	Revitalization of economy	x	x	x

▶ Return

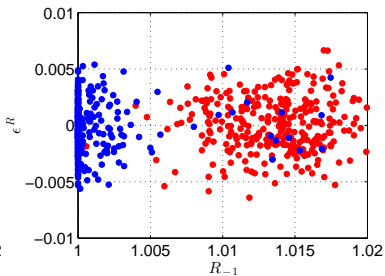
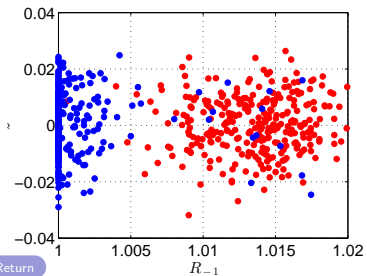
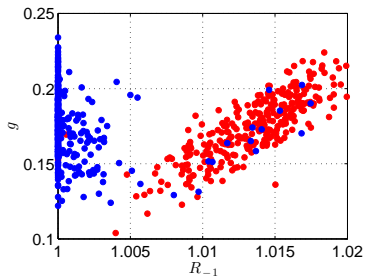
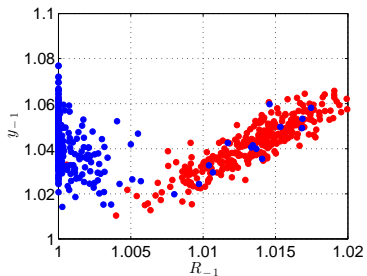
Solution Grid - U.S.



Return

• Ergodic Distribution ($s=1$) • Ergodic Distribution ($s=0$) • 2000-2008 • 2009-2013 ($s=0$) • 2009-2013

Solution Grid - Japan



Return

• Ergodic Distribution $(s=1)$

• Ergodic Distribution $(s=0)$