State-Dependent Pricing and the Paradox of Flexibility*

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Abstract

We study the effects of price-setting on shock amplification under a passive monetary policy in a general state-dependent pricing framework which nests the fixed menu cost model at one extreme and the Calvo model at the other. Since the type of price setting is crucial for the determination of the long-term real interest rate, it is a primary determinant of shock propagation under constant nominal interest rates, e.g. of the government spending multiplier. In order to have a multiplier substantially bigger than unity, under interest rates reacting according to a standard Taylor rule, both monetary policy has to be accommodative, and the aggregate price level has to be sufficiently sticky. State-dependent pricing in this case yield the lowest propagation. Conversely, when interest rate are constant (or at the zero lower bound), what matters is whether monetary policy stabilizes or not the price level. When a drift in the latter is allowed, as in most literature studying the ZLB or the effects of monetray forward guidance, more price flexibility results in a larger multiplier.We also find that idiodyncratic firm-level shocks matter for amplification. The paradox of flexibility thus holds across pricing models and under state-dependent pricing: increased flexibility of nominal prices, and thus a larger response of prices and inflation, reaises instead of reducing the real effects of shocks.

Keywords: firm heterogeneity, smoothly state-dependent pricing, constant interest rate

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1 Introduction

In the aftermath of the global "Great Recession" started in 2008, monetary policy rates were rapidly decreased to historically low levels in most advanced countries, and have been mostly kept unchanged since then. This unprecedented situation has spurred a host of studies investigating the consequences and policy implications of such a passive monetary policy. Building on the seminal contributions by Krugman (1998) and especially Eggertsson and Woodford (2003) (who analyzed the Japanese fall into a protracted period of low and unchanging interest rates), these papers are mostly based on the workhorse New Keynesian model, commonly used in monetary policy analysis. A key result of this literature is that a "passive" monetary policy that keeps the nominal interest rate constant, can result in a large amplification of shocks. Specifically, several recent papers have argued that a government spending shock can result in a very large multiplier under a passive monetary policy (see e.g. Christiano et. al., 2011, Eggertsson, 2011, Woodford, 2011, Werning, 2012). This result has been especially relevant given that in many countries by the end of 2008, the short-term nominal interest rate used as the main operating target for monetary policy had reached its effective lower bound. By the same token, announcements (forward guidance) about the future path of nominal interest rates can have a very large effect on real activity and inflation (see e.g. Del Negro et al. 2013).

The mechanism underlying this amplification revolves around the "paradox of flexibility": increased flexibility of nominal prices and wages, and thus a larger response of prices and inflation, instead of reducing the real effects of shocks, such as government spending shocks, raise them. The main idea is the following. Fiscal stimulus can have a large effect on aggregate output if private spending, such as consumption, increases with government purchases. In standard macroeconomic models, current consumption depends inversely on the long-term real interest rate (i.e. the sum of future expected short-term real rates). When the nominal interest rate is constant for some time, this long-term real rate will be a function of inflation expectations. To the extent that systematic monetary policy allows a sustained increase in future inflation in response to government purchases, the long-term rate will fall and current consumption will rise. Aggregate activity will thus increase in excess of government spending. Furthermore, the more flexible prices the larger the multiplier: Future inflation will be expected to be persistently higher.

Focusing on the case of the government spending multiplier, this paper studies the interaction between the price setting mechanism and a passive monetary policy, under which interest rates are constant. To the best of our knowledge all of the recent literature investigating these questions assumes that the timing of price changes is given exogenously, as in the Calvo (1983) - Yun (1996) model of price setting. However, the specifics of price setting at the micro level can be essential for the dynamics of the aggregate price level and output. For example, compared to time-dependent pricing, state-dependent pricing in a menu cost model can produce very different inflation and output responses to a monetary policy shock (Golosov and Lucas, 2007; Alvarez et al. 2011). Yet a study of fiscal multiplier, and of shock amplification with a passive monetary policy has been missing in the context of state-dependent pricing.

This paper's goal is to re-examine the consequences of constant interest rates within a framework of "smoothly state dependent pricing" by firms, as in Costain and Nakov (2011b). Pricing in this model is dubbed "smoothly state-dependent" because the probability of adjustment is a smoothly increasing function of the adjustment gain, rather than the 0–1 step function it is in the menu cost model. The framework is general in the sense that it nests the menu cost model at one extreme and the Calvo-Yun model at the opposite extreme, and its intermediate version is empirically more plausible than either of the extremes (Costain and Nakov, 2011a). In particular, the smoother intermediate model matches much better observed histograms of price changes such as that found in AC Nielsen's retail price data documented by Midrigan (2007) — see Figure 1.

A main difference of this model of price setting, relative to time-dependent models, concerns the behavior of "reset price inflation" (the rate of change of all desired prices) relative to actual inflation. As documented by Bils et al. (2009), time-dependent models imply unrealistically high persistence and stability of reset price inflation in response to shocks, relative to the data. But reset price inflation is a key determinant of future inflation in response to government spending shocks, and thus of the multiplier under a constant nominal interest rate.

The monetary policy reaction function also shapes the formation of (rational expectations of) future inflation, interacting with the price setting mechanism. Even under a constant nominal interest rate, it is well known that the specific form of the reaction function matters for the response of the economy to shocks, as shown by Eggertsson and Woodford (2003) in the case of the zero lower bound. Therefore, we explore the effects of different monetary policy rules, focusing on those that imply a constant nominal interest rate. After presenting the model setup in Section 2, we provide several analytical results in Section 3 for the Calvo-Yun version of our model. Following Woodford (2011),we start by analyzing a standard Taylor rule such that the nominal interest rate reacts to deviations of inflation *and* of government spending from their respective steady-state levels. The degree of accommodation to government spending is captured by the corresponding reaction coefficient in this rule. In the Calvo-Yun version of our model, we derive analytical conditions under which an appropriately chosen coefficient exactly delivers a constant nominal interest rate in response to government spending shocks. Such a rule also replicates the results under the zero lower bound in Woodford (2011), including the property that the multiplier is *locally* increasing in the degree of price flexibility. We also study the case in which the interest rate is kept constant for a given period of time by a series of anticipated shocks (dubbed by Galí (2012) the "modest interventions" rule), e.g. as in Erceg and Linde (2014). This is the standard approach in large policy models. We show that this rule delivers the largest multipliers, and magnifies the "paradox of flexibility".

However, there are other rules that imply a (unique) equilibrium with a constant nominal interest rate, which have very different implications for the size of the multiplier, and in particular the effects of the degree of price flexibility under time-dependent pricing. We turn to analyze three of these interest rate rules in Section 4: a Taylor rule with "extreme" interest rate smoothing (i.e. a coefficient on the lagged interest rate close to 1), and two further interest rules considered by Galí (2012), in addition to the "modest interventions" rule. We show analytically that under all these three rules, the multiplier is now *decreasing* in the degree of price flexibility with time-dependent pricing. These rules have very different consequences for the process of inflation than the lower-bound, modest intervention Taylor rule. The latter rule implies that there is a positive drift in the price level, which is proportional to the degree of price flexibility: the more desired prices set in the future by firms will react, the higher future inflation and the price level in the limit, and thus the lower the long-term real rate. Conversely, the other rules all imply that higher price flexibility will result in a *lower* expected inflation, and thus in a higher long-term rate. Specifically, we show that the two constant interest rate rules in Galí

(2012) impose a stationary or even negative drift on the price level, so that future desired price changes and thus inflation are negative: the more so, the more flexible prices in the Calvo-Yun framework. This finding relates and extends the debate on the different effects of government spending under a constant nominal interest rate, and under an exchange rate peg or in a monetary union (see Corsetti et al. (2009) and Nakamura and Steinsson (2010)).

Next, we turn to the numerical analysis of the model with state-dependent pricing and idiosyncratic shocks in Section 5, under the modest intervention rule. Exploring the implications of systematic monetary policy under state-dependent pricing is a further contribution of the paper (e.g. Golosov and Lucas, 2007 assumed a money growth rule). We present first results under a standard Taylor rule. We find that, in order to have a multiplier substantially bigger than unity, both the degree of monetary policy accommodation to the shock must be high, *and* the aggregate price level must be sufficiently sticky. When the rule is only allowed to respond to inflation, the multiplier is less than 1 across all models, and lowest under state dependent pricing.[1] Finally, we consider the case of the modest intervention rule. In this case, the multiplier is larger than 1 across all models, but shock amplification is largest under state dependent pricing. We also find that idiosyncratic shocks play a disproportionate role in the amplification mechanism. The paradox of flexibility thus holds across pricing models.

The following section describes the model, the parameterization and the method of computation. Section 3 and 4 derive analytical results under time-dependent pricing. Section 5 presents some numerical results, and Section 6 concludes.

2 A simple macroeconomic model nesting time- and state-dependent pricing

The model embeds state-dependent pricing by firms in an otherwise-standard New Keynesian general equilibrium framework. Following Costain and Nakov (2011a,b), the framework nests a continuum of

[1] Thus, the multiplier can be as large as 3.7 in the Calvo model but as small as 1 in the *ceteris paribus* menu cost model. In our best-fitting "smoothly state-dependent pricing" (SSDP) model, the multiplier under "no interest rate smoothing" is about 2.

pricing models, with the Calvo model on one extreme, and the Golosov and Lucas (2007) fixed menu cost model on the other. Besides the firms, there is a representative household, a monetary authority that implements a Taylor rule, and a government that levies lumps sum taxes to finance an exogenous stream of spending on goods.

Denoting the aggregate state of the economy at time t by Ω_t, time subscripts on aggregate variables will indicate dependence, in equilibrium, on aggregate conditions Ω_t. For example, consumption is denoted by $C_t \equiv C(\Omega_t)$, and the aggregate price level by $P_t \equiv P(\Omega_t)$.

2.1 Representative Household

The household's period utility is $\frac{1}{1-\gamma}C_t^{1-\gamma} - \chi\frac{1}{1+\eta}N_t^{1+\eta} + \nu\log(M_t/P_t)$, where C_t denotes consumption, N_t labor supply, and M_t/P_t real money balances. Utility is discounted by β. Consumption is a CES aggregate of differentiated products C_{it}, with elasticity of substitution ϵ:

$$C_t = \left\{\int_0^1 C_{it}^{\frac{\epsilon-1}{\epsilon}} di\right\}^{\frac{\epsilon}{\epsilon-1}}. \tag{1}$$

The household's nominal period budget constraint is

$$\int_0^1 P_{it}C_{it}di + M_t + R_t^{-1}B_t = W_tN_t + M_{t-1} + T_t + B_{t-1}, \tag{2}$$

where $\int_0^1 P_{it}C_{it}di$ is total nominal consumption. B_t is nominal bond holdings purchased at t, and paying interest rate $R_t - 1$ at time $t+1$. T_t is a nominal lump-sum transfer consisting of seigniorage revenues from the central bank plus dividend payments from the firms, net of lump-sum taxes to finance government spending on goods.

Households choose $\{C_{it}, N_t, B_t, M_t\}_{t=0}^{\infty}$ to maximize expected discounted utility, subject to the budget constraint (2). Optimal consumption across the differentiated goods implies

$$C_{it} = (P_t/P_{it})^{\epsilon}C_t, \tag{3}$$

where $P_t \equiv \left[\int_0^1 {P_{it}}^{1-\epsilon}di\right]^{\frac{1}{1-\epsilon}}$ is the relevant price index.

Optimal labor supply, consumption, and money use imply the following first-order conditions:

$$\chi C_t^{\gamma} N_t^{\eta} = W_t/P_t, \tag{4}$$

$$1 = \beta R_t E_t \left[P_t C_{t+1}^{-\gamma} / \left(P_{t+1} C_t^{-\gamma} \right) \right], \tag{5}$$

$$M_t/P_t = \nu C_t^{\gamma} R_t / \left(R_t - 1 \right). \tag{6}$$

2.2 Monopolistic firms

Firms are monopolistic competitors. Each firm i produces output Y_{it} using labor N_{it} as the only input, under a linear technology: $Y_{it} = A_{it} N_{it}$. Firm's productivity A_{it} is an idiosyncratic process, AR(1) in logs:

$$\log A_{it} = \rho_A \log A_{it-1} + \varepsilon_{it}^{a}, \tag{7}$$

where $0 \leq \rho_A < 1$ and $\epsilon_{it}^{a} \sim i.i.d.N(0, \sigma_a^2)$. Firm i charges a price P_{it} and faces demand from two sources, $Y_{it} = C_{it} + G_{it}$, where C_{it} is demand for goods by the households, and G_{it} is demand by the government. The government's consumption basket is also a CES aggregator with elasticity of substitution ϵ:

$$G_t = \left\{ \int_0^1 G_{it}^{\frac{\epsilon-1}{\epsilon}} di \right\}^{\frac{\epsilon}{\epsilon-1}}. \tag{8}$$

Optimal allocation of expenditure across goods on the part of households and the government implies that firm i faces the demand curve $Y_{it} = (C_t + G_t) P_t^{\epsilon} P_{it}^{-\epsilon}$. The firm fulfills all demand at its posted price. It hires in a competitive labor markets at wage rate W_t, generating period profits

$$U_{it} = P_{it} Y_{it} - W_t N_{it}. \tag{9}$$

Since firms are owned by the household, they discount nominal income between times t and $t+1$ at the rate $\beta \frac{P_t C_t{}^{-\gamma}}{P_{t+1} C_{t+1}{}^{-\gamma}}$, consistent with the household's marginal rate of substitution.

Costain and Nakov (2011) derive the following value function for a firm which produces with productivity A and sells at nominal price P:

$$V(P,A,\Omega)=U(P,A,\Omega)+\beta E\left\{\left.\tfrac{P(\Omega)C(\Omega')^{-\gamma}}{P(\Omega')C(\Omega)^{-\gamma}}\left[V(P,A',\Omega')+EG(P,A',\Omega')\right]\right|A,\Omega\right\}, \tag{10}$$

where

$$EG(P,A',\Omega')\equiv\lambda\left(\frac{\max_P V(P,A',\Omega')-V(P,A',\Omega')}{W(\Omega')}\right)\left(\max_P V(P,A',\Omega')-V(P,A',\Omega')\right) \tag{11}$$

is the *expected gain* from adjustment, and $\lambda\left(\frac{\max_P V(P,A',\Omega')-V(P,A',\Omega')}{W(\Omega')}\right)\in[0,1]$ is a mapping from the gain from price adjustment to the probability of adjustment, which is detailed in the following section (2.3).

Price stickiness means that the individual price process associated with the Bellman equation is

$$P\prime=\begin{cases}\arg\max_P V(P,A',\Omega') & \text{with probability } \lambda(P,A',\Omega')\\ P & \text{with probability } 1-\lambda(P,A',\Omega')\end{cases}. \tag{12}$$

2.3 Nesting alternative price-setting schemes

Following Costain and Nakov (2011a,b) we assume that the probability of price adjustment $\lambda(L)$, increases with the gain from adjustment L. Thus, the increasing function $\lambda(L)\in[0,1]$ that governs this probability is taken as a primitive of the model.[2] In particular, we postulate the following functional form:

$$\lambda\left(L\right)\equiv\frac{\bar{\lambda}}{(1-\bar{\lambda})+\bar{\lambda}\left(\alpha/L\right)^{\xi}} \tag{13}$$

where $L\equiv\left[\max_P V(P,A',\Omega')-V(P,A',\Omega')\right]/W(\Omega')$ is the relevant endogenous state, with α and ξ positive, and $\bar{\lambda}\in[0,1]$. This function is concave for $\xi\leq1$ and S-shaped for $\xi>1$ (see Fig. 1). The parameter ξ effectively controls the degree of state dependence. In the limit $\xi=0$, (13) nests Calvo (1983) so that $\lambda\left(L\right)=\bar{\lambda}$ regardless of L. At the opposite extreme, as $\xi\to\infty$, $\lambda\left(L\right)$ becomes the indicator function $\mathbf{1}\left\{L\geq\alpha\right\}$, which equals 1 whenever $L\geq\alpha$ and is zero otherwise. This is the

[2] Alternatively, λ can be viewed as an exogenously specified distribution of menu costs from which firms make random draws every period.

fixed menu cost model which implies "extreme state dependence", in the sense that the adjustment probability jumps from 0 to 1 as soon as the state L passes the threshold α. For intermediate values $0 < \xi < \infty$, the hazard increases "smoothly" with the state L. We call this intermediate version "smoothly state-dependent pricing" (SSDP) model.

2.4 Monetary policy and aggregate consistency

The monetary authority follows a Taylor rule,

$$\frac{R_t}{R^*} = \left[\left(\frac{P_t/P_{t-1}}{\Pi^*} \right)^{\phi_\pi} \left(\frac{C_t}{C_{t-1}} \right)^{\phi_C} \left(\frac{G_t}{G^*} \right)^{\phi_G} \right]^{1-\phi_R} \left(\frac{R_{t-1}}{R^*} \right)^{\phi_R} \prod_i^T \exp\left(\varepsilon_{t-i}^R\right) \tag{14}$$

where $\phi_\pi > 1$, and $0 \leq \phi_R < 1$. Fixing for the moment $\phi_C = 0$, parameter ϕ_G controls the degree of monetary policy accommodation to the government expenditure shock. The shocks ε_{t-i}^R are anticipated and are set so that, following a spending shock, the nominal interest is constant at its steady state value for T periods. This is a standard approach followed to examine the effects of constant interest rates, see e.g. Erceg and Linde (2014)

Further, we assume that government spending G_t follows an AR(1) process in logs:

$$\log\left(\frac{G_t}{G^*}\right) = \rho_G \log\left(\frac{G_{t-1}}{G^*}\right) + \varepsilon_t^G, \tag{15}$$

with persistence $0 < \rho_G < 1$ and $\epsilon_t^G \sim i.i.d.N(0, \sigma_G^2)$.

Seigniorage revenues are paid to the household as a lump-sum transfer, and the government budget is balanced each period. Bond market clearing is simply $B_t = 0$. When supply equals demand for each good i, total labor supply and demand satisfy

$$N_t = \int_0^1 \frac{Y_{it}}{A_{it}} di = P_t^\epsilon Y_t \int_0^1 P_{it}^{-\epsilon} A_{it}^{-1} di \equiv \Delta_t^p Y_t. \tag{16}$$

where $\Delta_t^p \equiv P_t^\epsilon \int_0^1 P_{it}^{-\epsilon} A_{it}^{-1} di$ is a measure of price dispersion which takes into account heterogeneous productivity.

It can be shown that the aggregate state of this economy is summarized by $\Omega_t \equiv (G_t, R_{t-1}, \Phi_{t-1})$, where $\Phi_{t-1}(P, A)$ is the lagged distribution of firms over prices and productivities (see Costain and Nakov, 2011).

2.5 Computation and parameterization

The equilibrium is computed following the two-step algorithm of Reiter (2009). Reiter's method is especially well suited to contexts such as this model, in which idiosyncratic shocks are large, but aggregate shocks are small. In a first step, the aggregate steady state is computed on a finite grid, using backwards induction. Second, the stochastic aggregate dynamics are computed by linearization around each grid point. Thus, the Bellman equation is treated as a large system of expectational difference equations, instead of a functional equation.[3]

We calibrate the three models (Calvo, fixed menu cost, and SSDP) to match salient features of the microdata on price changes, such as those found by Klenow and Kryvtsov (2008), Nakamura and Steinsson (2008), and Midrigan (2007, 2011). In particular, we seek price adjustment and productivity processes which are consistent with the histogram of price changes in AC Nielsen's monthly data documented by Midrigan (2007).

The model is one of "regular" price changes, excluding temporary "sales", and the working frequency is monthly. We set the growth rate of money to 0%, consistent with the zero average price change found in AC Nielsen's data. Other macro parameters are set to standard values in the RBC literature. Thus, the discount factor is $\beta = 1.04^{-1/12}$. Consumption utility is CRRA, $u(C) = \frac{1}{1-\gamma}C^{1-\gamma}$, with $\gamma = 2$, while labor disutility is $\chi\dfrac{N^{1+\eta}}{1+\eta}$, with $\eta = 1$ and $\chi = 11$, which implies that 35% of the time is dedicated to work. The elasticity of substitution in the consumption aggregator is $\epsilon = 7$. The utility of real money holdings is logarithmic, $v(m) = \nu \log(m)$, with $\nu = 1$. And we set $G^* = 0.07$, consistent with an average share of government spending of 20% of GDP.

This leaves us with five parameters to pin down the aggregate steady-state: the persistence ρ_A and the volatility of σ_a^2 of the idiosyncratic productivity process; and the three parameters $(\bar{\lambda}, \xi, \alpha)$ of the adjustment function (13). We estimate these parameters by minimizing a distance criterion between the data and the model's steady state. The criterion sums two terms, scaled for comparability: the

[3] See Costain and Nakov (2011) for a detailed exposition of Reiter's method as applied to sticky price models.

first is the absolute difference between the adjustment frequencies in the data and the simulation, while the second is the Euclidean distance between the frequency vectors associated with the histograms of nonzero price adjustments in the data and the simulation.[4] The left panel of figure 1 shows the fit of the three models to the histogram of price changes from AC Nielsen's data. The right panel shows the adjustment hazard function for different parameterizations. Clearly, the SSDP model provides the best fit to the observed histogram.

To simulate the model's dynamics, we need to specify two additional sets of parameters. One is related to the monetary policy rule (14). For our benchmark model, we set $\phi_R = 0$, $\phi_C = \phi_G = 0$, $\phi_\pi = 2$. The other set of parameters calibrates the exogenous process for government spending (15). We set $\rho_G = 0.9$, which similar to Erceg and Linde (2014), and pick the shock size so that, on impact, the increase in government spending equals 1% of GDP.

3 The spending multiplier, systematic monetary policy and nominal rigidities

In this section we analyze a log-linearized version of the model under the assumption of Calvo pricing, deriving some useful analytical results. Consider first the log-linearized representative household Euler equation (5), where small letters denote percentage deviations from steady state values and $\sigma = \gamma^{-1}$:

$$y_t - g_t = E_t\left(y_{t+1} - g_{t+1}\right) - \sigma\left(r_t - \overline{r} - E_t\pi_{t+1}\right).$$

Solving this equation forward for $c_t = y_t - g_t$, under stationarity of consumption and the government spending shock, it is easy to obtain the following expression for current output in excess of government purchases (both in deviations from steady state output):

$$y_t - g_t = -\sigma E_t \sum_{j=1}^{\infty}\left(r_{t+j-1} - \overline{r} - \pi_{t+j}\right).$$

[4]See Costain and Nakov (2011a,b) for more details about the estimation.

The response of output in excess of government purchases (equal to the spending multiplier minus one) depends on whether the sum of expected short-term real rates, or the long-term real rate, is positive or negative (see Woodford, 2011). Under nominal rigidities, the degree to which systematic monetary policy reacts to the government spending shock and interacts with future inflation formation will determine the size of the multiplier.

For instance, if we assume that the Taylor rule (14) is such that $r_{t+j-1} - \overline{r} = \phi_\pi \pi_{t+j}$, $\phi_\pi > 1$, the expression above becomes:

$$y_t - g_t = -\sigma\left(\phi_\pi - 1\right) E_t \sum_{j=1}^{\infty} \pi_{t+j} = -\sigma\left(\phi_\pi - 1\right)\left(E_t \sum_{j=0}^{\infty} \pi_{t+j} - \pi_t\right) = \sigma\left(\phi_\pi - 1\right)\left(p_t - \lim_{T\rightarrow\infty} E_t p_{t+T}\right).$$

As $\sigma\left(\phi_\pi - 1\right) > 0$, the size of the multiplier will be increasing in the current price level response, and decreasing in the long-run expected response of the price level (equal to cumulated inflation). Under the interest rate rule considered, the price level will not return to steady state. We show below that under Calvo-Yun price setting, the multiplier is generally smaller than one, as cumulated inflation will be larger than the current inflation rate when government spending follows an AR(1) process (see also Woodford (2011); Corsetti et al. (2010) analyze more general spending processes). We will show numerically that this result also holds under more general price-setting mechanism.

Results however change when the monetary policy rule entails a nominal interest rate less responsive to inflation. If we assume that systematic monetary policy is such that the nominal interest rate is constant for the whole horizon of the shock, $r_t = \overline{r}$, the long-term real rate will depend only on future expected inflation as follows:

$$y_t - g_t = \sigma E_t \sum_{j=1}^{\infty} \pi_{t+j} = \sigma\left(E_t \sum_{j=0}^{\infty} \pi_{t+j} - \pi_t\right) = \sigma\left(\lim_{T\rightarrow\infty} E_t p_{t+T} - p_t\right).$$

Interestingly, now the multiplier will be larger than one if expected cumulated inflation exceeds the current inflation response. Therefore, whether or not systematic monetary policy allows for a drift in the price level is a key determinant of the size of the multiplier under a constant nominal rate. For instance, the kind of monetary policy rules analyzed under the zero lower bound by e.g. Christiano et al. (2011) and Woodford (2011) — which can be thought of as equivalent to the constant interest rate rules Galí (2012) dubs "modest intervention" rules — imply that the price level has a positive drift and exceeds current inflation, resulting in a large multiplier in the Calvo-Yun model. In this case, the more 'flexible' prices, the bigger the difference between cumulated inflation and current inflation, as we show below. This is because the rate of inflation is determined by the rate of change of desired prices ("reset inflation"), which under time-dependent pricing is stable and persistent.

Specifically, it is easy to show that in the Calvo-Yun model, actual inflation is proportional to reset inflation $\left(\pi_t^* \equiv \log\left(p_t^*/p_{t-1}^*\right)\right)$:

$$\pi_t = \bar{\lambda}\pi_t^* + \left(1-\bar{\lambda}\right)\pi_{t-1},$$

where $\bar{\lambda}$ is the probability of price change, and the desired price p_t^* in response to a government spending shock is given by:

$$\log\left(p_t^*/P_t\right) = \beta E_t \log\left(p_{t+1}^*/P_{t+1}\right) + \left(1-\left(1-\bar{\lambda}\right)\beta\right)(\gamma+\eta)\left(y_t - \frac{\gamma}{\gamma+\eta}g_t\right).$$

But as shown by Bils et al. (2009), the link between actual and reset inflation is quite differently under state-dependent pricing. Endogenous price changing, and especially selection of changers, breaks the simple translation from π_t^* to π_t of time-dependent models. On the one hand, reset inflation tends to react more strongly to shocks on impact. On the other hand, it is much less stable and persistent, resulting in different properties of cumulated actual inflation. Even assuming the same frequency of price adjustment, state-dependent pricing can have dramatic implications for the effects of government purchases on aggregate economic activity under the lower bound-constant interest rate.

Likewise, when the constant interest rate results from a rule implying a stationary price level, i.e. $\lim_{T\to\infty} E_t p_{t+T} = 0$, such as a price level target, the price setting mechanism will affect the size of the multiplier. As we show below, the latter is in general smaller than unity, and decreasing in the degree of price flexibility, as the response of current (reset) inflation will be larger the more 'flexible' prices. In this case, state-dependent pricing should imply a smaller multiplier than time-dependent pricing, for the same frequency of price adjustment.

We have argued that the interaction between the monetary policy rule and price setting is paramount to understand the effects of fiscal stimulus, even when the nominal interest rate is constant and at its lower bound. In the next section we derive some analytical results for the version of our economy with Calvo-Yun pricing.

4 Constant nominal interest rates, price flexibility and the size of the multiplier

Consider the log-linearized Calvo-Yun model in Woodford (2011), akin to its counterpart in Section 2 above:

$$\begin{aligned} \pi_t &= \beta E_t \pi_{t+1} + \kappa (y_t - \Gamma g_t) \\ y_t - g_t &= E_t (y_{t+1} - g_{t+1}) - \gamma^{-1} (i_t - \overline{r} - E_t \pi_{t+1}) \end{aligned}$$

where

$$\begin{aligned} \kappa &= \bar{\lambda} \left(1 - \left(1 - \bar{\lambda}\right) \beta\right) (\gamma + \eta) / \left(1 - \bar{\lambda}\right) \\ \Gamma &= \frac{\gamma}{\gamma + \eta}, \end{aligned}$$

with $0 < \Gamma < 1$ denoting the multiplier under flexible prices. We will characterize the size of the multiplier, $\gamma_y = \frac{\partial y_t}{\partial g_t}$, already expressed as an elasticity, for different monetary policy rules. Since the focus is on the case of a constant nominal interest rate, it is useful to recall the key results under the zero lower bound. It is easy to show that under a shock inducing a ZLB lasting with probability p (e.g. as in Eggertsson and Woodford, 2003 or Christiano et al., 2011), with the zero-inflation steady state an absorbing state, the multiplier for a spending shock with same binomial process (and thus persistence) is given by:

$$\begin{aligned} \gamma_y - 1 &= \frac{(1 - \Gamma) \kappa p}{(1 - p)(1 - \beta p) \gamma - p\kappa} \\ \gamma_y - \Gamma &= \frac{(1 - p)(1 - \beta p) \gamma}{(1 - p)(1 - \beta p) \gamma - p\kappa} (1 - \Gamma), \end{aligned}$$

where in order to have a unique bounded solution it must be that $(1-p)(1-\beta p)\gamma p^{-1} > \kappa$.[5]

The multiplier is larger than unity (and its flex-price counterpart Γ), and also quite large for a sufficiently persistent shock. It is also increasing in κ, and particularly in the degree of price flexibility $1-\bar{\lambda}$. This is the "paradox of flexibility " stressed by e.g. Eggertsson and Krugman (2012). Finally, the multiplier is also *decreasing* in Γ, and particularly equal to one when $\Gamma = 1$, the case with linear labor disutility (as in this case the only allocation with a constant nominal interest rate coincides with that under flexible prices with a constant real rate). This is nevertheless an interesting case to consider as it is the benchmark specification in the study of money growth shocks in Golosov and Lucas (2007).

4.1 Taylor rules

A rule without smoothing Rather than imposing a constant interest rate, we start assuming in line with Section 2 that monetary policy follows the following interest rate feedback rule

$$r_t - \overline{r} = \phi_\pi \pi_t + \phi_C (y_t - \Gamma g_t) - \phi_G g_t,$$

with $\phi_R = 0$, and that government spending follows the process $g_t = \rho g_{t-1} + \varepsilon_t^G$, where obviously ρ plays the same role as p. Guessing the following decision rules under the unique bounded solution (for $\phi_\pi > 1$)[6]:

$$\begin{aligned} \pi_t &= \gamma_\pi g_t \\ y_t &= \gamma_y g_t \\ r_t - \overline{r} &= \gamma_r g_t, \end{aligned}$$

[5] This can be easily verified by checking that both eigenvalues of the characteristic equation of the following difference equation are larger than 1 when $(1-p)(1-\beta p)\gamma p^{-1} > \kappa$.

$$\beta p^2 (y_{t+1} - g_{t+1}) - p\left(1+\beta+\kappa\gamma^{-1}\right)(y_t - g_t) + (y_{t-1} - g_{t-1}) = p\kappa\gamma^{-1}(1-\Gamma) g_t,$$

[6] This can be easily verified by checking that both eigenvalues of the characteristic equation of the following difference equation in inflation are larger than 1:

$$(1+\kappa\sigma\phi_\pi)\pi_t - (1+\beta+\kappa\sigma) E_t\pi_{t+1} + \beta E_t\pi_{t+2} = 0.$$

it is straightforward to find the following solutions for their coefficients:

$$\begin{aligned}\gamma_r &= \phi_\pi \gamma_\pi + \phi_C \left(\gamma_y - \Gamma\right) - \phi_G \\ \gamma_\pi &= \frac{\kappa \left(\gamma_y - \Gamma\right)}{(1-\beta\rho)} \\ \gamma_y - 1 &= \rho \left(\gamma_y - 1\right) - \sigma \left[(\phi_\pi - \rho) \frac{\kappa \left(\gamma_y - \Gamma\right)}{(1-\beta\rho)} + \phi_C \left(\gamma_y - \Gamma\right) - \phi_G \right].\end{aligned}$$

Importantly the last equation implies that the multiplier is larger than 1 only to the extent that the short-term real interest rate $(r_t - \overline{r} - E_t \pi_{t+1}) = \left[(\phi_\pi - \rho) \frac{\kappa \left(\gamma_y - \Gamma\right)}{(1-\beta\rho)} + \phi_C \left(\gamma_y - \Gamma\right) - \phi_G \right]$ is *negative.*

Solving explicitly for γ_y yields:

$$\left(\gamma_y - 1\right) = \frac{\phi_G - \left(\frac{(\phi_\pi - \rho)}{(1-\beta\rho)} \kappa + \phi_C \right) (1 - \Gamma)}{(1-\rho)\,\sigma^{-1} + \left[\frac{(\phi_\pi - \rho)}{(1-\beta\rho)} \kappa + \phi_C \right]}.$$

First, the multiplier is larger than 1 if and only if $\phi_G > \left(\frac{(\phi_\pi - \rho)}{(1-\beta\rho)} \kappa + \phi_C \right) (1 - \Gamma) > 0$, where it can be shown that the latter expression is indeed proportional to the real interest rate that would prevail with a unitary multiplier. For instance, for a rule with $\phi_G = \phi_C = 0$, it is clear that the multiplier is always smaller than unity: as argued above, the response of cumulated inflation is larger than that of current inflation, resulting in a positive long-term real interest rate.

Second, for $\gamma_y \geq 1$ the multiplier is increasing in σ, Γ and ρ (to the extent that $\phi_\pi > \beta^{-1}$), and decreasing in $\phi_C, \phi_\pi, 1 - \bar{\lambda}$. Therefore, the steeper the Phillips Curve, e.g. the more flexible prices (the smaller $\left(1 - \bar{\lambda}\right)$), the smaller the multiplier, in contrast with the finding under the ZLB.

We can also derive conditions under which the nominal interest rate is constant for the duration of the shock, namely for $\gamma_r = 0$ (similarly to the modest interventions rule of Galí (2012)):

$$\begin{gathered}\gamma_r = \phi_\pi \gamma_\pi + \phi_C \left(\gamma_y - \Gamma\right) - \phi_G = 0 \\ \left[\phi_C + \phi_\pi \frac{\kappa}{(1-\beta\rho)} \right] \left(\gamma_y - \Gamma\right) = \phi_G;\end{gathered}$$

combined with the above solution for the decision rules it yields the following expression:

$$\left[\phi_C+\phi_\pi\frac{\kappa}{(1-\beta\rho)}\right]\left(1+\frac{\phi_G-\left(\frac{(\phi_\pi-\rho)}{(1-\beta\rho)}\kappa+\phi_C\right)}{(1-\rho)\,\sigma^{-1}+\frac{(\phi_\pi-\rho)}{(1-\beta\rho)}\kappa+\phi_C}\right)(1-\Gamma)=\phi_G.$$

Interestingly, for $\Gamma = 1$, which is the case with linear disutility of labor, the nominal interest rate is constant when $\phi_G = 0$, namely for a constant real interest rate and zero inflation, which corresponds to the flexible price equilibrium and a unity multiplier. This is also the value of the multiplier under the ZLB in this case.

For the general case in which $0 < \Gamma < 1$, the nominal interest rate is constant when the following holds:

$$[\phi_C(1-\beta\rho)+\phi_\pi\kappa](1-\Gamma)(1-\rho)=\phi_G[(1-\rho)(1-\beta\rho)-\sigma\rho\kappa].$$

This implies that in response to spending shocks, ϕ_G is such that the nominal interest rate adjusts accordingly as to stay constant over the horizon of the shock. This is the (conditional) constant interest rate rule Galí (2012) dubs a "modest intervention" rule, as it is equivalent to one with a shock to the interest rate rule perfectly correlated with the spending shock. However, the ϕ_G needs to be chosen as a function of the model's parameters to insure the interest rate is constant.

We thus obtain our first result, namely that the expression for the multiplier under the "modest intervention" rule is the same as under the ZLB:

$$\begin{aligned}(\gamma_y-1)&=\frac{(1-\Gamma)\,\rho\kappa}{(1-\rho)(1-\beta\rho)\gamma-\rho\kappa}\\(\gamma_y-\Gamma)&=\frac{(1-\rho)(1-\beta\rho)\gamma}{(1-\rho)(1-\beta\rho)\gamma-\rho\kappa}(1-\Gamma),\end{aligned}$$

for $(1-\rho)(1-\beta\rho)\gamma \neq \rho\kappa$ (when the latter expression holds with equality and $\phi_C = 0$, the interest rate is constant only if $\phi_\pi = 0$, which would result in multiple bounded solutions). Similarly to the ZLB case, the multiplier with a constant interest rate is larger than under flexible prices and also larger than one, if the NKPC is *flat* enough, i.e. if $(1-\rho)(1-\beta\rho)\gamma > \rho\kappa$. However, in contrast with the ZLB, the multiplier is defined also when the latter inequality is reversed and it changes sign, so that the multiplier is initially negative (for $\Gamma\rho\kappa < (1-\rho)(1-\beta\rho)\gamma < \rho\kappa$), but also always *smaller* than under flexible prices (and thus also smaller than 1). Away from the discontinuity, in which its value falls discretely, the spending multiplier is always *increasing* the steeper the NKPC (in κ):

$$\frac{\partial \gamma_y}{\partial \kappa} = \frac{(1-\rho)(1-\beta\rho)(1-\Gamma)}{\left[(1-\rho)(1-\beta\rho)-\rho\sigma\kappa\right]^2}\rho\sigma > 0,$$
$$(1-\rho)(1-\beta\rho)\gamma \neq \rho\kappa.$$

Therefore, while the "paradox of flexibility" continues to hold, nevertheless the spending multiplier is larger than its flexible price counterpart only when prices are sufficiently sticky that the inequality $(1-\rho)(1-\beta\rho)\gamma > \rho\kappa$ is satisfied.

The intuition for this result is straightforward: with a constant nominal interest, the long term real interest rate is a function of expected inflation, which in turn is increasing in the slope of the NKPC. Specifically, it is easy to show that the price level has a positive drift with a constant nominal interest rate under the modest intervention rule:

$$\begin{aligned} E_t \sum_{j=1}^{\infty} \pi_j &= \lim_{T\to\infty} p_{t+T} - p_0 = \gamma_\pi g_0 \sum_{j=1}^{\infty} \rho^j = \frac{\kappa\left(\gamma_y - \Gamma\right)}{(1-\beta\rho)}\frac{\rho}{1-\rho} g_0, \\ \lim_{T\to\infty} p_{t+T} &= \frac{1}{1-\rho}\frac{\kappa\left(\gamma_y-\Gamma\right)}{(1-\beta\rho)} g_0, \end{aligned}$$

where

$$\left(\gamma_y - \Gamma\right) = \frac{(1-\rho)(1-\beta\rho)}{(1-\rho)(1-\beta\rho)-\rho\sigma\kappa}(1-\Gamma).$$

The latter expression is positive if the multiplier is larger than Γ, its flexible price counterpart, for $(1-\rho)(1-\beta\rho) > \sigma\rho\kappa$. Therefore, in the Calvo model under the ZLB or the modest intervention rule, the multiplier will also increase in $\left(1-\bar{\lambda}\right)$, and thus in the slope of the NKPC. This is because in the Calvo model, expected inflation is proportional to current inflation, and thus to "reset inflation", and thus the more responsive the steeper the Phillips curve.

The case of interest rate smoothing Consider now instead a Taylor rule with interest rate smoothing:

$$r_t - \overline{r} = (1-\phi_R)\left[\phi_\pi \pi_t + \phi_C\left(y_t - \Gamma g_t\right) - \phi_G g_t\right] + \phi_R\left(r_{t-1} - \overline{r}\right),$$

and guess the following decision rules:

$$\begin{aligned}
\pi_t &= \gamma_\pi g_t + \lambda_\pi\left(r_{t-1} - \overline{r}\right) \\
y_t &= \gamma_y g_t + \lambda_y\left(r_{t-1} - \overline{r}\right) \\
r_t - \overline{r} &= \gamma_r g_t + \lambda_r\left(r_{t-1} - \overline{r}\right).
\end{aligned}$$

The government spending multiplier is now given by:

$$\left(\gamma_y - 1\right) = \frac{\sigma\kappa\rho\left(1-\Gamma\right)}{\left(1-\rho\right)\left(1-\beta\rho\right) - \sigma\kappa\rho} + \beta^{-1}\left[\frac{\left(\beta-\lambda_\pi\right)\left(1-\beta\rho\right)\beta + \sigma\left(\lambda_\pi - \left(1-\beta\rho\right)\beta\right)}{\left(1-\rho\right)\left(1-\beta\rho\right) - \sigma\kappa\rho}\right]\gamma_r,$$

$$\begin{aligned}
\gamma_r &= \left(1-\phi_R\right)\left[\phi_\pi\gamma_\pi + \phi_C\left(\gamma_y - \Gamma\right) - \phi_G\right] \\
\gamma_\pi &= \frac{\kappa\left(\gamma_y - \Gamma\right) + \lambda_\pi\gamma_r}{\left(1-\beta\rho\right)},
\end{aligned}$$

and

$$\begin{aligned}
\lambda_y &= \beta - \lambda_\pi \\
\lambda_r &= \phi_R + \left(1-\phi_R\right)\left[\left(\phi_\pi - \phi_C\right)\lambda_\pi + \phi_C\beta\right],
\end{aligned}$$

while λ_π is the stable root (inside the unit circle) of the following polynomial in λ_π :

$$h\left(\lambda_\pi\right) = \beta\kappa - \left[1 + \kappa + \beta\left(\phi_R + \left(1-\phi_R\right)\phi_C\beta\right)\right]\lambda_\pi + \beta\left(1-\phi_R\right)\left(\phi_\pi - \phi_C\right)\lambda_\pi^2 = 0.$$

One may think that for $\gamma_r \rightarrow 0$, which now can be obtained also by setting $\phi_R \rightarrow 1$ (although the case in which $\gamma_r = 0, \phi_R = 1$ cannot obviously be studied as the equilibrium would not be determined), it would be possible to obtain the same results as under the modest intervention rule, but now for any ϕ_G. However, it can be shown that this is not the case: even if the interest rate is virtually constant also in this case, the multiplier in general will be different from the one under the modest intervention rule, unless ϕ_G is again chosen appropriately.

To summarize our findings so far, the key driver of the stimulus effect of government spending is the response of the real interest rate. When the monetary policy rule is such that the nominal interest rate reacts to endogenous variables, e.g. as in the case of a standard Taylor rule, the real interest rate response will be a function of the current and future effects of government spending. To the extent that the interest rate responds to current inflation, the steeper the Phillips Curve (the more flexible prices) the smaller the decrease in the real interest rate. This is because the response to current inflation ($\phi_\pi > 1$) necessary to ensure determinacy is larger than the persistence of inflation expectations.

Conversely, when the monetary policy rule is such that the interest rate does not (or cannot under the ZLB) respond to endogenous variables, and is thus constant, as it is case under the rule studied above, the real interest rate is purely a function of expected inflation. The latter in the Calvo model, under a stationary solution, is always proportional to current inflation, and thus the more responsive the steeper the Phillips curve. In the next section we show that the expansionary effects of the spending shock depend on some subtle properties of the policy rule, rather than on interest rate being simply constant. This is already known from the literature on government spending shocks in open economies (see e.g. Corsetti et al. 2009).

4.2 Two more constant nominal interest rules

As shown by Galí (2012), an allocation with a constant nominal interest rate can always be implemented by using either of the following two interest rate rules:

$$
\begin{aligned}
r_t &= \overline{r} + \phi_i \left[\sigma \pi_t + \Delta \left(y_t - g_t\right)\right], \quad \phi_i > 1. \\
r_t &= \overline{r} + \phi_i \left[\pi_t + r_{t-1}\right]
\end{aligned}
$$

4.2.1 A constant interest rate rule as a money growth rule

Starting with the first rule, in terms of the rest of the rest of the system we have that

$$\phi_i \left[\sigma\pi_t + \Delta\left(y_t - g_t\right)\right] = E_t\left[\Delta\left(y_{t+1} - g_{t+1}\right) + \sigma\pi_{t+1}\right]$$

$$\phi_i > 1 => \sigma\pi_t + \Delta\left(y_t - g_t\right) = 0 = r_t - \overline{r}.$$

Alternatively, we can also implement the same allocation by using a money growth rule. In the case of the log-log money demand function assumed in our specification

$$m_t = \sigma^{-1}\left(y_t - g_t\right) - \frac{\beta^2}{1-\beta}\left(r_t - \overline{r}\right),$$

a constant money growth rule delivers a constant nominal interest rate:

$$\mu_t = \sigma\pi_t + \Delta\left(y_t - g_t\right) = 0.$$

This is the monetary policy reaction function studied by Golosov and Lucas (2007), which would thus imply an unconditionally constant nominal interest rate, but in the case of a money growth shock.

Using the fact that $\sigma\pi_t = -\Delta\left(y_t - g_t\right)$ and substituting out the NKPC into the Euler equation we get a 2nd order difference equation in y_t:

$$\beta E_t\left(y_{t+1} - g_{t+1}\right) - \left(1 + \beta + \kappa\sigma\right)\left(y_t - g_t\right) + \left(y_{t-1} - g_{t-1}\right) = \kappa\sigma\left(1 - \Gamma\right)g_t,$$

which has the following general solution:

$$y_t = \gamma_0^y g_t + \gamma_1^y g_{t-1} + \lambda_1 y_{t-1} + a_2\lambda_2^t,$$

where λ_1, λ_2 are the zeros of the following polynomial in λ :

$$1 - \left(1 + \beta + \kappa\sigma\right)\lambda + \beta\lambda^2 = 0.$$

It is clear that for $\kappa\sigma > 0$ we always have $0 < \lambda_1 < 1 < \lambda_2$, implying that for $a_2 = 0$ there will be a unique bounded solution.

This solution is given by:

$$
\begin{aligned}
y_t &= \gamma_0^y g_t + \gamma_1^y g_{t-1} + \lambda_1 y_{t-1} \\
\gamma_0^y - 1 &= -\frac{\kappa\sigma(1-\Gamma)}{\beta(1-\rho) + 1 - \lambda_1 + \kappa\sigma} \\
\gamma_1^y &= -\frac{1 - \beta\lambda_1\rho}{\beta(1-\rho) + 1 - \lambda_1 + \kappa\sigma}.
\end{aligned}
$$

Despite the fact that the nominal interest rate is constant, the impact multiplier, although still larger than Γ (unless $\Gamma = 1$) is now always lower than 1, and decreasing in $\left(1 - \bar{\lambda}\right)$. The intuition for this result is straightforward: the monetary policy rule implies a price-level target, i.e. $\sigma p_t = -(y_t - g_t)$. This entails a long-run real interest rate that is now much higher than under the interest rate rule one considered in the previous section, in which an inflation target was indeed assumed.

More formally, since the price level is stationary, it is easy to show that

$$
y_0 - g_0 = \sigma E_t \sum_{j=1}^{\infty} \pi_j = \sigma \left(E_t \sum_{j=0}^{\infty} \pi_j - \pi_0 \right) = \sigma \left(\lim_{T\to\infty} p_{t+T} - \pi_0 \right),
$$

where $p_{t+T} \to 0$ implies $E_t \sum_{j=0}^{\infty} \pi_j = 0$. The multiplier is then decreasing in the contemporaneous response of inflation, so that a steeper NKPC will always result in a smaller multiplier. Likewise, the more 'flexible' are prices across price setting models, the lower the multiplier.

4.2.2 A constant interest rate rule as an inflation target

Considering now the second rule, in terms of the rest of the rest of the system we have that

$$
\phi_r \left[\pi_t + r_{t-1}\right] = E_t \left[\pi_{t+1} + r_t\right]
$$

$$
\phi_r > 1 => \pi_t + r_{t-1} = 0 = i_t - \overline{r}
$$

$$
\pi_t = \beta E_t \pi_{t+1} + \kappa \left(y_t - \Gamma g_t\right).
$$

Let's check first under which conditions the above equilibrium is unique. Using the fact that $\pi_t = -r_{t-1}$ and substituting out the NKPC into the Euler equation we get the following 2nd order difference equation in r_t:

$$\kappa\left(y_t - g_t\right) = \beta r_t - r_{t-1} - \kappa\left(1-\Gamma\right) g_t$$

$$E_t \Delta\left(y_{t+1} - g_{t+1}\right) = \sigma r_t$$

$$\kappa E_t \Delta\left(y_{t+1} - g_{t+1}\right) = E_t\left(\beta r_{t+1} - r_t - \kappa\left(1-\Gamma\right)\rho g_t\right) - \left(\beta r_t - r_{t-1} - \kappa\left(1-\Gamma\right) g_t\right) = \kappa\sigma r_t$$

$$\beta E_t r_{t+1} - \left(1+\beta+\kappa\sigma\right) r_t + r_{t-1} = -\kappa\left(1-\Gamma\right)\left(1-\rho\right) g_t,$$

which has the following general solution:

$$r_t = \gamma_r g_t + \lambda_1 r_{t-1} + a_2 \lambda_2^t,$$

where λ_1, λ_2 are the zeros of the following polynomial in λ :

$$1 - \left(1+\beta+\kappa\sigma\right)\lambda + \beta\lambda^2 = 0,$$

which coincide with those above. It is clear that for $\kappa\sigma > 0$ we always have $0 < \lambda_1 < 1 < \lambda_2$, implying that for $a_2 = 0$ there will be a unique bounded solution given by:

$$\begin{aligned} r_t &= \gamma_r g_t + \lambda_1 r_{t-1} \\ \gamma_r &= \frac{\kappa\left(1-\Gamma\right)\left(1-\rho\right)}{\beta\left(1-\rho\right) + 1 - \beta\lambda_1 + \kappa\sigma} > 0. \end{aligned}$$

The real interest rate increases in response to the spending shock. Plugging this in the NKPC we can derive the response of output:

$$\begin{aligned} y_t - g_t &= \frac{\beta}{\kappa}\left(\gamma_r g_t + \lambda_1 r_{t-1}\right) - \kappa^{-1} r_{t-1} - \left(1-\Gamma\right) g_t \\ &= -\left[\frac{1 - \beta\lambda_1 + \kappa\sigma}{\beta\left(1-\rho\right) + 1 - \beta\lambda_1 + \kappa\sigma}\right]\left(1-\Gamma\right) g_t - \frac{1}{\kappa}\left(1-\beta\lambda_1\right) r_{t-1} \end{aligned}$$

Clearly the impact multiplier is again always lower than 1, though still larger than Γ, and decreasing in κ. The intuition for this result is straightforward: the monetary policy rule implies a negative drift in inflation as $\pi_t = -r_{t-1}$. This means that the long-run real interest rate is even higher than under the interest rate rule one considered in the previous subsection, so that the impact multiplier is lower, as

$$-\frac{\kappa\sigma}{\beta(1-\rho)+1-\lambda_1+\kappa\sigma} > -\frac{1-\beta\lambda_1+\kappa\sigma}{\beta(1-\rho)+1-\beta\lambda_1+\kappa\sigma},$$

given that λ_1 is the same.

Formally, because $\pi_t = 0$ we have that:

$$E_t\sum_{j=1}^{\infty}\pi_{t+j} = E_t\sum_{j=0}^{\infty}\pi_{t+j} = E_t\lim_{T\to\infty}p_{t+T};$$

higher price flexibility will imply a more negative long run price level, making the multiplier smaller:

$$E_t\lim_{T\to\infty}p_{t+T} = -E_t\sum_{j=0}^{\infty}r_{t+j} = -\frac{\gamma_r g_t}{\lambda_1-\rho}\lambda_1\sum_{j=0}^{\infty}\lambda_1^j\left(1-\frac{\rho^{j+1}}{\lambda_1^{j+1}}\right) = -\frac{\kappa(1-\Gamma)(1-\rho)}{\beta(1-\rho)+1-\beta\lambda_1+\kappa\sigma}\frac{g_t}{(1-\lambda_1)(1-\rho)} < 0$$

To summarize, we have shown that the key driver of the stimulus effect of government spending, is really the response of the long-run real interest rate, and thus when the nominal interest rate is constant, of the difference between expected cumulated inflation and current inflation. While the monetary policy reaction function will determine whether the price level is stationary or not, the relation between current and cumulated inflation will be shaped by the model of price setting. The Calvo-Yun model implies a very tight and specific relation between the two concepts of inflation: this is related to the concept of reset inflation introduced by Bils et al. (2009). However, this property of reset inflation and of the inflation process of expected inflation does not seem a general one that would apply to other, more plausible models of price stickiness, with relevant consequences for the size of the multiplier. This is what we want to check in the rest of the paper.

4.3 The case of a constant rate for T periods

We conclude this section by also considering the case of a sequence of anticipated shocks which keep the interest rate constant for T periods. As of $T+1$, monetary policy returns to the whatever rule it followed. This is a standard approach to approximate the effects of the ZLB, especially in models with many state variables, in which the modest intervention approach cannot be implemented easily with an AR(1) shocks. However, we will show that this approach in general does not provide the same results as in Woodford (2011) and Christiano et al. (2011). Specifically, under this approach the "paradox of flexibility" is actually amplified and the multiplier can be very large even when $(1-\rho)(1-\beta\rho)\gamma < \rho\kappa$.

Again, it is easy to show that over the period for which the interest is anticipated to be constant, the dynamics of the economy under an AR(1) spending shock is fully described by the following second order difference equation:

$$\beta(y_{t+1}-g_{t+1}) - \left(1+\beta+\kappa\gamma^{-1}\right)(y_t-g_t) + (y_{t-1}-g_{t-1}) = \kappa\gamma^{-1}(1-\Gamma)\rho g_t,$$

with general solution given by

$$\begin{aligned}(y_t-g_t) &= \frac{(1-\Gamma)\kappa\rho}{(1-\rho)(1-\beta\rho)\gamma-\rho\kappa}g_t + a_1\lambda_1^t + a_2\lambda_2^t,\\ \lambda_i &= \frac{1+\beta+\kappa\gamma^{-1} \pm \sqrt{(1+\beta+\kappa\gamma^{-1})-4\beta}}{2\beta},\end{aligned}$$

for $T-1 \geq t \geq 0$, where the eigenvalues satisfy $0<\lambda_1<1<\lambda_2$. In turn, the constants a_1, a_2 are determined by the following two terminal conditions:

$$\gamma(y_T-g_T) = \gamma(y_{T+1}-g_{T+1}) + \pi_{T+1}$$

$$\begin{aligned}\gamma(y_{T-1}-g_{T-1}) &= \gamma(y_T-g_T)+\pi_T\\ \gamma(y_{T-1}-g_{T-1}) &= \gamma\left(1+\kappa\gamma^{-1}\right)(y_{T+1}-g_{T+1}) + \left(1+\beta+\kappa\gamma^{-1}\right)\pi_{T+1} + (1-\Gamma)\kappa\rho g_{T-1},\end{aligned}$$

yielding the following solution

$$
\begin{aligned}
a_1 &= -\frac{(1-\Gamma)\kappa\rho}{(1-\rho)(1-\beta\rho)\gamma-\rho\kappa}\frac{1-\rho\beta\lambda_1}{1-\beta\lambda_1^2}\left(\frac{\rho}{\lambda_1}\right)^T g_0+\frac{1}{1-\beta\lambda_1^2}\frac{(1-\beta\lambda_1)\gamma(y_{T+1}-g_{T+1})+\pi_{T+1}}{\gamma\lambda_1^T}\\
a_2 &= \frac{(1-\Gamma)\kappa\rho}{(1-\rho)(1-\beta\rho)\gamma-\rho\kappa}\frac{\beta\lambda_1(\lambda_1-\rho)}{1-\beta\lambda_1^2}\left(\frac{\rho}{\lambda_2}\right)^T g_0+\frac{\beta\lambda_1}{1-\beta\lambda_1^2}\frac{(1-\lambda_1)\gamma(y_{T+1}-g_{T+1})-\lambda_1\pi_{T+1}}{\gamma\lambda_2^T}.
\end{aligned}
$$

These constants are uniquely defined if after the interest rate is free to move the equilibrium is uniquely defined, pinning down $(y_{T+1}-g_{T+1})$ and π_{T+1}. Observe that since $\lambda_2>1$, a_2 will play a small role in the initial dynamics, especially when T is large. Conversely, since $\lambda_1<1$, a_1 could be quite important, when the equilibrium after liftoff is such that $(y_{T+1}-g_{T+1})$ and π_{T+1} are not small, or when $\rho>\lambda_1$. In particular, it can be shown that this is always the case when $(1-\rho)(1-\beta\rho)\gamma<\rho\kappa$. Moreover, recall that λ_1 is decreasing in κ and thus in the frequency of price adjustment.

To better appreciate the consequences for the spending multiplier, we can write the solution for the impact response of output as follows

$$
\begin{aligned}
y_0-g_0 &= \frac{(1-\Gamma)\kappa\rho}{(1-\rho)(1-\beta\rho)\gamma-\rho\kappa}g_0+a_1+a_2\\
&= \frac{(1-\Gamma)\kappa\rho}{(1-\rho)(1-\beta\rho)\gamma-\rho\kappa}\left[1-\frac{(1-\rho\beta\lambda_1)\left(\frac{\rho}{\lambda_1}\right)^T-\beta\lambda_1(\lambda_1-\rho)\left(\frac{\rho}{\lambda_2}\right)^T}{1-\beta\lambda_1^2}\right]g_0+\\
&\quad \frac{1}{1-\beta\lambda_1^2}\left[\left(1-\beta\lambda_1^2\left(\frac{\lambda_1}{\lambda_2}\right)^T\right)\frac{\pi_{T+1}}{\gamma\lambda_1^T}+\left(1-\beta\lambda_1\left(1-(1-\lambda_1)\left(\frac{\lambda_1}{\lambda_2}\right)^T\right)\right)\frac{(y_{T+1}-g_{T+1})}{\lambda_1^T}\right].
\end{aligned}
$$

The following considerations are in order. First, the multiplier is very similar to that under the ZLB experiments examined by Woodford (2011) and Christiano et al. (2011) when, in addition to $\rho<\lambda_1$, T is sufficiently large, and $(y_{T+1}-g_{T+1})$ and π_{T+1} tend to zero. When prices become more flexible in this range, and κ increases, the multiplier increases. Secondly, similarly to the analysis above, the multiplier is also defined for values of price flexibility such that $\rho>\lambda_1$. In this case, for T sufficiently large, the multiplier can also be very large, and is increasing in κ and thus in the frequency of price adjustment. Namely, the paradox of flexibility will arise for all values of $1-\overline{\lambda}$, resulting in an ever growing multiplier. Finally, the state of the economy after liftoff of the constant rate can also affect the impact multiplier if active monetary policy manipulates the values of output and inflation – see Del Negro et al. (2013) and Cochrane (2013). In particular, if monetary policy implements the flexible price solution, we will have $\pi_{T+1} = 0$ and $(y_{T+1} - g_{T+1}) = -(1-\Gamma)\rho^T g_0$, so that for T sufficiently large the following obtains:

$$y_0 - g_0 = \frac{(1-\Gamma)\kappa\rho}{(1-\rho)(1-\beta\rho)\gamma - \rho\kappa} g_0 - \frac{(1-\Gamma)(1-\rho)}{1-\beta\lambda_1^2}\left[\frac{\rho\kappa\beta\lambda_1 + (1-\beta\lambda_1)(1-\beta\rho)\gamma}{(1-\rho)(1-\beta\rho)\gamma - \rho\kappa}\right]\left(\frac{\rho}{\lambda_1}\right)^T g_0.$$

Again, as κ increases, the multiplier will grow ever larger.

5 Numerical Results

5.1 Price setting and the spending multiplier under a Taylor rule

We start by looking at the effects of a government spending shock under a standard interest rate rule. As anticipated in the previous section, with sticky prices and a Taylor rule for monetary policy, an increase in government spending can have an additional effect on output depending on the systematic reaction of monetary policy to the shock. In particular, only if the Taylor rule reacts to a measure of economic activity which includes government spending, then monetary policy could either reinforce the output effect of the government spending shock. The channel through which this happens is the standard one in New Keynesian models: the expenditure shock affects inflation expectations and hence the long-run real interest rate faced by households. And while the effects of a government spending shock have been analyzed in time-dependent models such as Calvo (Christiano et. al., 2011) or pre-set prices (Woodford, 2011), our interest is in how the fiscal multiplier might change if pricing is instead state-dependent. It is known that, in the presence of large idiosyncratic shocks to firms, the menu cost model can produce quite different dynamic responses to a money growth shock (Golosov-Lucas, 2007). For this reason, we also report impulse responses for a version of the Calvo model with idiosyncratic shocks to firms.

These results are presented in Figure 2. The figure plots the responses to our AR(1) benchmark government spending shock, under a rule that only responds to current inflation, of six variables: the government spending process, consumption, GDP, inflation, the nominal interest rate, and the real interest rate (all annualized). In the last row the figure also reports the responses of three statics related to price setting: the "intensive margin", the "selection effect" and a measure of the dispersion of price changes. The green lines with squares plot the responses with fixed menu costs, the blue lines with circles plot those under Calvo pricing and idiosyncratic shocks, an the red lines with dots plot those under the textbook Calvo model. The shock to government spending is scaled so that on impact, government expenditure increases by 1% of GDP.

In all three models the government spending shock raises inflation but depresses consumption. This is consistent with the nominal interest rate rising by more than the rise in expected future inflation, so that the *long run*, ex-ante, real interest rate increases in response to the shock.

However, important quantitative differences emerge when looking across the three models. Thus, inflation rises by 1.5 percentage points on impact under the menu cost model, while it rises only by 0.5 percentage points in the Calvo model. This is just the "selection effect" emphasized by Golosov-Lucas (2007): in the menu cost model the firms that adjust are those for which adjustment is most valuable and the aggregate price level is quite flexible. In particular, many firms that were just inside the lower S,s bound and were not going to adjust in the absence of the shock, now choose to make a large upward price change. Similarly, many firms just outside the upper S,s bound that were contemplating a large price decline, are now discouraged from it in the aftermath of the shock. Because of this major change in the distribution of "adjusters", the aggregate price level is much more flexible in the fixed menu cost model than in the Calvo model in which "adjusters" are drawn randomly from the population of firms. This is clearly visible in the last row, showing that this selection effect contributes a great deal to inflation in the menu cost model, while it is absent by construction in the Calvo model. Therefore, while in the Calvo model with idiosyncratic shocks the "extensive margin" channel (namely the change in the size of price changes) is very close to that of the fixed menu cost model, the response of inflation is virtually identical to that in the textbook Calvo version without idiosyncratic shocks. The dispersion of price changes, measured by the interquartile range also increase more in the menu cost model than in the idiosyncratic Calvo model — we do not compute this measure in the model without firm-level shocks. This statistic has been recently stressed by Vavra (2012), who shows that in the US data it is positively correlated with the frequency of price adjustment. Interestingly, this also what we find in the menu cost model under the Taylor rule.

On the flip side of that, consumption falls by much more under fixed menu costs than it does under Calvo pricing. As a result, the impact multiplier (dY_1/dG_1) in the menu cost model is close to its flexible price counterpart of 2/3. Under Calvo, although less than one as anticipated above, the multiplier exceeds its flexible price counterpart. Overall, these experiments confirm that under a standard Taylor the endogenous response of prices and inflation is larger in the state-dependent model, leading to smaller real effects of shocks. We now turn to the case of a constant interest rate, under which the paradox of flexibility has been established under time-dependent pricing.

5.2 The multiplier under constant interest rates

Here we report results under a constant interest rate, which we assume it is anticipated not to react to the shock for 36 months (Results would be very similar with an interest rate fixed for 24 months). In Figures 3 and 4, the blue lines with circles plot the responses with fixed menu costs, the green lines with squares plot those under Calvo pricing and idiosyncratic shocks, an the red lines with dots plot those under the textbook Calvo model. The shock to government spending is again scaled so that on impact, government expenditure increases by 1% of GDP.

Across all models, the fact that monetary policy does not react produces a persistent expansionary effect. This is consistent with the nominal interest rate not rising with the rise in expected future inflation, so that the *long run*, ex-ante, real interest rate drops in response to the shock. The result is that consumption rises and thus the multiplier exceeds one in all three models. Strikingly, the state-dependent model now delivers a much larger multiplier than the time-dependent models, an order of magnitude larger. Obviously this is reflected in a very large response of inflation (above 1000 percent) and in a dramatic fall in the short term real rate. Intuitively, under a constant interest rate, more flexible prices lead to a larger response of the expected price level upon exit. As we have shown in the previous section, contrary to the stochastic ZLB case analyzed by Woodford (2011), when the interest rate is kept constant as in our experiment, an equilibrium is defined for any admissible flexibility of prices. Under fixed menu costs, the effects of a constant interest rate are much bigger than under

Calvo pricing with the same average frequency of price adjustment. Indeed, we would get a similar multiplier with a higher frequency of price adjustment in the Calvo model, of around 0.2 instead of .1. The "paradox of flexibility" thus holds under state-dependent pricing and accounts for these very large responses.

This is not only a result of selection in the fixed menu cost model. Also idiosyncratic shocks play a role. Figure 4 compares the responses of the Calvo model with and without idiosyncratic shocks, where the standard deviation of the latter gradually increases from zero to its benchmark level of 0.01. Strikingly, the multiplier increases with the volatility of idiosyncratic shocks. Effectively, the presence of firm-level shocks makes prices more volatile, resulting in an amplification mechanism. This mechanism is quantitatively important when the interest rate is constant, while it is not operative under a standard Taylor rule, as shown in Figure 2. Interestingly, this mechanism has an interesting implication for disaggregated data, as it leads to a sharp increase in the cross-sectional dispersion of prices. Evidence on such increased dispersion in the latest recession is provided by Vavra (2012).

6 Conclusion

We have studied the effects of state-dependent pricing on the amplification of shocks under a constant interest rate, focusing on the government spending multiplier. Since the type of price setting by firms is crucial for the determination of the long-term real interest rate in the workhorse DSGE model, it is a primary determinant of the size of the spending multiplier. To have a multiplier substantially bigger than unity under a standard interest rate rule, both monetary policy has to be accommodative, and the aggregate price level has to be sufficiently sticky. Conversely, when the nominal interest is constant or at the zero lower bound, if the monetary policy is such that the price level has a positive drift, a paradox of flexibility arises: the multiplier, and thus shock amplification, is increasing in the degree of price flexibility. We thus find much larger multipliers in the fixed menu cost model than under Calvo pricing, when the nominal interest rate is constant, for the same frequency of price adjustment. We also find that firm-level shocks play an important role in amplification even in the standard Calvo model.

In concluding, it is however important to remember that our results concern the size of policy multipliers in the workhorse New Keynesian model, which is the core of DSGE models also used to inform policy debates, but do not consider the importance of the many additional factors in the latter. These other factors, such as distortionary taxes, credit and financial constraints for households and firms, and many others, are important for specific quantitative answers and for a full welfare evaluation.

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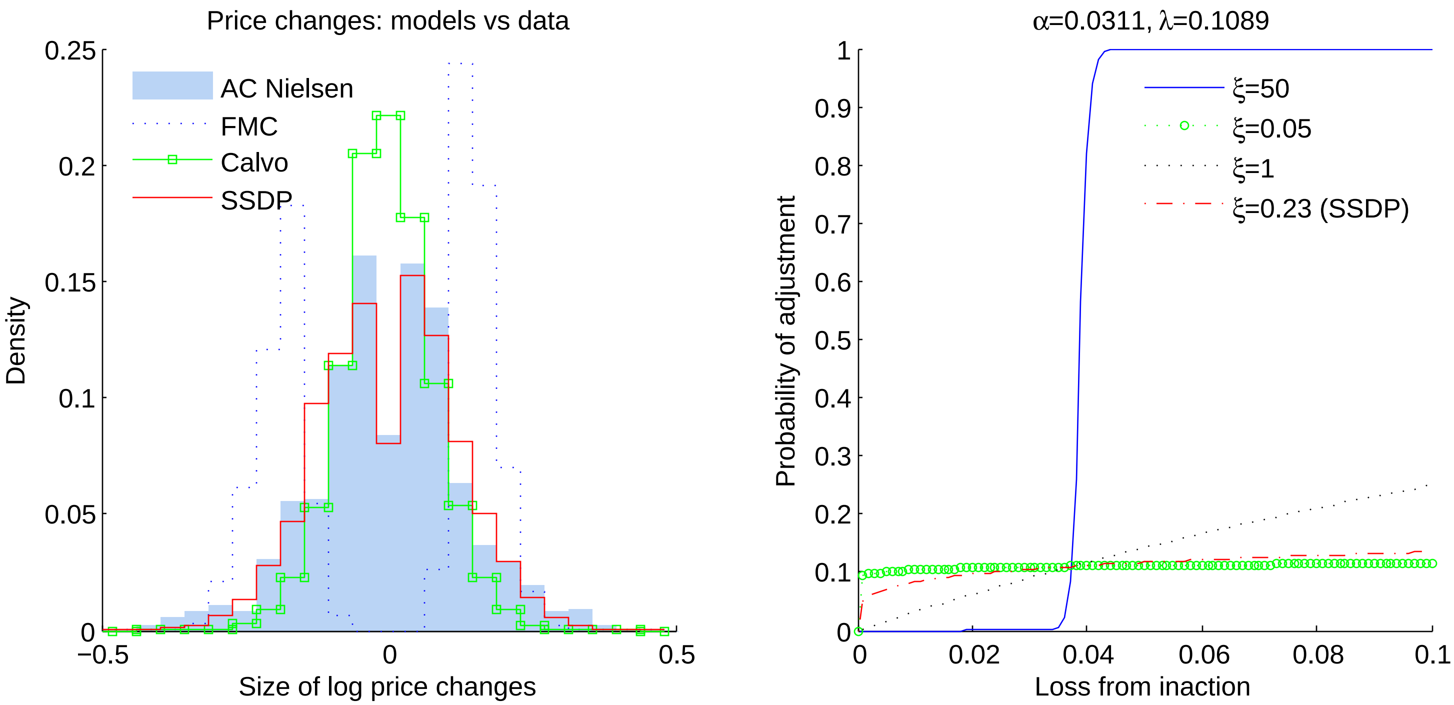
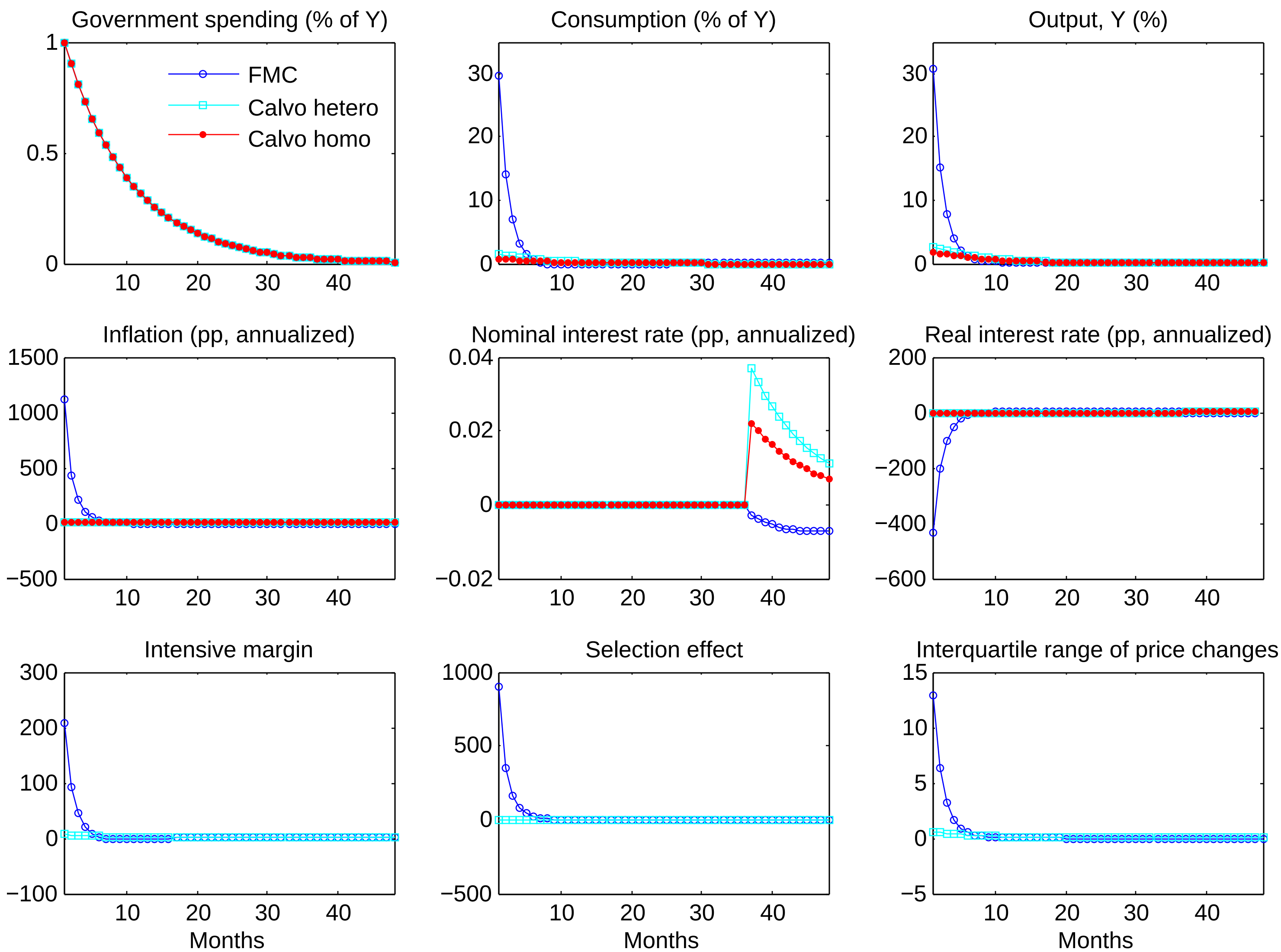
Figure 1



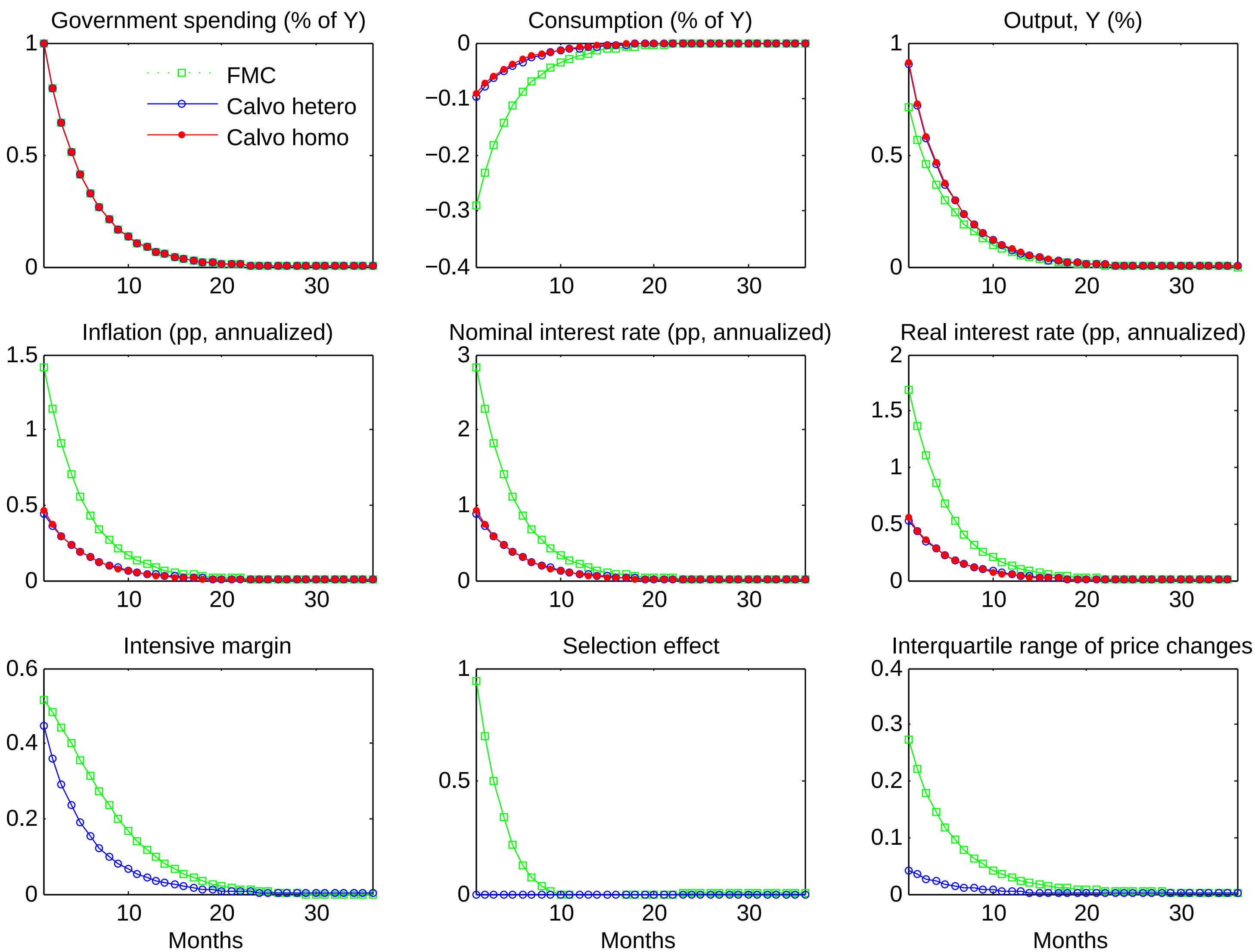
Price changes: models vs data
AC Nielsen
FMC
Calvo
SSDP
Density
Size of log price changes
0.25
0.2
0.15
0.1
0.05
0
−0.5
0
0.5
α=0.0311, λ=0.1089
ξ=50
ξ=0.05
ξ=1
ξ=0.23 (SSDP)
Probability of adjustment
Loss from inaction
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.02
0.04
0.06
0.08
0.1

Figure 2



Government spending (% of Y)
FMC
Calvo hetero
Calvo homo
Consumption (% of Y)
Output, Y (%)
Inflation (pp, annualized)
Nominal interest rate (pp, annualized)
Real interest rate (pp, annualized)
Intensive margin
Selection effect
Interquartile range of price changes
Months
Months
Months

Figure 3



Government spending (% of Y)
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Inflation (pp, annualized)
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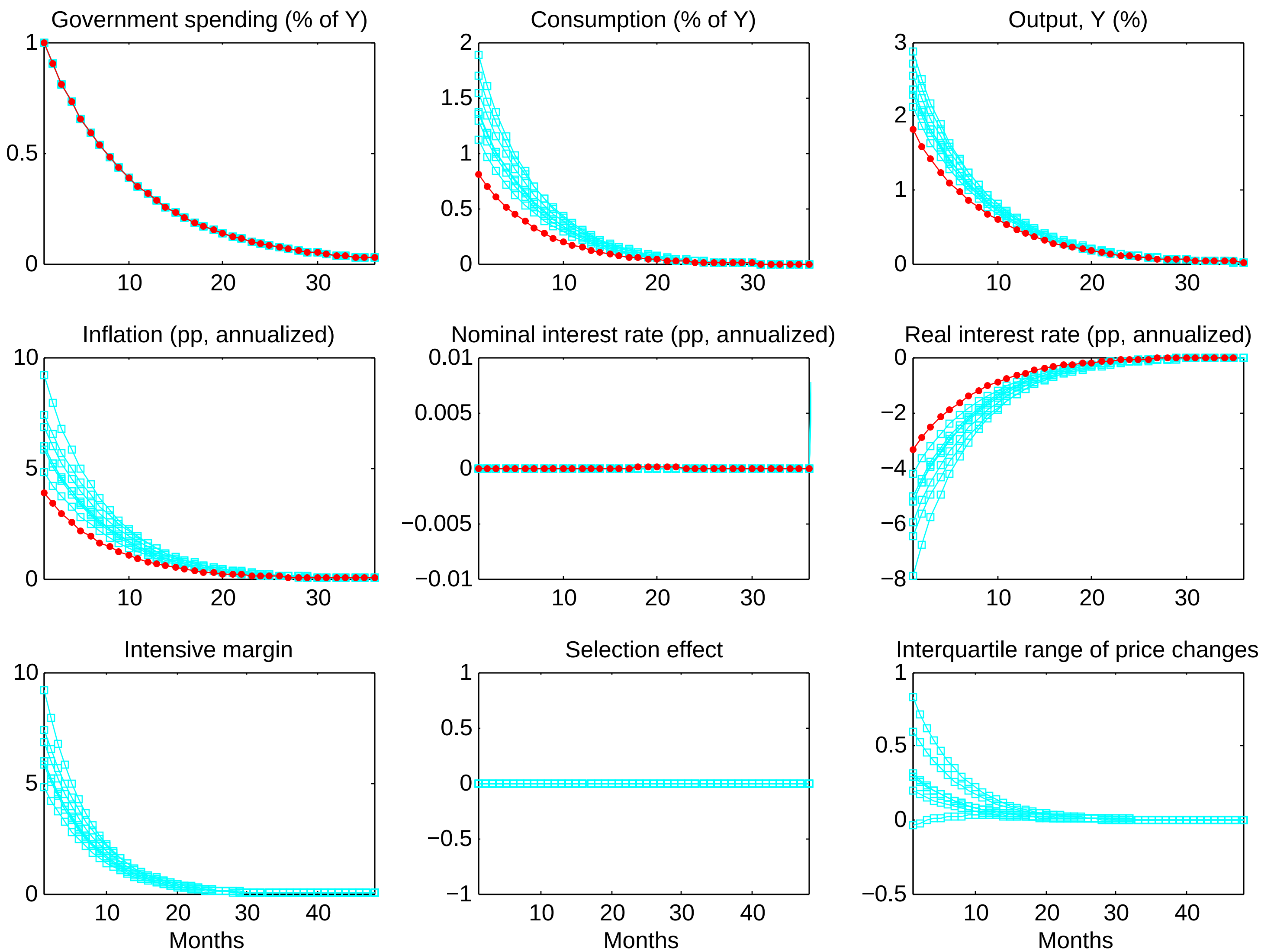
Real interest rate (pp, annualized)
Intensive margin
Selection effect
Interquartile range of price changes
Months
Months
Months

Figure 4


Government spending (% of Y)
Consumption (% of Y)
Output, Y (%)
Inflation (pp, annualized)
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Months
Months
Months