Monetary Policy and Debt Fragility*

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Abstract

The valuation of government debt is subject to strategic uncertainty, stemming from investors' sentiments. Pessimistic lenders, fearing default, bid down the price of debt. This leaves a government with a higher debt burden, increasing the likelihood of default and thus confirming the pessimism of lenders. This paper studies the interaction of monetary policy and debt fragility. It asks: do monetary interventions mitigate debt fragility? The answer depends in part on the nature of monetary policy, particularly the ability to commit to future state contingent actions. With commitment to a state contingent policy, the monetary authority can indeed overcome strategic uncertainty. Under discretion, debt fragility remains.

Keywords: *monetary policy, seignorage, inflation, sovereign debt, self-fulfilling debt crisis, sunspot equilibria.*

JEL classification: E42, E58, E63, F33.

1 Introduction

> *But there is another message I want to tell you. Within our mandates, the ECB is ready to do whatever it takes to preserve the euro. And believe me, it will be enough.* [Mario Draghi, July 2012][1]

This paper studies the interaction of fiscal and monetary policy in the presence of strategic uncertainty over the value of government debt. In real economies, beliefs of investors about the likelihood of government default, and hence the value of its debt, can be self-fulfilling. Pessimistic investors, fearing government

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[1]This statement is an excerpt from the address of Mario Draghi, President of the European Central Bank, at a financial conference, in July 2012.

default, will only purchase government debt if there is a sufficient risk premium. The resulting increase in the cost of funds makes default more likely.[2] Pessimism can be self-fulfilling even if fundamentals are sound enough that an equilibrium without default exists as well.

These results hold for real economies, in which the intervention of a monetary authority is not considered. Does this debt fragility exist in a nominal economy? The presence of a monetary authority can provide an alternative source of revenue through an inflation tax and perhaps use its influence to stabilize real interest rates. Can the monetary authority act to eliminate strategic uncertainty over the value of sovereign debt? If so, will it have an incentive to do so? The answers to these questions are relevant for assessing the relevance of these results on strategic uncertainty in debt markets and for guidance on the conduct of monetary policy.

The overlapping generations model with active fiscal and monetary interventions provides a framework for analysis. The model is structured to highlight strategic uncertainty in the pricing of government debt stemming from the default choice of a government. By construction, there is an equilibrium without default, and in general there are other equilibria with state contingent default.

The monetary authority intervenes through transfers to the fiscal entity, financed by an inflation tax. The monetary intervention has a number of influences. First, the inflation tax delivers real resources to the government, thus reducing the debt burden from taxation. Second, the realized value of inflation alters the real value of debt and consequently the debt burden left to the fiscal authority. Third, it may impact expectations of future inflation and thus the tax base for seignorage.

Given these transfers and its outstanding obligations, the fiscal authority chooses to default or not. Our analysis emphasizes the dependence of this default decision, and thus the extent of strategic uncertainty, on the conduct of monetary policy.

A central element of the analysis lies in the ability of the monetary authority to commit to a state contingent policy. If there is complete discretion in monetary policy or if the monetary authority commits to a strict inflation target, the strategic uncertainty in the real economy is present in the monetary economy.

However, if the monetary authority can commit to a particular state contingent transfer function, given an inflation target, then its intervention can stabilize debt valuations. Specifically, this policy is designed to eliminate all equilibria with state contingent default, preserving the one in which debt is risk-free. Interestingly, this desired intervention does not "bail-out" the fiscal authority. Rather, the countercyclical nature of this policy induces an accommodative fiscal stance in times of low productivity. This intervention leans against negative sentiments of investors and preserves the fundamental price of debt.

This policy is reminiscent of the commitment of the European Central Bank, reflected in the above quote of Mario Draghi, to undertake *whatever it takes* to counter pessimistic self-fulfilling expectations on Eurozone sovereign debt markets. Under the intervention we design, the central bank uses its commitment power to have a stabilizing influence on sovereign's debt valuations. In equilibrium, no actual intervention is required and debt is uniquely valued. Moreover, this policy does not endanger the primary objective of the central bank, to anchor inflation expectations around an inflation target.

Other analysis examine possible strategies for central banks to address self-fulling debt crises. Calvo

[2] This interaction between beliefs and default is central to Calvo (1988); and other contributions that followed, including Cole and Kehoe (2000), Roch and Uhlig (2012) and Cooper (2012).

(1988) extends his real economy to include a discussion of inflation as a form of partial default. He imposes an exogenous motive of money demand and an explicit cost of inflation function that affects net output. Calvo (1988) argues that there may exist multiple equilibria in the determination of inflation and the nominal interest rate on government debt. For this analysis, there is no interaction between fiscal and monetary debt repudiation.

Corsetti and Dedola (2013) augments Calvo's framework to study the interaction of fiscal and monetary policy. Their analysis retains some of the central features of Calvo's model, including exogenous money demand and costly *ex post* inflation. They argue that monetary interventions through the printing press will not generally resolve debt fragility. But, the central bank, through its holding of government debt, can have a stabilizing influence.

Aguiar, Amador, Farhi, and Gopinath (2013) build a nominal economy with a debt roll-over crisis, as in Cole and Kehoe (2000). They investigate the optimal degree of conservativeness of the central bank (as in Rogoff (1985)) as a tool to address inefficient debt crisis. Moderate inflation aversion contains the occurrence of self-fulfilling debt crisis and restrains the inflation bias in normal times.

The paper is structured as follows. Section 2 describes the economic environment and the fiscal problem of the government. Section 3 displays debt fragility in a benchmark real economy. Section 4 defines the relevant equilibrium concept in the nominal economy and investigates the presence of debt fragility under two monetary policy frameworks: delegation and discretion. Section 5 characterizes a monetary policy rule that can eliminate debt fragility. Section 6 concludes.

2 Economic Environment

Consider an overlapping generation economy with domestic and foreign agents. Agents live two periods. Time is discrete and infinite.

There are a couple of key components of the model. First, agents differ in productivity in young age and form a demand for savings. Relatively poor agents hold money as a store of value rather than incurring a cost to save through an intermediary. Importantly, money demand is endogenous, thus making the tax base for seignorage dependent on inflation expectations of young agents.

Second the government issues debt each period and faces a choice on how to finance the repayment of its obligations. In particular, the government can tax labor income, print money or default on its debt.

The environment is structured to highlight debt fragility: there are multiple self-fulfilling values of government debt. In this section, we describe the choices of private agents and the fiscal environment.

2.1 Private Agents

Every period, a continuum of mass 1 of domestic agents (households) is born and lives two periods. These agents consume only when old and their preferences are linear in consumption and labor disutility is quadratic. This restriction is introduced to neatly capture the reaction of agents to government policy choices.

Domestic agents produce a perishable good in both young and old age. Production is linear. In youth, productivity is heterogenous. A mass ν^m of agents have low productivity $z^m = 1$. A mass $\nu^I = 1 - \nu^m$ of agents have high productivity $z^I = z > 1$. In old age, productivity A is stochastic, i.i.d., and common to all old agents.[3]

Agents have access to two technologies to store value: money or financially intermediated claims. Access to the latter is costly: agents pay a participation cost Γ for access to intermediaries. Limited financial market participation sorts agents in two groups. For convenience, we will refer to *poor* agents, who will hold only *money* in equilibrium, and *rich* agents, who hold *intermediated claims* in equilibrium. Intermediated claims are invested either in nominal government bonds or in a risk-free asset, e.g. storage, that delivers a real return R.

2.1.1 Poor Households

Poor households have low labor productivity $z^m = 1$ in youth. Their savings between young and old age are composed only of money holdings, whose real return is given by $\tilde{\pi}'$, the inverse of the gross inflation rate.[4] Their labor supply decisions in young and old age solve:

$$\max_{n,n'} \; E\big[u(c') - g(n')\big] - g(n), \tag{1}$$

subject to young and old age real budget constraints:

$$m = n \tag{2}$$

$$c' = A'n'(1-\tau') + m\tilde{\pi}' + t'. \tag{3}$$

In youth, poor agents supply labor n and have real money holdings, m, carried on from young to old age. Return on money is given by the gross inverse inflation rate $\tilde{\pi}'$. In old age, poor agents supply labor n', which is augmented by aggregate productivity A'. τ' is the tax rate on labor income of old agents and $t' \geq 0$ a possible lump-sum transfer. Denote by n_y^m and n_o^m the optimal labor supply decision of young and old poor agents. With $u(c) = c$ and $g(n) = \frac{n^2}{2}$, labor supply decisions are:

$$n_y^m = E(\tilde{\pi}') \;\text{ and }\; n_o^m = A'(1-\tau'). \tag{4}$$

Labor supply in both young and old age are driven by real returns to working. In youth, agents form expectations $\tilde{\pi}^e = E(\tilde{\pi}')$, and supply labor accordingly: if agents expect high inflation, i.e. a low $\tilde{\pi}^e$, they will reduce labor supply and the associated demand for real money holding. Similarly, tax on old age labor income is distortionary: a high tax rate reduces return to working and hence the labor supply of old agents.

In contrast to, for example, Calvo (1988), money demand is endogenous in our model, reflecting a labor supply and an asset market participation decision. This is important since the impact of expected monetary

[3]Formally, the distribution of A has full support on the closed and compact set $[A_l, A_h]$. $F(\cdot)$ is the associated cumulative distribution function, and $f(\cdot) = F'(\cdot)$.

[4]We verify later that these agents prefer to save via money rather than costly intermediaries in equilibrium.

interventions is to influence the magnitude of the *ex post* tax base created by money holdings. This interaction between the tax base and the inflation tax rate generates an inflation Laffer curve.

2.1.2 Rich Households and Financial Intermediation

Rich households differ from poor agents by their productivity in youth, $z^I = z > 1$. This higher productivity induces them to pay the fixed cost Γ to access intermediated saving. A parametric restriction ensures that young rich agents save via the financial sector for any positive expected inflation rate.[5] Formally,

Assumption 1.

$$z^2 > \frac{R\Gamma}{R^2 - 1} > 1. \tag{\textbf{A.1}}$$

The rich solve:

$$\max_{n,n'} \; E\big[u(c') - g(n')\big] - g(n), \tag{5}$$

subject to young and old age real budget constraints:

$$m + s = zn - \Gamma \tag{6}$$

$$s = b^I + k \tag{7}$$

$$c' = A'n'(1 - \tau') + \tilde{\pi}'m + \mathbb{1}_D(1 + i')\tilde{\pi}'b^I + Rk + t'. \tag{8}$$

In youth, rich agents supply labor n and produce zn. After incurring the fixed cost Γ, they invest a per capita amount s in intermediated claims. These claims are invested in government bonds b^I and risk-free assets k so that $s = b^I + k$, where b^I denotes the per-capita holding of government debt of domestic rich agents. Government debt is nominal and pays an interest rate i' next period if there is no default. When old, these agents supply labor n', contingent on the realization of A' and the tax rate τ'. Consumption in old age depends on the decision $D \in \{r, d\}$ of the government to repay or default on its debt, captured here by the operator $\mathbb{1}_D$ in (8): $\mathbb{1}_r$=1 and $\mathbb{1}_d$=0.

Finally, given linear utility of consumption, the portfolio decision between intermediated saving s and money holding m is only driven by expected returns. As long as expected return on money holding $\tilde{\pi}^e$ is strictly inferior to the real return R on the risk-free asset, rich households do not hold money. The portfolio for intermediated savings will include both nominal government debt and risk-free asset as long as the expected return on government debt equals that on the asset:

$$(1 + i')\tilde{\pi}^e\big(1 - P^d\big) = R, \tag{9}$$

where P^d is the probability of default, determined in equilibrium. We refer to this as the 'no-arbitrage condition'. Denote by n_y^I and n_o^I the optimal labor supply decisions of intermediated agents in young and

[5]We verify this in characterizing equilibria.

old age. The solution to (5) implies:

$$n_y^I = Rz \text{ and } n_o^I = A'(1-\tau'). \tag{10}$$

Labor supply n_y^I of young agents is determined by the expected return R on intermediated savings. In old age though, the effective return on intermediated savings will depend on the realized inverse inflation rate $\tilde{\pi}'$, the nominal interest rate i' and the default decision of the government.

2.1.3 Foreign Households

In addition to domestic agents, there are also foreign households who hold domestic debt. Like rich households, they save through intermediaries that hold government debt and consume the domestic good. The details of the foreign economy are not important for this analysis except that foreign households are risk neutral and have access to domestic debt as a store of value. In equilibrium, they hold a fraction $(1-\theta)$ of domestic debt.[6]

2.2 The Government

The government is composed of a treasury and a central bank. Every period, it has to finance a constant and exogenous flow of real expenses g. Government expenditures do not enter into agents utility. To finance these expenses, it can raise taxes on old agent labor income, print money and issue nominal debt B'.[7] Alternatively, it can default on its inherited debt obligation.

Under repayment, the real budget constraint of the government is:

$$(1+i)\tilde{\pi}b + g = \tau\big(\nu^m A n_o^m(\tau) + \nu^I A n_o^I(\tau)\big) + \frac{\Delta M}{P} + b'. \tag{11}$$

The left hand side contains the real liabilities of the government, net of realized inflation $\tilde{\pi}$, where b is real debt outstanding. On the right hand side, $n_o^j(\tau)$ is the labor supply decision of old agents of type $j \in \{I, m\}$, ΔM is the change in the total money supply (M) and P is the price level. Denote by σ the rate of money creation that implements the change in money supply ΔM.

Assume $g = b'$, so that new expenses are financed exclusively by debt. With this restriction, fiscal policy has no intergenerational element. Instead, debt created when agents are young is paid for or defaulted on when these agents are old.[8] We return to this restriction in our concluding comments. So the government budget constraint under repayment becomes generation specific:

$$(1+i)\tilde{\pi}b = \tau\big(\nu^m A n_o^m(\tau) + \nu^I A n_o^I(\tau)\big) + \frac{\Delta M}{P}. \tag{12}$$

[6]Given the indifference of risk neutral agents regarding their portfolio of government debt and storage, θ is not determined in equilibrium. Thus equilibria will be characterized for given values of θ.

[7]The assumption of no taxation of income when young is just a simplification that allows us to neatly disentangle demand for money, for intermediated claims and labor supply driven by taxation.

[8]This use of generational budget balance appears in Chari and Kehoe (1990) and Cooper, Kempf, and Peled (2010), for example. An alternative, as in Cole and Kehoe (2000), could add more strategic uncertainty through debt rollover. For an analysis of self-fulfilling debt crisis with active debt issuance and maturity management, see Lorenzoni and Werning (2013).

Instead of repayment, the government can fully renege on its debt. But there are two costs of default for domestic agents. First, direct costs of default are born by old rich agents, who hold a fraction θ of government debt. Second, if the government repudiates its debt, the country suffers from a deadweight loss, as commonly assumed in the literature on strategic default.[9] Formally, aggregate productivity contemporaneously drops by a proportional factor γ. The model excludes punishments involving exclusion from future capital markets. This is partly to ensure that default effects are contained within a generation but also reflects the quantitative finding that the main force preventing default is the direct output loss.[10]

As the government budget constraint holds over time for a given generation, a decision to default on period t debt has no direct effect on future generations. That is, default affects only the welfare of current old agents, who otherwise are taxed via seignorage or labor tax.

The government weights the welfare burden of tax distortions against the direct costs and penalty induced by the default decision. Denote by $W^r(\cdot)$ the welfare of the economy under *repayment* and by $W^d(\cdot)$ under *default*. The decision to default is optimal whenever $\Delta(\cdot) = W^d(\cdot) - W^r(\cdot) \geq 0$.

Given aggregate productivity A, nominal interest rate i, real money tax base m_{-1}, tax rate τ, money printing rate σ that satisfy (12) and the induced inverse inflation rate $\tilde{\pi}$, the welfare criterion $W^D(\cdot)$ for $D \in \{r, d\}$ is:

$$W^D(A, i, m_{-1}, \tau, \sigma, \tilde{\pi}) = \nu^m \Big(c_o^m(D) - \frac{n_o^m(D)}{2}^2 \Big) + \nu^I \Big(c_o^I(D) - \frac{n_o^I(D)}{2}^2 \Big). \tag{13}$$

The levels of $\tilde{\pi}$ are chosen under each of the options, as a function of the monetary regime under which the economy operates.

Specifically, under repayment, $D = r$, the welfare of old agents is:

$$W^r(A, i, m_{-1}, \tau, \sigma, \tilde{\pi}^r) = \frac{\big[A(1-\tau)\big]^2}{2} + \nu^m m_{-1} \tilde{\pi}^r + \big((1+i)\tilde{\pi}^r - R\big)\theta b + \nu^I R(Rz^2 - \Gamma). \tag{14}$$

Here the inflation is created by the printing of money that is transferred directly to the treasury.

The option to default, $D = d$, triggers penalties but no tax need be raised. In keeping with the generational view of the budget constraint, any money creation in the current period is transferred lump-sum to the current old. The amount of this transfer will depend on the monetary regime. In this case, the welfare of old agents becomes:

$$W^d(A, i, m_{-1}, \sigma, \tilde{\pi}^d) = \frac{\big[A(1-\gamma)\big]^2}{2} + \nu^m m_{-1} \tilde{\pi}^d - R\theta b + \nu^I R(Rz^2 - \Gamma) + T(\sigma, m_{-1}, \tilde{\pi}^d), \tag{15}$$

where $T(\sigma, m_{-1}, \tilde{\pi}^d)$ is the aggregate lump sum transfer to old agents that implements $\tilde{\pi}^d$.[11]

[9] Penalties and direct sanctions are central theoretic concepts for enforcement of international asset trade. See the seminal work by Eaton and Gersovitz (1981). For an extensive review, see Eaton and Fernandez (1995).

[10] Empirical evidence regarding reputation costs of default are mixed: exclusion from international credit markets are short-lived and premium following defaults are usually found to be negligible. An extensive discussion can be found in Trebesch, Papaioannou, and Das (2012). From a theoretical point of view, Bulow and Rogoff (1989) show that reputation mechanisms cannot enforce international asset trade, if the government can buy foreign assets as an alternative source of insurance.

[11] Computations are detailed in Appendix 7.1.

2.3 Assumptions

The following two assumptions are used for characterizing equilibria. The first places a lower bound on γ so that default is costly, especially when no debt is held by domestic agents.

Assumption 2.

$$\frac{A_l^2\gamma(2-\gamma)}{2} > \nu^m. \tag{A.2}$$

Under this assumption, default is not a desirable option when seignorage revenue alone could service principal and interest on debt.[12] The next assumption ensures that the fundamentals of the economy are compatible with a risk-free outcome, i.e. given the real level of debt b, a real interest rate of R, the debt will be repaid for all A. That is, there is a solution to (9) without default. Formally,

Assumption 3. $b < \bar{b}$ *where*

$$\bar{b} = \frac{A_l^2(1-\gamma)\gamma}{R}. \tag{A.3}$$

Note that Assumption 3 is stated in the extreme case where there is no seignorage revenue, and all debt is held by foreigners.[13] The presence of an equilibrium without default provides a convenient benchmark for the analysis.

3 Debt Fragility in a Real Economy

To explicitly illustrate debt fragility, first consider this economy without money and nominal quantities. The equilibrium in the monetary economy will be constructed on the foundation of the multiplicity in the real economy.

The government issues real debt and can raise only taxes on labor income. Given A and real interest rate i, its budget constraint under repayment is simply:

$$(1+i)b = A^2(1-\tau)\tau. \tag{16}$$

Let τ be the smallest tax rate satisfying (16) so that tax revenue are locally increasing in the tax rate.

Private agents can save only through intermediation. For simplicity, set $\Gamma = 0$ so that there are no costs associated to saving through the holding of government debt. Further, assume $\theta = 0$ so that all debt is held by foreigners and all domestic saving is through storage.

The labor supply choices of the rich are given by (10). Since the poor access the intermediary for saving, their labor supply decisions are given by

$$n_y^m = R \quad \text{and} \quad n_o^m = A'(1-\tau'). \tag{17}$$

[12] This is established in the construction of equilibria.

[13] This assumption is derived using the government budget constraint with no inflation, no fiscal resource from seignorage and all debt is held by foreigners ($\theta = 0$). It implies that there will be an equilibrium without default risk when some of the debt is held by domestic agents and when money printing does provide resources to the fiscal authority. Indeed, domestic holding of public debt or a higher money printing rate relaxes the willingness of the fiscal authority to default rather than repay its debt.

The government defaults whenever $\tilde{W}^d(\cdot) \geq \tilde{W}^r(\cdot)$, where these values for the real economy are defined by:

$$\tilde{W}^r(A, i, \tau) = \frac{\big[A(1-\tau)\big]^2}{2} + \nu^m R^2 + \nu^I (Rz)^2, \tag{18}$$

and

$$\tilde{W}^d(A, i) = \frac{\big[A(1-\gamma)\big]^2}{2} + \nu^m R^2 + \nu^I (Rz)^2. \tag{19}$$

For the real economy with $\theta = 0$, the government will default whenever $\tau > \gamma$. Equivalently, using (16), the government defaults for any realization of $A < \bar{A}$, where $\bar{A}$ satisfies:

$$\bar{A}^2 = \frac{(1+i)b}{\gamma(1-\gamma)}. \tag{20}$$

This expression defines $\bar{A}(i)$, the default threshold of the government as a function of the real interest rate i.

The probability of default given i is $F\big(\bar{A}(i)\big)$. Using this, the no-arbitrage condition (9) becomes:

$$(1+i)\Big(1 - F\big(\bar{A}(i)\big)\Big) = R. \tag{21}$$

This equation may have several solutions.[14] Default arises both because of fundamental shocks (low A) and strategic uncertainty: the probability of default depends on the interest rate, and in equilibrium on the beliefs of investors which determine this probability. Hence the multiplicity.

Proposition 1. *If government debt has value, then there are multiple interest rates that solve the no-arbitrage condition (21).*

Proof. An equilibrium of the debt financing problem is characterized by an interest rate i and a default threshold, $\bar{A}$ solving (20) and (21). Combining these expressions yields

$$\bar{A}^2\big(1 - F(\bar{A})\big) = \frac{Rb}{\gamma(1-\gamma)}. \tag{22}$$

Any $\bar{A}$ solving this equation is part of an equilibrium.

Denote by $G(A)$ the left side and by Z the right side of (22). $G(\cdot)$ is continuous on $[A_l, A_h]$, $G(A_l) = A_l^2 > 0$ and $G(A_h) = 0$.

Consistent with Assumption 3, if $Z < A_l^2 = G(A_l)$, there is a risk free interest rate: $\bar{A} = A_l$ and $(1+i) = R$ is a solution to (22). Also, by continuity of $G(\cdot)$, there is $\bar{A} \in (A_l, A_h)$ such that $G(\bar{A}) = Z$. Hence there is also an interest rate that carries a risk-premium and that solves the no-arbitrage condition.

Relaxing Assumption 3, if $Z > A_l^2 = G(A_l)$, all equilibria include default risk and thus a risk premium. If b is very high, then Z will be large as well and there may be no equilibrium in which debt is valued. If debt is valued so that there is a solution to (22), then $G(A) > Z$ for some $A > A_l$. Again, by continuity, there is a second equilibrium. ∎

[14] This source of multiplicity is at the heart of Calvo (1988) and Cooper (2012).

The multiple equilibria of the debt financing problem identified here are welfare ordered. The *fundamental* equilibrium with certain repayment provides higher utility than any other equilibrium with higher interest rate and state contingent default. In the fundamental equilibrium, repayment is preferred to default in those states where default is optimal in the other equilibria. And in repayment states, lower interest rate in the fundamental equilibrium requires lower taxes, hence higher welfare, than under the other equilibria.

Importantly, note that Proposition 1 does not directly restrict the level of debt, b. As long as debt has value, there are multiple equilibria. The level of debt though cannot be too large. Else an equilibrium with valued debt will not exist, since the government would default for all realizations of A.

The underlying source of strategic uncertainty is aptly captured by Proposition 1 for the real economy. It is a building block for the analysis of a monetary economy. The subsequent developments allow debt to be held both internally and externally.

4 Debt Fragility in a Monetary Economy

This section studies the interaction of monetary interventions and debt fragility. Intuitively, monetary policy acts via three channels. First, it can collect seignorage taxes and supplement the resources collected through labor taxation. Second, by adjusting the realized inflation rate, it can lower the real value of debt.

But, third, there are potential resource costs of funding the government through the inflation tax: young agents perceiving high inflation in the future will work less, reducing the monetary tax base. This effect though depends on the extent of discretion in the conduct of monetary policy. Also, the mean inflation rate $\tilde{\pi}^e$ is priced into the nominal interest rate, which makes attempt to deliver surprise inflation difficult.

Accordingly, this section of the paper is constructed around two polar cases, distinguished by the ability of the monetary authority to commit.

i. *Monetary delegation*: monetary policy decisions are made by an independent central bank that pursues a known and explicit rule, independent of fiscal considerations. We consider the case of strict inflation targeting: the central bank is committed to an unconditional inflation rate.

ii. *Monetary discretion*: monetary and fiscal decisions are linked. Given the state of the economy, money creation and taxes are set so as to minimize welfare costs and tax distortions of old agents given a budget constraint. Default is also chosen optimally *ex post*.

Before analyzing debt fragility under these monetary regime, we formally define the equilibrium concept of the nominal economy.

4.1 State Variables and Equilibrium Definition

The strategic uncertainty identified in Proposition 1 is modeled through a sunspot variable, denoted s, that corresponds to confidence of domestic and foreign households about the repayment of government debt next period.

- If $s = s^o$, agents are "optimists" : they coordinate on the risk free (fundamental) price of the government debt.
- If $s = s^p$, agents are "pessimists" : they coordinate on higher risk / lower price equilibria with state contingent default.

The distribution of sunspot shocks is i.i.d. Denote by $p \in (0, 1)$ the probability of $s = s^o$. In the event there is a unique equilibrium price, then the fundamental price obtains regardless of the sunspot realization. Note that we only consider cases where debt has value.[15]

The state of the economy is $\mathcal{S} = (A, i, m_{-1}, s, s_{-1})$. Aggregate productivity, A, is realized and directly affects the productivity of the old. There are two endogenous predetermined state variables, m_{-1} and i, respectively real money holdings of current old agents, and the nominal interest rate on outstanding public debt. Both the sunspot shock last period, s_{-1}, and the current realization, s, may impact fiscal policy, monetary policy and the choices of private agents.

To define a Stationary Rational Expectations Equilibrium (SREE), it is necessary to be precise about market clearing conditions, the link between money printing, inflation and seignorage revenue and from these, the government budget constraint. These conditions are used in the equilibrium definition and in constructing various types of equilibria.

4.1.1 Market Clearing

In every state, the markets for money and bonds must clear. The condition for money market clearing is

$$\nu^m m(\mathcal{S}) = \frac{M(\mathcal{S})}{P(\mathcal{S})} \; \forall \mathcal{S}, \tag{23}$$

where $P(\mathcal{S})$ is the state dependent money price of goods and $M(\mathcal{S})$ is the stock of fiat money. This equation implies that the real money demand of the current young equals the real value of the supply.

The market for government debt clears if the no-arbitrage condition (9) holds and the savings of the rich plus the demand from the foreigners is not less than the real stock of government debt. We assume that the foreigners' endowment is large enough to clear the market for bonds as long as (9) is met.

4.1.2 Government Budget Constraint, Inflation and Seignorage

The SREE version of the government budget constraint, (12), requires a couple of building blocks. The inverse inflation rate, $\tilde{\pi}$, is given by:

$$\tilde{\pi}(\mathcal{S}) = \frac{P(\mathcal{S}_{-1})}{P(\mathcal{S})} = \frac{m(\mathcal{S})}{m(\mathcal{S}_{-1})} \frac{1}{1 + \sigma(\mathcal{S})}, \tag{24}$$

using (23). Revenue from seignorage is:

$$\frac{\Delta M}{P(\mathcal{S})} = \sigma(\mathcal{S}) \nu^m m(\mathcal{S}_{-1}) \tilde{\pi}(\mathcal{S}) = \nu^m m(\mathcal{S}) \left(\frac{\sigma(\mathcal{S})}{1 + \sigma(\mathcal{S})} \right). \tag{25}$$

[15] The case of "market shutdown", where debt has no value, is not of direct interest for our analysis.

Here $m(\mathcal{S}_{-1})$ represents the real money holdings of the current old. Importantly, these equations imply a one-to-one mapping between the rate of money creation $\sigma(\mathcal{S})$ and realized inverse inflation $\tilde{\pi}(\mathcal{S})$. This reflects the fact that $m(\mathcal{S}_{-1})$ is given in (24) and the employment and money demand for the current generation, $m(\mathcal{S})$, is, as we verify below, independent of the current rate of money creation. Accordingly, our equilibrium definition is stated with the government setting inflation $\tilde{\pi}(\mathcal{S})$.

Embedded in (25) is an interaction between inflation expectations, that determines the real money holding $m(\mathcal{S}_{-1})$, and realized inflation. This element will give rise to a monetary Laffer curve and strategic interactions between expected inflation and delivered inflation, as unveiled in the rest of the analysis.

Substituting these expressions for seignorage and the inverse inflation rate into the government budget constraint:

$$(1+i)\tilde{\pi}(\mathcal{S})b = \tau(\mathcal{S})\big(\nu^m A n_o^m(\tau(\mathcal{S})) + \nu^I A n_o^I(\tau(\mathcal{S}))\big) + \nu^m m(\mathcal{S})\left(\frac{\sigma(\mathcal{S})}{1+\sigma(\mathcal{S})}\right), \tag{26}$$

and using the labor supply policy functions of old agents:

$$(1+i)\tilde{\pi}(\mathcal{S})b = A^2\big(1-\tau(\mathcal{S})\big)\tau(\mathcal{S}) + \nu^m m(\mathcal{S})\left(\frac{\sigma(\mathcal{S})}{1+\sigma(\mathcal{S})}\right). \tag{27}$$

4.1.3 Stationary Rational Expectations Equilibrium

Definition 1. *A Stationary Rational Expectations Equilibrium (SREE) is given by:*

- *The labor supply and savings decisions of private agents,* $\big(n_y^m(\mathcal{S}), n_o^m(\mathcal{S}), n_y^I(\mathcal{S}), n_o^I(\mathcal{S}), m(\mathcal{S}), k(\mathcal{S}), b(\mathcal{S})\big)$, *who form rational expectations in youth, supply labor in young and old age, solve* (1) *and* (5) *subject to their respective budget constraints* (2),(3) *and* (6),(8), *given state contingent monetary and fiscal policies* $(\{\tau(\mathcal{S}), \tilde{\pi}(\mathcal{S}), D(\mathcal{S})\})$, *for all* $\mathcal{S}$.

- *The government maximizes its welfare criterion by choosing a policy* $(\{\tau(\mathcal{S}), \tilde{\pi}(\mathcal{S}), D(\mathcal{S})\})$ *subject to the government budget constraint,* (27) *for all* $\mathcal{S}$.

- *All markets clear (goods, money, bonds) for all* $\mathcal{S}$.

The welfare criterion of the government will depend on the monetary policy framework, as detailed below. The polar cases of delegation and discretion are studied within this framework; the conduct of monetary policy determines what the government takes as given in choosing its policy.[16] Also, we characterize equilibria for given θ, share of government debt held by domestic agents, as its value is not pinned down in equilibrium.

4.2 Monetary Delegation

In this institutional structure, the treasury has discretionary power over fiscal policy, choosing fiscal policy *ex post* given the monetary intervention. In contrast, the monetary authority is endowed with a commitment technology. We find that under monetary delegation to a strict inflation target, debt fragility remains.[17]

[16] Aguiar, Amador, Farhi, and Gopinath (2013) and Corsetti and Dedola (2013) study discretionary monetary authorities. Our analysis will also highlight particular forms of commitment by the central bank.

[17] The analysis with commitment is extended beyond strict inflation targeting in Section 5.

The key intuition behind this result is that strict inflation targeting turns a nominal debt contract into a real security. Hence the presence of debt fragility in real economy identified in section 3 persists in the nominal economy with strict inflation targeting.

One interpretation of this structure is that the government of an individual country delegates its monetary policy to an independent central bank, by joining a monetary union for instance. The central bank of the union pursues an independent policy of strict inflation targeting and the fiscal authority is left with discretionary tax policy choices (taxes or default).

Specifically, the central bank commits to an inflation target $0 < \tilde{\pi}^* \leq 1$ and delivers it by printing money. By doing so, the central bank does not accommodate productivity shocks. Revenue from seignorage is transferred to the treasury. Formally, the policy of the central bank is:

$$\tilde{\pi}(\mathcal{S}) = \tilde{\pi}^* \qquad \forall \mathcal{S}. \tag{28}$$

As the central bank is bound to deliver its inflation target $\tilde{\pi}^*$, agents' expectations are $\tilde{\pi}^e = \tilde{\pi}^*$.[18] In a stationary equilibrium, there is a stationary rate of money creation, σ^*, directly linked to the target inflation: $\frac{1}{1+\sigma^*} = \tilde{\pi}^*$.

Using (25), modified to reflect the equilibrium under an inflation target $\tilde{\pi}^*$, revenue obtained from seignorage is:

$$\frac{\Delta M}{P(\mathcal{S})} = \nu^m m(\mathcal{S}) \left(\frac{\sigma(\mathcal{S})}{1 + \sigma(\mathcal{S})} \right) = \nu^m \tilde{\pi}^* (1 - \tilde{\pi}^*), \tag{29}$$

as $m = m_{-1} = \tilde{\pi}^e = \tilde{\pi}^*$. This is maximized at $\tilde{\pi}^L \equiv \frac{1}{2}$ which is the top of the seignorage "Laffer curve". At $\tilde{\pi}^* > \tilde{\pi}^L$, a reduction in $\tilde{\pi}^*$ (i.e. an increase in the rate of inflation) will increase revenue.[19] Within this monetary set-up, the government budget constraint under repayment becomes:

$$(1 + i)\tilde{\pi}^* b = A^2(1 - \tau)\tau + \nu^m \tilde{\pi}^* (1 - \tilde{\pi}^*). \tag{30}$$

To formally derive the result that debt fragility persists in this monetary regime, we establish the existence of several interest rates that solve the no-arbitrage condition. To do so, we first verify that the default decision in the monetary economy has the same monotonicity property as in the real economy: if the government defaults for a given realization of technology $\bar{A}$, then it would default for any lower realization $A \leq \bar{A}$.

Lemma 1. *Under Assumption 2, given a level of real obligations $(1+i)\tilde{\pi}^* b$, there is a unique $\bar{A}(i) \in [A_l, A_h]$ such that if $A \leq \bar{A}(i)$, then the treasury defaults on its debt. Otherwise it repays its debt.*

Proof. Given a nominal interest rate i, the decision to repay or default on debt is given by $\Delta(\cdot) = W^d(\cdot) - W^r(\cdot)$, where the relevant welfare criteria are given by (14) and (15) and the lump-sum transfer under default

[18] In particular, if there is default, the monetary authority prints money and transfers it to old agents to meet this target.

[19] The determination of the optimal inflation target $\tilde{\pi}^*$ is not part of the present analysis. The model could provide a positive theory of inflation, where the inflation target would be set to minimize distortions associated to tax revenue. Given the Laffer curve property of seignorage, any inflation target $0 < \tilde{\pi}^* < \tilde{\pi}^L$ is inefficient, but this does not affect the essential results regarding debt fragility.

by $T(\tilde{\pi}^*) = \nu^m \tilde{\pi}^*(1-\tilde{\pi}^*)$. Hence, a point of indifference between default and repayment, $\bar{A}(i)$ solves:

$$\frac{[A(1-\gamma)]^2}{2} - \frac{[A(1-\tau)]^2}{2} = (1+i)\tilde{\pi}^*\theta b - \nu^m \tilde{\pi}^*(1-\tilde{\pi}^*), \tag{31}$$

where τ satisfies the government budget constraint (30). Denote by $G(A)$ the left side of (31). Clearly if $G(A)$ is monotonically decreasing in A, then the default decision satisfies the desired cut-off rule. Rewrite $G(A)$ as follow:

$$G(A) = \frac{[A(1-\gamma)]^2}{2} - \frac{A^2}{2} - \frac{A^2\tau(\tau-2)}{2}. \tag{32}$$

Using the government budget constraint, (32) rewrites:

$$G(A) = \frac{A^2\gamma(\gamma-2)}{2} - \big[(1+i)\tilde{\pi}^* b - \nu^m \tilde{\pi}^*(1-\tilde{\pi}^*)\big]\frac{(\tau-2)}{2(1-\tau)}. \tag{33}$$

The first term is negative since $\gamma < 1$. If seignorage revenue is enough to service debt, then no tax need be raised and $\bar{A}(i) = A_l$, by Assumption 2.[20] Otherwise, $(1+i)\tilde{\pi}^* b - \nu^m \tilde{\pi}^*(1-\tilde{\pi}^*) > 0$. Finally, we need to derive the monotonicity of $\frac{\tau-2}{1-\tau}$ with respect to A. Its derivative is:

$$\frac{-1}{(1-\tau)^2}\frac{d\tau}{dA} > 0, \tag{34}$$

which is positive since $\frac{d\tau}{dA} < 0$ for the lowest value of τ that solves the budget constraint. Overall, we have $G'(A) < 0$. Hence, the cut-off value $\bar{A}(i)$ is unique and default occurs if and only if $A \le \bar{A}(i)$.

Note that if $\bar{A}(i) \le A_l$, then debt is risk free. Finally, $\bar{A}(i) = A_h$ is inconsistent with the assumption that debt has value. ∎

From this result, the probability of default P^d becomes $F(\bar{A}(i))$. This probability also implicitly depends on $\tilde{\pi}^*$, which appears on the right side of (31). Altogether, an interest rate for the government debt solves:

$$(1+i)\tilde{\pi}^*\Big(1 - F\big(\bar{A}(i)\big)\Big) = R. \tag{35}$$

This expression outlines the interplay between beliefs, probability of default and best-response of the government. As in the real economy, the probability of default depends on the interest rate, and in equilibrium on the beliefs of investors which determine this probability.

Lemma 2. *Under Assumptions 2 and 3, for any inflation target $0 < \tilde{\pi}^* \le 1$, there are multiple interest rates that solve the no-arbitrage condition (35).*

Proof. An equilibrium of the debt financing problem is characterized by an interest rate i and a default threshold $\bar{A}$. Importantly, an equilibrium is such that *beliefs* of investors are consistent with the *best response* of the government.

[20] To see this, set $\theta = 0$, $m = 1$ and $\tilde{\pi}^r = 0$ in (14) and (15) and verify that $\Delta(\cdot) = W^d(\cdot) - W^r(\cdot) < 0$ under Assumption 2.

Investors believe that the government defaults with probability $P^d = F(\bar{A})$. This belief induces $\bar{A}^b(i)$, the default threshold consistent with P^d:

$$(1+i)\tilde{\pi}^*\big(1 - F(\bar{A})\big) = R \Rightarrow \;\bar{A}^b(i). \tag{36}$$

Given i, the government decision to repay or default induces $\bar{A}^g(i)$, the realization of A for which the government is indifferent between default and repayment:[21]

$$\Delta(A,i) = W^d(A,i) - W^r(A,i) = 0 \Rightarrow \bar{A}^g(i). \tag{37}$$

An equilibrium requires $\bar{A}^b(i) = \bar{A}^g(i)$. The nominal interest rate i can takes value on $[\underline{i}, +\infty)$ where $\underline{i}$ is the nominal interest rate consistent with risk-free debt. Formally, it satisfies $(1+\underline{i})\tilde{\pi}^* = R$. We study the monotonicity properties of $\bar{A}^b(\cdot)$ and $\bar{A}^g(\cdot)$.

The default threshold $\bar{A}^b(i)$ induced by belief of investors has the following properties. First, $\bar{A}^b(\underline{i}) = A_l$: if investors charge $\underline{i}$, it means that they expect no default. Second, differentiating (36) with respect to $\bar{A}$ and i, one gets:

$$\frac{d\bar{A}^b(i)}{di} = \frac{\big(1 - F(\bar{A})\big)}{f(\bar{A})(1+i)} > 0, \tag{38}$$

since $f(\cdot) > 0$. Finally, $\lim\limits_{i\to+\infty} \bar{A}^b(i) = A_h$.

The best response of the government to i is captured by $\bar{A}^g(i)$, the default threshold. Given Assumption 3, for low values of i, debt is risk free.[22] Hence, there is $\epsilon > 0$ such that $\bar{A}^g(\underline{i} + \epsilon) = A_l$. Second, by differentiating (31) with respect to $\bar{A}$ and i, one gets:

$$\tilde{\pi}^* b\Big[\frac{1-\tau}{1-2\tau} - \theta\Big]di + \bar{A}\Big[(1-\gamma)^2 - \frac{(1-\tau)^2}{1-2\tau}\Big]d\bar{A} = 0. \tag{39}$$

The factor of di is positive since $\frac{1-\tau}{1-2\tau} > 1$ and the factor of $d\bar{A}$ is negative since $\frac{(1-\tau)^2}{1-2\tau} > 1$. Hence:

$$\text{if } \bar{A}^g(i) \in (A_l, A_h), \text{ then } \frac{d\bar{A}^g(i)}{di} > 0. \tag{40}$$

Finally, there is an upper bound $\bar{i}$ such that default occurs for all A if $i \geq \bar{i}$:

$$\forall i > \bar{i},\; \bar{A}^g(i) = A_h. \tag{41}$$

By continuity of the functions $\bar{A}^g(\cdot)$ and $\bar{A}^b(\cdot)$, there is a value $i > \underline{i}$ that satisfies $\bar{A}^g(i) = \bar{A}^b(i)$. ∎

The monotonicity properties of $\bar{A}^g(i)$ and $\bar{A}^b(i)$ are summarized in Figure 1. Under Assumption 3, there is

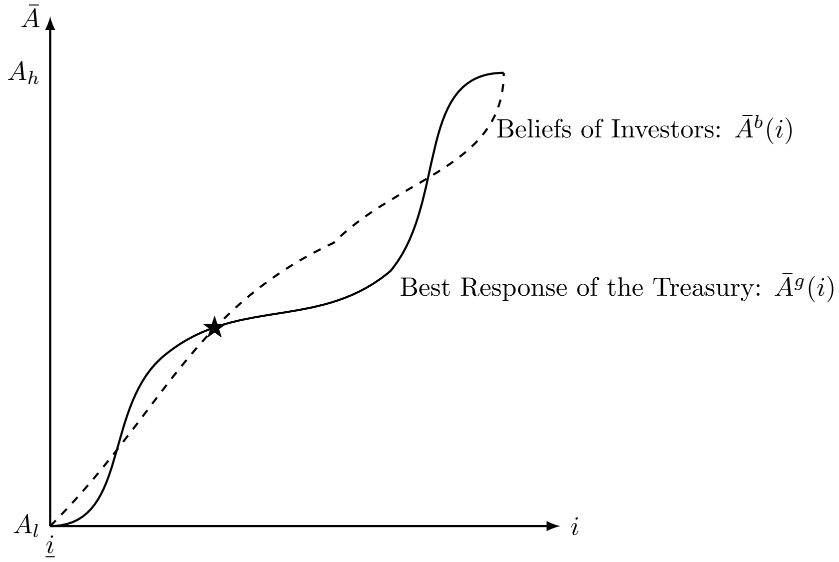
[21] Lemma 1 established that this threshold is unique. To determine $\Delta(A,i)$ from (14) and (15), set $\tilde{\pi} = m = \tilde{\pi}^*$ and set τ from (30) if the government decides to repay.

[22] Relaxing Assumption 3 and allowing a fundamental equilibrium with positive probability of default does not change the central result that several interest rates are compatible with the no-arbitrage condition. This is explicit in the real environment, as in Proposition 1, and can be established in the nominal economy as well.

always an equilibrium with certain repayment. In addition, there will exist an equilibrium in which the debt is never repaid and, accordingly, investors place zero probability on repayment.[23] Lemma 2 characterizes additional interior equilibria in which default arises with a positive probability: there is $\bar{A} \in (A_l, A_h)$ and $i > \underline{i}$ that satisfy the no-arbitrage condition with state contingent default.

Figure 1 illustrates the multiplicity of equilibria, including three interior equilibria. The equilibrium of the debt financing problem labeled ★ is a locally stable equilibrium with a positive probability of default. Here local stability refers to best-response dynamics and is used for comparative statics.[24]

Figure 1: Multiplicity of Interest Rates under Monetary Delegation



This figure represents the mapping from interest rate i to default threshold $\bar{A}$, both for investors and the fiscal authority. Investors associate an interest rate i to a default threshold via the probability of default in the no-arbitrage condition. This is the dashed line. Given the interest rate i, the optimal decision of the fiscal authority to service its debt or default is captured by the default threshold, indicated by the solid line. An equilibrium is reached when beliefs of investors are consistent with the best-response of the fiscal authority. The figure highlights the existence of several equilibria, one of them being risk-free. The equilibrium indicated with a ★ is locally stable under best response dynamics.

This lemma provides the basis for the existence of a SREE in which sunspots matter, i.e. the value of government debt is dependent upon the beliefs of investors. In equilibrium there are sunspot dependent variations in employment, output and consumption.

Proposition 2. *Under Assumption 2 and 3, for any $0 < \tilde{\pi}^* \leq 1$, there is a SREE with the following characteristics:*

1. *If $s_{-1} = s^o$, the government security is risk free and the treasury reimburses with probability 1.*
2. *If $s_{-1} = s^p$, the interest rate incorporates a risk-premium and the treasury defaults on its debt with positive probability.*

[23] As mentioned previously, we discard this "market shutdown" case, which always exists.

[24] Specifically, 'best response dynamics' points to the dynamics induced by investors responding to the treasury, followed by the treasury responding to investors. To see why the ★ equilibrium is locally stable, suppose the interest rate i is lower than the equilibrium value. Given i, the treasury decision is captured by a threshold level for default, $\bar{A}^g(i)$, along the solid line. Given this, investors will 'set' an interest rate such that the no-arbitrage conditions holds, i.e. $\bar{A}^b(i)$ along the dashed line. Following this dynamic will lead to the locally stable equilibrium.

Proof. The characterization of the SREE directly comes from Lemma 2 and the existence of several interest rates compatible with the no-arbitrage condition in equilibrium. We describe the optimal behavior of agents consistent with the equilibrium definition.

As $\tilde{\pi}^e = \tilde{\pi}^* \in (0, 1]$, poor agents save only with money holding and rich young agents invest in intermediated claims. Indeed, consider a young household with productivity z. It can either save with money holding or via the financial sector, incurring the fixed cost Γ.

If it chooses to hold money, its labor supply when young is $n = z\tilde{\pi}^e$, its real demand for money holding is $zn = z^2\tilde{\pi}^e$ and the net expected contribution to consumption: $\big(z\tilde{\pi}^e\big)^2$. If it chooses the intermediated savings, its labor supply when young is $n = Rz$, its savings net of the intermediation cost $s = Rz^2 - \Gamma$ and the net expected contribution to consumption: $R(Rz^2 - \Gamma)$. Hence, intermediated saving dominates money holding if and only if:

$$z^2 > \frac{R\Gamma}{R^2 - (\tilde{\pi}^e)^2}, \tag{42}$$

which is true for any $\tilde{\pi}^e \in (0, 1]$ as long as Assumption 1 holds. An aggregate fraction $\theta \in [0, 1]$ of the government security is being held by domestic rich agents.

If $s_{-1} = s^o$, then young agents form expectations $P^d = 0$ and $\tilde{\pi}^e = \tilde{\pi}^*$. They supply labor accordingly. Consequently, the interest rate on debt satisfies the no-arbitrage condition (9) with $P^d = 0$ and $\tilde{\pi}^e = \tilde{\pi}^*$. Given i, seignorage revenue $\nu^m\tilde{\pi}^*(1 - \tilde{\pi}^*)$ and using Assumption 3, the optimal policy of the treasury is then to raise labor taxes τ for all A so as to satisfy its budget constraint and repay its debt.

All markets clear. The money demand of the young poor agents is constant at $\tilde{\pi}^*$. The price level adjusts to ensure market clearing. From this, $\tilde{\pi}^* = \frac{1}{1+\sigma^*}$. In this equilibrium, inflation targeting and setting fixed money growth rate are equivalent. Given the no-arbitrage condition, the bond market clears assuming the foreign lenders have enough endowment to buy the government debt not purchased by domestic rich agents.

For the case $s_{-1} = s^p$, we outline only differences with the previous case. From Lemma 2, there is an interest rate i that carries a risk premium and satisfy the no-arbitrage condition, such that $(1 + i)\tilde{\pi}^* > R$. Young agents form expectations $P^d > 0$ and $\tilde{\pi}^e = \tilde{\pi}^*$. They price the government debt according to $P^d > 0$ and $\tilde{\pi}^e = \tilde{\pi}^*$. Given i and seignorage revenue $\nu^m\tilde{\pi}^*(1 - \tilde{\pi}^*)$, there is a unique threshold $\bar{A}(i)$ such that the optimal policy of the treasury is to raise labor taxes τ for all $A \geq \bar{A}(i)$ to satisfy its budget constraint and default otherwise. Finally, expectations are consistent with the best response of the government: $P^d = F\big(\bar{A}(i)\big)$. ∎

Aguiar, Amador, Farhi, and Gopinath (2013) find a similar result if there is a very high perceived cost of inflation: when the central bank is very inflation averse, it never chooses to inflate the nominal value of debt, it is *de facto* committed to no inflation, converting nominal debt into real debt. Roll-over crises occur for a larger range of debt.

Overall this section, particularly Proposition 2, makes clear that debt fragility, as identified in real economies (Proposition 1), extends to economies with nominal debt. In effect, the inflation target of the monetary authority converts the nominal obligation to a real one. Seignorage does reduce the real debt burden left to the fiscal authority, but without eliminating the underlying strategic uncertainty. The choice of the inflation target does not allow the monetary authority to peg the real interest rate. Instead the real interest rate on debt continues to reflect the sentiments of investors.

This does not imply though that the equilibrium is independent of the inflation target. The inflation target will influence seignorage revenue and affect the fiscal burden. The size and magnitude of these effects will depend on the target inflation relative to the peak of the "Laffer curve".

Proposition 3. *In the equilibrium characterized in Proposition 2, for $\tilde{\pi}^* \geq \tilde{\pi}^L$, an increase in the target inflation rate will increase seignorage and lower the probability of default if and only if the equilibrium of the debt financing problem is locally stable.*

Proof. The proof relies on three expressions. The point of indifference between repayment and default, (i.e. the default threshold $\bar{A}$) is given in (31), the government budget constraint given in (30), and the no-arbitrage condition, given in (35). These are all evaluated at a given inflation target and thus $\tilde{\pi}^*$. Substituting the no-arbitrage condition into (31) gives:

$$\frac{[\bar{A}(1-\gamma)]^2}{2} - \frac{[\bar{A}(1-\tau)]^2}{2} + \nu^m \tilde{\pi}^*(1-\tilde{\pi}^*) - \frac{R\theta b}{1-F(\bar{A})} = 0, \tag{43}$$

where τ satisfies the government budget constraint evaluated at $\bar{A}$.

Taking the derivative of (43) w.r.t. $\bar{A}$ and π^*:

$$\Big[\bar{A}(1-\gamma)^2 - \bar{A}(1-\tau)^2 + \bar{A}^2(1-\tau)\tau_A - \frac{R\theta b f(\bar{A})}{\big(1-F(\bar{A})\big)^2}\Big] d\bar{A} + \Big[\bar{A}^2(1-\tau)\tau_{\tilde{\pi}^*} + \nu^m(1-2\tilde{\pi}^*)\Big] d\pi^* = 0, \tag{44}$$

where τ_A and $\tau_{\tilde{\pi}^*}$ are given by the derivative of the government budget constraint evaluated in $\bar{A}$. Substituting the no-arbitrage condition into the budget constraint (30) and taking the derivative w.r.t. τ, π^*, A, one gets:

$$\tau_A = \frac{1}{A^2(1-2\tau)}\Big[\frac{Rbf(A)}{\big(1-F(A)\big)^2} - 2A(1-\tau)\tau\Big] \qquad \tau_{\tilde{\pi}^*} = -\frac{\nu^m(1-2\tilde{\pi}^*)}{A^2(1-2\tau)}. \tag{45}$$

Rearranging (44), one gets:

$$\Big[\bar{A}(1-\gamma)^2 - \bar{A}\frac{(1-\tau)^2}{1-2\tau} + \frac{Rbf(\bar{A})}{\big(1-F(\bar{A})\big)^2}\Big(\frac{1-\tau}{1-2\tau} - \theta\Big)\Big] d\bar{A} + \Big[\nu^m(1-2\tilde{\pi}^*)\Big(1 - \frac{1-\tau}{1-2\tau}\Big)\Big] d\tilde{\pi}^* = 0. \tag{46}$$

The factor of $d\tilde{\pi}^*$ is positive since $\tilde{\pi}^* \geq \tilde{\pi}^L$ and $\frac{1-\tau}{1-2\tau} > 1$. Hence the sign of the factor of $d\bar{A}$ is critical to derive the response of the default threshold to a change in the inflation target.

This sign is determined by the condition of local stability. An equilibrium is locally stable under best response dynamics if and only if $\frac{d\bar{A}^g(i)}{di} < \frac{d\bar{A}^b(i)}{di}$. Rewriting (38) with the no-arbitrage condition and using (39), the condition for local stability becomes:

$$\bar{A}(1-\gamma)^2 - \bar{A}\frac{(1-\tau)^2}{1-2\tau} + \frac{Rbf(A)}{\big(1-F(A)\big)^2}\Big[\frac{1-\tau}{1-2\tau} - \theta\Big] < 0. \tag{47}$$

Hence under local stability, $\frac{d\bar{A}}{d\pi^*} > 0$. ∎

Proposition 3 is essentially a comparative statics result and thus holds for only a subset of equilibria of the debt financing problem, i.e. those that are locally stable under best response dynamics. A locally stable equilibrium is indicated in Figure 1 and refers to the determination of the interest rate on debt and the default cut-off. The relative slopes of the two curves at this point are used in the proof of Proposition 3.

4.3 Monetary Discretion

Does monetary discretion insulate against debt fragility? Intuitively, a discretionary policy maker could adjust inflation and seignorage to accommodate variations in the price of government debt driven by strategic uncertainty and avoid default.

In a monetary discretion regime, the government has full discretionary power over both monetary and fiscal policy. It designs its policy $(\tau(\mathcal{S}), \tilde{\pi}(\mathcal{S}), D(\mathcal{S}))$ in every state, as a best response to realized productivity shock A, the sunspots (s_{-1}, s) and predetermined variables of the economy m_{-1} and i. The government maximizes the welfare of home agents. This is, in effect, the same as minimizing the cost of its policy to taxpayers, hence to old agents, since they contribute to government's resources via the tax on labor income and seignorage on money holding.

In an environment with discretion, money creation provides an *ex post* source of revenue without creating any distortion. This low social cost of revenue ought to reduce the likelihood of default and stabilize debt values.

But, an essential element of this environment is the interaction between expected and realized inflation. Specifically, if agents anticipate high inflation (low $\tilde{\pi}^e$), they would reduce labor supply in youth and their real money holdings m_{-1} accordingly. To collect revenue from seignorage, the central bank then has to deliver a higher inflation rate (low $\tilde{\pi}$), consistent with the initial beliefs of agents. Hence, under discretion, the capacity of the central bank to support a stressed fiscal authority may be compromised by the strategic complementarity between expected inflation and delivered inflation: if agents anticipate the willingness of the central bank to resort to inflation, the real money tax base would decrease, which in turn reduces the capacity of the central bank to intervene.

The formation of expectations by young agents reflects these *ex post* policy choices. Let $\tilde{\pi}^e(\mathcal{S})$ denote the expectation of future (inverse) inflation given the current state $\mathcal{S}$. Then the requirement of rational expectations is $\tilde{\pi}^e(\mathcal{S}) = E_{\mathcal{S}'|\mathcal{S}}\tilde{\pi}(\mathcal{S}')$ where the expectation is over the future state given $\mathcal{S}$. This condition will be used in the construction of equilibria under discretion.

To characterize a SREE under discretion, we first determine the policy choices of a discretionary policy maker, then analyze the debt pricing dimension of the equilibrium and the associated stationary inflation expectations. The SREE combines these essential elements.

4.3.1 Choice Problem of a Discretionary Government

Given the productivity shock A, real money holding m_{-1} and nominal interest rate i on debt, the government chooses the money printing rate σ and whether to default $(D = d)$ or raise taxes τ and repay its debt $(D = r)$.

Under this regime, the government sets the money growth rate σ^D and collects seignorage given by:

$$\frac{\Delta M}{P} = \nu^m m_{-1} \sigma^D \tilde{\pi}^D, \tag{48}$$

where m_{-1} is the real money holding of current old agents and $\tilde{\pi}^D$ is the inverse rate of inflation for $D \in \{r, d\}$ induced by the choice of σ^D. It is given by:

$$\tilde{\pi}^D = \frac{1}{1+\sigma^D} \frac{m}{m_{-1}}. \tag{49}$$

Here $m \equiv m(\mathcal{S})$ is the real money demand of current young agents. As seen in (4), the money demand of the young is driven by current inflation expectations that are entirely independent of the current choices of the discretionary policy maker. Further, m_{-1}, real money held by the current old, is predetermined when the monetary authority decides on σ^D. Thus (49) captures a direct link from money growth to inverse inflation $\tilde{\pi}^D$. The government budget constraint under repayment is:

$$(1+i)\tilde{\pi}^r b = A^2(1-\tau)\tau + \nu^m m_{-1} \sigma^r \tilde{\pi}^r. \tag{50}$$

Hence, the government solves

$$D \in \{r, d\} = \text{argmax}\Big[\max_{\tau, \sigma^r} W^r(A, i, m_{-1}, \tau, \sigma^r, \tilde{\pi}^r), \max_{\sigma^d} W^d(A, i, m_{-1}, \sigma^d, \tilde{\pi}^d)\Big], \tag{51}$$

subject to its budget constraint (50), a non-negativity constraint on labor tax $\tau \geq 0$ and the following restriction on the realized inverse inflation rate: $\tilde{\pi}^D \in [\underline{\tilde{\pi}}, 1]$. The solution generates a default choice as well as a tax rate τ in the event of repayment and money growth rates σ^D dependent on the default decision, $D = d, r$. As mentioned above, the money growth rate induces a realization of the inflation rate, hence we describe monetary policy as the choice of $\tilde{\pi}^D(\cdot)$.

Following Calvo (1978), we assume that money printing is bounded so that the effective inverse inflation rate cannot be lower than $\underline{\tilde{\pi}} > 0$. Without this restriction, the monetary authority will always resort to the inflation tax and never tax labor income. Chari, Christiano, and Eichenbaum (1998) impose a similar restriction.[25] Both Corsetti and Dedola (2013) and Aguiar, Amador, Farhi, and Gopinath (2013) introduce an *ex-post* cost of inflation to limit money creation.

If the government chooses to repay the debt, the real money tax base m_{-1} is given. Its policy is naturally biased toward inflation since taxing money holdings does not distort labor supply decisions of current money holders. If this tax revenue is sufficient to cover its obligations, there is no labor tax imposed, and, using Assumption 2, repayment is preferred over default. Else, if seignorage does not generate enough revenue to cover its obligations, the government must impose a labor tax if it chooses to avoid default. This characterization is summarized in Lemma 3.

[25] In the appendix of that paper, this restriction is rationalized by the presence of an alternative technology such that agents can bypass the cash-in-advance constraint. In effect, the return on this alternative technology pins down the worst sustainable equilibrium and thus $\tilde{\pi}$. In our framework, the poor could store at a return of $r < 1$ instead of holding money and a parallel argument could be made for $\underline{\tilde{\pi}}$.

In the event of default, the choice of the inflation rate is welfare neutral given the specified social welfare function: when default occurs, monetary policy is implemented via lump-sum transfers which are purely redistributive, and consequently has no influence on the choices of the government. We set $\tilde{\pi}^d = \underline{\tilde{\pi}}$ in the event of default so that this rate is consistent with the inflation chosen whenever the government is indifferent between default and repayment.

The following lemma summarizes the state contingent choices of the government under discretion.

Lemma 3. *Under Assumption 2, given $\mathcal{S}$, the policy choices of the discretionary government are:*

1. *if the government chooses to repay its debt, then*

   a. $\tilde{\pi}^r = \max\left\{\underline{\tilde{\pi}}, \Pi(\mathcal{S})\right\}$, *where* $\Pi(\mathcal{S}) = \frac{\nu^m m(\mathcal{S})}{\nu^m m_{-1}+(1+i)b}$,

   b. $\tau > 0$ *and solves the government budget constraint* (50) *if and only if* $\tilde{\pi}^r = \underline{\tilde{\pi}}$.

2. *if the government chooses to default, then* $\tau = 0$ *and* $\tilde{\pi}^d = \underline{\tilde{\pi}}$.

3. *the government chooses to default if and only if*

$$\Delta(\cdot) = \frac{[A(1-\gamma)]^2}{2} - \frac{[A(1-\tau)]^2}{2} - (1+i)\underline{\tilde{\pi}}\theta b + T(\mathcal{S}, \underline{\tilde{\pi}}) > 0,$$

   where τ solves (50) *given* $\tilde{\pi}^r = \underline{\tilde{\pi}}$ *under repayment and $T(\cdot)$ is the lump-sum transfer that implements* $\underline{\tilde{\pi}}$ *under default.*

Proof. If the government repays, it will first use the inflation tax to obtain revenue since this tax is not distortionary. It will use labor taxation only if needed to repay the debt. Hence, if the real inflation tax base is large enough to service debt, then its labor tax policy is $\tau = 0$.

We derive first the condition under which seignorage alone is enough to service debt. From the budget constraint (50), if $\tau = 0$, then $(1+i)\tilde{\pi}^r b = \nu^m \tilde{\pi}^r m_{-1}\sigma$ implying $\sigma = \frac{(1+i)b}{\nu^m m_{-1}}$. Using (49), we get that under repayment $\tilde{\pi}^r = \frac{m(\mathcal{S})}{m_{-1}}\frac{1}{1+\sigma}$, where m_{-1} is real money held by the old and $m(\mathcal{S})$ is the level of real money demand of the current young. The resulting inverse rate of inflation is given by $\Pi(\mathcal{S}) = \frac{\nu^m m(\mathcal{S})}{(1+i)b+\nu^m m_{-1}}$. Hence, resource from seignorage is enough to service debt if $\Pi(\mathcal{S}) \geq \underline{\tilde{\pi}}$.

We next verify that $\Pi(\mathcal{S}) \geq \underline{\tilde{\pi}}$ implies the treasury chooses to service its debt rather than default, i.e. $\Delta(\cdot) \equiv W^d(\cdot) - W^r(\cdot) < 0$. With $\tau = 0$, $\Delta(\cdot)$ is:

$$\Delta(\cdot) = \frac{[A(1-\gamma)]^2}{2} - \frac{A^2}{2} + \nu^m m_{-1}(\underline{\tilde{\pi}} - \tilde{\pi}^r) - (1+i)\tilde{\pi}^r\theta b + T(\mathcal{S}, \underline{\tilde{\pi}}). \tag{52}$$

Here $T(\cdot) = \nu^m m_{-1}\sigma^d \underline{\tilde{\pi}}$ is the lump-sum transfer that implements $\tilde{\pi}^d = \underline{\tilde{\pi}}$, with $\sigma^d = \frac{m(\mathcal{S})}{\underline{\tilde{\pi}} m_{-1}} - 1$. Also, as seignorage is sufficient to service principal and interest on debt, $(1+i)b = \sigma^r \nu^m m_{-1}$, with $\sigma^r = \frac{m(\mathcal{S})}{\tilde{\pi}^r m_{-1}} - 1$. Finally, by the definition of $\tilde{\pi}^D$, $\nu^m m_{-1}\tilde{\pi}^D(1+\sigma^D) = \nu^m m(\mathcal{S})$ for $D = r, d$. Rearranging (52), one gets:

$$\begin{aligned}\Delta(\cdot) &= \frac{[A(1-\gamma)]^2}{2} - \frac{A^2}{2} - \nu^m m_{-1}\tilde{\pi}^r\sigma^r(\theta - 1) \\ &= \frac{[A(1-\gamma)]^2}{2} - \frac{A^2}{2} - \nu^m m(\mathcal{S})\frac{\sigma^r}{1+\sigma^r}(\theta - 1).\end{aligned} \tag{53}$$

This is negative by Assumption 2 as long as $\frac{m(\mathcal{S})\sigma^r}{1+\sigma^r}(1-\theta) < 1$. With $\theta \leq 1$ and $\sigma \geq 0$ under both optimism and pessimism, $\tilde{\pi} \leq 1$ so that $m(\mathcal{S})= \tilde{\pi}^e(\mathcal{S}) \leq 1$. Hence $\frac{m(\mathcal{S})\sigma}{1+\sigma}(1-\theta) < 1$. We get $\Delta(\cdot) < 0$, i.e. when seignorage is enough to service principal and interest, the government chooses not to default.

If resource from seignorage is not enough to service principal and interest on debt, then positive labor taxes are implemented: $\tau > 0$ if and only if $\underline{\tilde{\pi}} > \Pi(\mathcal{S})$. In this case, default is possible. Using these elements together with (14) and (15), one gets the expression for $\Delta(\cdot)$ stated in the Lemma. ∎

Equation (53) highlights an interesting aspect of redistribution in this economy. If all debt is held internally, $\theta = 1$, the default decision in this monetary economy is independent of the rate of money creation. This reflects the fact that the inflation tax is not distortionary and simply redistributes between rich and poor, both of whom are risk neutral. In this case, $\Delta < 0$ and there is surely no default. But if $\theta < 1$, then the inflation tax borne by the poor old agents is redistributed to the foreign holders of the debt, which is welfare reducing. Still, as argued in the proof, default does not occur if the debt obligation can be financed entirely by seignorage.

4.3.2 Price of Government Debt

Building on this characterization, we next investigate whether debt fragility remains under monetary discretion. First, we show that the multiplicity of interest rates consistent with the no-arbitrage condition (9) persists and interacts with inflation expectations.

Lemma 4. *Under Assumptions 2 and 3, under monetary discretion, there are multiple interest rates that solve the no-arbitrage condition (9).*

Proof. Consider the debt pricing building block of the equilibrium. We show that there are several possible outcomes, and consistent with our equilibrium definition, these different outcomes are driven by the realization of the sunspot s_{-1}. Using Assumption 3 and Lemma 3, there is a risk-free equilibrium of the debt financing problem, with inflation expectations $\tilde{\pi}^e(s^o) \geq \underline{\tilde{\pi}}$.[26] This may arise with $\tau = 0$ and $\tilde{\pi}^e(s^o) \geq \underline{\tilde{\pi}}$ or, from Lemma 3, with $\tau > 0$ and $\tilde{\pi}^e(s^o) = \underline{\tilde{\pi}}$.

Suppose investors believe the government will default on its debt with positive probability. If the belief is self-fulfilling, then the optimal policy of the government must be to set the inflation level to $\underline{\tilde{\pi}}$ for all A whether it reimburses its debt or defaults. Otherwise, resources from seignorage would be enough to cover principal and interest on debt for all realization of A, and default would be avoided. Hence, inflation expectations of agents are consistent with the best response of the government at $\tilde{\pi}^e(s^p) = \underline{\tilde{\pi}}$. The no-arbitrage condition pricing public debt becomes:

$$(1+i)\underline{\tilde{\pi}}\Big(1 - F\big(\bar{A}(i)\big)\Big) = R, \tag{54}$$

where $\bar{A}(i)$, defined in Lemma 1, is the boundary of the default region given i.

[26] In general $\tilde{\pi}^e(\mathcal{S})$ denotes expected (inverse) inflation. The notation $\tilde{\pi}^e(s)$ highlights the dependence of expectations on the sunspot, s. This is the expectation held by young agents regarding the future value of $\tilde{\pi}$. This value determines the labor supply and real money demand of young poor agents. It also influences the nominal interest rate, see (9).

From Lemma 2, we know that there are at least two interest rates i that are consistent with this equilibrium condition, one of which carries a risk-premium and induces the government to default for some realizations of A. Hence the initial pessimistic beliefs are self-fulfilling and support the existence of an interest rate that carries a positive probability of default. ∎

The key is that inflation expectations and probability of default are jointly linked by the anticipation of the best response of the discretionary government. In particular, the interest rate with a risk-premium that solves the no-arbitrage condition is systematically associated with the lowest real money tax base $m = \tilde{\pi}^e(s^p) = \underline{\tilde{\pi}}$, which in turn prevents the central bank from inflating away the real value of debt.

4.3.3 Inflation Expectations

We now turn to the third component of the equilibrium: the determinants of inflation expectations $\tilde{\pi}^e(s)$. This section establishes two results. First, there exist inflation expectations, contingent on the sunspot realization, that are consistent with the choices of the government. Second, there may be multiple such levels of inflationary expectations. This last point arises from the complementarities between monetary expectations and policy response.

As shown in the proof of Lemma 4, in the event of pessimism, young agents expect high inflation, i.e. $\tilde{\pi}^e(s^p) = \underline{\tilde{\pi}}$. That is, whenever the equilibrium of the debt financing problem induces state-contingent default, the inflation rate is maximal. So, regardless of the current state $\mathcal{S}$, given pessimism in the previous period, $s_{-1} = s^p$, the inverse inflation rate is $\tilde{\pi}(\mathcal{S}) = \underline{\tilde{\pi}}$. This is then consistent with the initial expectations of the current old, formed when they were young in the previous period, i.e. $\tilde{\pi}^e(s^p) = \underline{\tilde{\pi}}$.

The issues of existence and multiplicity of inflation expectations arise when $s_{-1} = s^o$. From Lemma 4, young agents anticipate the government will service its debt obligation for all $\mathcal{S}$. The question is then how the government will repay its obligation. Given the bias toward inflationary financing of debt, what determines $\tilde{\pi}^e(s^o)$ is whether seignorage resource is enough to service principal and interest on debt for all (A, s). If the debt is not too large, then the inflation tax alone is sufficient to cover debt obligations: i.e. $\tilde{\pi}^r(A, s, \cdot) > \underline{\tilde{\pi}}$ for all (A, s) and $\tilde{\pi}^e(s^o) > \underline{\tilde{\pi}}$. In this case, $\tau(A, s) = 0$ for all (A, s). Else, the inflation tax will be maximal, $\tilde{\pi}^e(s^o) = \underline{\tilde{\pi}}$, and supplemented by a labor tax. In both cases $D(A, s,) = r$ for all (A, s).

Formally, Lemma 3 established that with $s_{-1} = s^o$, given the real money tax base $\nu^m m_{-1} = \nu^m \tilde{\pi}^e(s^o)$, the inflation delivered by the discretionary government satisfies:

$$\tilde{\pi}^r(A, s, \cdot) = \max\left\{\frac{\nu^m \tilde{\pi}^e(s)}{\nu^m \tilde{\pi}^e(s^o) + (1+i)b}; \underline{\tilde{\pi}}\right\} \qquad \forall A \ \forall s \in \{s^o, s^p\}, \tag{55}$$

where the max operator captures whether seignorage resource is enough to service principal and interest on debt, and $\tilde{\pi}^e(s) = m(\mathcal{S})$ is the real money demand of current young agents, conditional on the realization of the current sunspot s. The no-arbitrage condition gives: $(1+i)\tilde{\pi}^e(s^o) = R$. Accordingly, $\tilde{\pi}^e(s^o) \geq \underline{\tilde{\pi}}$ can be part of a stationary equilibrium if and only if it satisfies:

$$\tilde{\pi}^e(s^o) = p \max\left\{\frac{\nu^m \tilde{\pi}^e(s^o)}{\nu^m \tilde{\pi}^e(s^o) + \frac{R}{\tilde{\pi}^e(s^o)} b}; \underline{\tilde{\pi}}\right\} + (1-p) \max\left\{\frac{\nu^m \tilde{\pi}^e(s^p)}{\nu^m \tilde{\pi}^e(s^o) + \frac{R}{\tilde{\pi}^e(s^o)} b}; \underline{\tilde{\pi}}\right\}, \tag{56}$$

where p is the stationary probability of optimism.

The following lemma establishes the existence of stationary inflation expectations under optimism that are consistent with the policy choices of the government for all $b \in (0, \bar{b})$.

Lemma 5. *Given Assumptions 2 and 3, under monetary discretion, there is a debt threshold* $\hat{b} = \frac{\nu^m \tilde{\pi}(1-\tilde{\pi})}{R}$ *such that:*

1. *If* $0 < b < \hat{b}$, *then* $\tilde{\pi}^e(s^o) > \underline{\tilde{\pi}}$ *is consistent with the government choice* $\tilde{\pi}^r(A, s, \cdot) > \underline{\tilde{\pi}}$, *for all* (A, s).
2. *If* $\hat{b} \leq b < \bar{b}$, *then* $\tilde{\pi}^e(s^o) = \underline{\tilde{\pi}}$ *is consistent with the government choice* $\tilde{\pi}^r(A, s, \cdot) = \underline{\tilde{\pi}}$, *for all* (A, s).

Proof. Computation details are provided in Appendix 7.2. ∎

In the first case, the level of debt and inflation expectations are such that seignorage is sufficient to service debt for any realization of s. In the second case, the level of debt and inflation expectations are such that seignorage is not sufficient to service debt, and must be complemented with labor taxes for any realization of s.[27]

In fact, for some debt level $b \geq \hat{b}$, $\tilde{\pi}^e(s^o)$ can take several values. This is due to the interactions between expected inflation and delivered inflation, present in (56). It can give rise to a seignorage Laffer curve, where several rates of inflation in excess of $\underline{\tilde{\pi}}$ generate the same real resource.[28] In addition, there is another possible outcome with maximum inflation, i.e. $\tilde{\pi}^e(s^o) = \underline{\tilde{\pi}}$, and positive labor taxes. These elements are summarized in Figure 2. Lemma 5 establishes the existence of $\tilde{\pi}^e(s^o)$ for all $b < \bar{b}$, but does not describe all the possible regimes. Indeed, as explained in the proof of Lemma 4, the inflation regime under optimism does not influence the existence of multiple valuations of debt. Hence, without loss of generality, we select in Lemma 5 an inflation regime for $b \geq \hat{b}$, marked in blue in Figure 2.

4.3.4 Equilibrium characterization

The analysis has established the government budget constraint under discretion, the potential for multiple solutions to the debt valuation equation and the existence of inflation expectations consistent with monetary policy. Taken together, these elements create the basis for sunspot equilibria associated with the valuation of government debt. Formally,

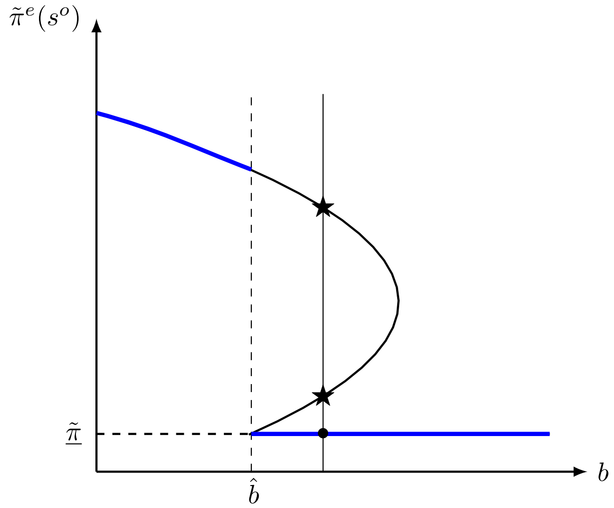
Proposition 4. *Under Assumptions 2 and 3, there is a SREE under discretion with the following properties:*

1. *If* $s_{-1} = s^o$, *government debt is risk free as the treasury reimburses with probability 1, with either:*
   a. *if* $0 < b < \hat{b}$, *then* $\tilde{\pi}^e(s^o) > \underline{\tilde{\pi}}$ *and for all* A *all* s, $\tilde{\pi}(A, s, \cdot) > \underline{\tilde{\pi}}$, $\tau(A, s, \cdot) = 0$, $D(A, s, \cdot) = r$,
   b. *if* $\hat{b} \leq b < \bar{b}$, *then* $\tilde{\pi}^e(s^o) = \underline{\tilde{\pi}}$ *and for all* A *all* s, $\tilde{\pi}(A, s, \cdot) = \underline{\tilde{\pi}}$, $\tau(A, s, \cdot) > 0$, $D(A, s, \cdot) = r$.
2. *If* $s_{-1} = s^p$, *the interest rate incorporates a risk-premium. For all* A, $\tilde{\pi}(A, \cdot) = \underline{\tilde{\pi}}$. *The treasury defaults on its debt for all* $A < \bar{A}$ *where* $\bar{A} \in (A_l, A_h)$ *and* $\tilde{\pi}^e(s^p) = \underline{\tilde{\pi}}$.

[27] We do not impose further parametric restriction to ensure that $\hat{b} < \bar{b}$, where $\bar{b}$ is defined by Assumption 3. This requires the lower bound on productivity A_l or the cost of default γ to be high enough or the share of money holder ν^m to be low enough. If it were the case that $\hat{b} \geq \bar{b}$, then only case 1 of Lemma 5 would apply, our results would not be affected.

[28] This is the standard textbook Laffer curve, as in Theorem 26.2 of Azariadis (1993), and discussed in Appendix 7.2.

Figure 2: Multiplicity of Inflation Regime under Optimism



The figure illustrates the possibility of multiple stationary inflation expectations compatible with the policy of the government. For $b \geq \hat{b}$, several outcomes are possible: the ones marked ★ correspond to the seignorage Laffer curve, where several rates of inflation are consistent with the repayment of debt, with no labor taxes. In addition, the outcome labelled • can arise: agents reduce their real money demand to $\underline{\tilde{\pi}}$, which in turn induces the discretionary government to generate $\underline{\tilde{\pi}}$ and complement seignorage resource with positive labor taxes to service debt. The inflation regime under optimism selected in this analysis are indicated in blue.

Proof. We describe the optimal behavior of agents consistent with the equilibrium definition. This proof builds on Lemma 4 and the existence of several interest rates (and associated inflation expectations) consistent with the equilibrium definition.

If $s_{-1} = s^o$, then by Assumption 3, debt is risk free. Two cases need to be distinguished, as established in Lemma 5. If $b < \hat{b}$, then inflation expectations under optimism $\tilde{\pi}^e(s^o)$ allow seignorage resource to be sufficient to service principal and interest on debt. Young agents form expectations of no default and $\tilde{\pi}^e(s^o) > \underline{\tilde{\pi}}$. They supply labor accordingly, young agents with low productivity save with money, young rich agents save via intermediated claims; the interest rate i on the government security satisfies the no-arbitrage condition (9) with a zero probability of default, i.e. $P^d = 0$, and $\tilde{\pi}^e(s^o)$. The optimal policy of the government is then to set for all A, all s, $\tilde{\pi}(A, s, \cdot) > \underline{\tilde{\pi}}$, $\tau(A, s, \cdot) = 0$ and repay the debt.

On the other hand, if $\hat{b} \leq b < \bar{b}$, then there is an equilibrium with $\tilde{\pi}^e(s^o) = \underline{\tilde{\pi}}$, seignorage resource is not sufficient and taxes need be raised to service debt. Using Lemma 3 and Assumption 3, for all A, all s, $\tilde{\pi}(A, s, \cdot) = \underline{\tilde{\pi}}$, $\tau(A, s, \cdot)$ solves the government budget constraint (50) and debt is repaid. Accordingly, young agents form expectations $P^d = 0$, $\tilde{\pi}^e(s^o) = \underline{\tilde{\pi}}$, the government security is priced according to (9). In both cases, all markets clear.

For $s_{-1} = s^p$, we detail only the differences with the previous case. Independently of the level of b, young agents form rational expectations in which there is a positive probability of default, i.e. $P^d > 0$, and $\tilde{\pi}^e(s^p) = \underline{\tilde{\pi}}$. The government security is priced accordingly. Given i and seignorage revenue $\nu^m \underline{\tilde{\pi}}(1 - \underline{\tilde{\pi}})$, there is a unique threshold $\bar{A}(i) > A_l$ such that the optimal policy is to raise labor taxes τ for all $A \geq \bar{A}(i)$ so as to satisfy the budget constraint (50) and default otherwise. Finally, expectations are consistent with the best response of the government: $P^d = F(\bar{A}(i))$. ∎

Does monetary discretion provide a shield against debt fragility? Can the government inflate the real value of debt and generate additional resources to service its debt? The answer is negative. Indeed, when pessimism hits the economy, the interplay between inflation expectations and real money tax base *corners* the central bank into a high inflation regime with no more capacity to inflate debt or provide additional resources to the treasury. Hence, under monetary discretion, the sunspot shock to investors confidence triggers a joint shift in inflation expectations and debt sustainability.

The potential for inflation multiplicity discussed in sub-section 4.3.3 is not explicit in Proposition 4. The point of the proposition, and of this paper, is to study the interaction of debt fragility and monetary policy. Accordingly, the analysis in Proposition 4 rests on the selection of a particular equilibrium in the event of optimism, i.e. $s_{-1} = s^o$. The equilibrium selection is orthogonal to the multiple solutions of the debt pricing equation, as argued before. Of course, it is possible to add another, independent, dimension to the problem by introducing strategic uncertainty over the the inflation rate, given optimism.

The equilibria characterized in Proposition 4 depend on $\underline{\tilde{\pi}}$. If the ceiling on maximal inflation is lowered (i.e. $\underline{\tilde{\pi}}$ is higher), then the revenue collected from seignorage will change. Whether revenue increases or decreases depends on the response of money demand, i.e. on which side of the Laffer curve is the economy operating. For this model, the upper slope of the Laffer curve corresponds to $\underline{\tilde{\pi}} > \tilde{\pi}^L$. When more revenue is collected from inflation, then the probability of default will decrease. Sunspot equilibria will remain.

Proposition 5. *In the sunspot equilibrium characterized in Proposition 4, if $\underline{\tilde{\pi}} > \tilde{\pi}^L$, then a reduction in the inflation ceiling (i.e. an increase in $\underline{\tilde{\pi}}$) will (i) increase the magnitude of taxes when $s_{-1} = s^o$ and (ii) increase the probability of default under $s_{-1} = s^p$, if and only if the equilibrium of the debt financing problem is locally stable.*

Proof. An increase in $\underline{\tilde{\pi}}$ will, if $\underline{\tilde{\pi}} > \frac{1}{2}$, lower seignorage revenue. From Proposition 4, if $s_{-1} = s^o$ and $b \geq \hat{b}$, the government sets $\tilde{\pi}(\cdot) = \underline{\tilde{\pi}}$ and relies on labor taxes to pay the remainder of its obligations. If $\underline{\tilde{\pi}}$ increases, then the government has to increase τ to finance its debt obligation.

In the case $s_{-1} = s^p$, the inflation rate is $\tilde{\pi}(\cdot) = \underline{\tilde{\pi}}$. Using Proposition 3, we get that an increase in $\underline{\tilde{\pi}}$, if $\underline{\tilde{\pi}} > \tilde{\pi}^L$, i.e. a reduction in the inflation ceiling on the upward sloping part of the Laffer curve, will increase the default threshold if and only if the equilibrium is locally stable. ∎

5 Stabilization through Commitment: Leaning Against the Winds

These results make clear that the monetary authority may be unable to prevent debt fragility. If there is a commitment to an unconditional inflation tax (or inflation target), the environment is similar to a real economy, thus exposing the debt to multiple valuations. If the monetary authority has complete discretion, then it will use the inflation tax to raise revenue *ex post* and again fiscal tools may be needed to finance debt repayments. In both cases, when productivity is low, the tax burden can become excessive leading to default. In equilibrium, the valuation of debt will be subject to investor sentiments in a sunspot equilibrium.

This section returns to the commitment case. Instead of imposing an inflation target, we allow the central bank to choose a state-contingent inflation policy that alters the real debt burden and distributes resources from seignorage across states. As in Chari, Christiano, and Eichenbaum (1998) this is a one period commitment, allowing the monetary authority to commit to inflation next period, contingent on the current state. By carefully choosing the distribution of realized future inflation, the central bank can provide a shield against debt fragility.

Suppose the monetary authority commits to a rule given by $\tilde{\pi}(A, i, s_{-1})$: the rate of inflation in the current period depends on current productivity, the interest rate on outstanding debt as well as the sunspot realization from previous period.[29] This rule is devised with a couple of key properties. First, to induce agents to hold money, the rule will deliver a target rate of inflation. Second, it will support the fundamental equilibrium by using monetary tools to counter pessimistic expectations so that equilibria with strategic uncertainty no longer exist.[30] In this way, the monetary authority responds to variations in current beliefs, reflected in the sunspot and the interest rate, by appropriately setting policy for the future. Importantly, if investors were pessimistic in the previous period, this policy responds to variations in productivity: the rate of inflation is inversely related to current productivity. Specifically, when A is high, the rate of inflation is relatively low and fiscal policy, through the setting of tax rates, bears more of the burden of financing debt obligations. But during times of low productivity, when default is likely, the monetary authority inflates the real value of debt and generates seignorage revenue. Both effects allow the fiscal authority to set low taxes and avoid default.

We first describe the desired properties of this policy, derive its existence and properties in Lemma 6. Then, we characterize the stationary equilibrium of the economy under $\tilde{\pi}(A, i, s_{-1})$ and argue that such monetary policy rule stabilizes debt valuations.[31]

Specifically, suppose the central bank commits to a rule in which $\tilde{\pi}(A, i, s^o) = \tilde{\pi}^*$ for all (A, i): under optimism, there is an inflation target as in *monetary delegation*. Delivered inflation $\tilde{\pi}^*$ is independent of both current productivity A and the interest rate on debt. When $s_{-1} = s^p$, the central bank implements a state dependent (on (A, i)) monetary policy. This policy satisfies two key properties.

First, given pessimism the policy rule anchors inflation expectations: $\tilde{\pi}(A, i, s^p)$ meets the inflation target *on average*:

$$\int_A \tilde{\pi}(A, i, s^p) dF(A) = \tilde{\pi}^*, \tag{57}$$

for all i. Combined with the policy under optimism, $\tilde{\pi}(A, i, s^o) = \tilde{\pi}^*$, unconditional inflation expectations are anchored at $\tilde{\pi}^*$. Thus, the real money tax base is invariant and resources from seignorage are given by:

$$\frac{\Delta M}{P} = \nu^m \tilde{\pi}^* \big(1 - \tilde{\pi}(A, i, s^p)\big). \tag{58}$$

[29]This commitment is independent of other elements of the state vector.

[30]To be clear, the policy is designed to eliminate equilibria with state contingent default. The equilibrium with certain default remains.

[31]As written, the intervention depends on (A, i, s_{-1}). In the equilibrium constructed below, optimism is equivalent to an interest rate satisfying $(1 + i)\tilde{\pi}^* = R$. Hence there is only one interest rate conditional on optimism. If there is pessimism, we condition monetary policy on the interest rate on the outstanding debt in order to specify the monetary intervention both on and off the equilibrium path. An alternative would write the equilibrium conditions solely as a function of the interest rate, not the sunspots. This is used in the discussion of policy implementation.

The government budget constraint under repayment becomes:

$$(1+i)\tilde{\pi}(A,i,s^p)b = A^2(1-\tau)\tau + \nu^m\tilde{\pi}^*\big(1-\tilde{\pi}(A,i,s^p)\big). \tag{59}$$

Second, $\tilde{\pi}(A,i,s^p)$ deters state contingent default: given A and i, the treasury either reimburses its debt with probability 0 or 1. For low values of debt obligations, the fiscal authority will choose to repay its debt, for all A. For high values of these obligations, the fiscal authority will default, again for all A. Of course, the size of the debt obligations are determined in equilibrium, based upon investor beliefs and central bank policy. Formally, Lemma 6 establishes that there is a monetary policy rule under pessimism that satisfies these two properties.

Lemma 6. *Given an inflation target* $0 < \tilde{\pi}^* \leq 1$*, there is a monetary policy rule* $\tilde{\pi}(A,i,s^p)$ *that satisfies the inflation target and deters state contingent default. Moreover,* $\tilde{\pi}(A,i,s^p) > 0$ *for all* (A,i) *and is increasing in* A*.*

Proof. We derive a state-contingent monetary policy rule $\tilde{\pi}(A,i,s^p)$ that satisfies (57) and deters state contingent default. Consider the case $\theta = 0$, where all debt is held abroad, and $\nu^m \approx 0$, which makes seignorage a negligible source of income for the fiscal authority. This simplified framework outlines clearly that the capacity of the central bank to influence the default decision of the treasury does not primarily rely on providing more or less resources, but rather on its capacity to alter the real return to debt across states. The proof is extended to the general case $\theta \geq 0$ and $\nu^m \geq 0$ in Appendix 7.3.

Given $s_{-1} = s^p$, we focus on the dependence of inflation on the interest rate on outstanding debt and the realization of the technological shock A. Consider a state contingent rule $\tilde{\pi}(A,i,s^p)$, denoted $\tilde{\pi}^p_A$ in the following analysis. This rule induces a unique interest rate cut-off i^δ such that if $i < i^\delta$ then the fiscal authority is induced to repay its debt for all A, i.e. with probability 1. If $i > i^\delta$, then the fiscal authority defaults for all A, i.e. with probability 1. For $i = i^\delta$, the fiscal authority is indifferent between repayment and default for all A. This condition for indifference is:

$$\Delta(A,i^\delta,m_{-1},\tau,\tilde{\pi}^p_A) = W^d(\cdot) - W^r(\cdot) = 0 \qquad \forall A, \tag{60}$$

where $m_{-1} = \tilde{\pi}^*$ using the inflation target condition (57), $\tilde{\pi}^p_A$ is defined below and τ satisfies the government budget constraint (59) given $(A,i^\delta,\tilde{\pi}^p_A)$:

$$(1+i^\delta)\tilde{\pi}^p_A b = A^2(1-\tau)\tau. \tag{61}$$

Using $\theta = 0$ and $\nu^m \approx 0$, (60) implies $\tau = \gamma$ for all A. From the government budget constraint:

$$\tilde{\pi}^p_A = \frac{A^2(1-\gamma)\gamma}{(1+i^\delta)b} \qquad \forall A. \tag{62}$$

Applying the inflation target requirement (57), the nominal interest rate cut-off i^δ is:

$$1 + i^\delta = \frac{(1-\gamma)\gamma}{\pi^* b} \int_A A^2 dF(A), \tag{63}$$

which gives:

$$\tilde{\pi}^p_A = \frac{A^2 \pi^*}{\int_A A^2 dF(A)}. \tag{64}$$

We verify that this monetary rule deters state contingent default:

$$\frac{d\Delta(A, i, m, \tau, \tilde{\pi}^p_A)}{di} = A^2(1-\tau)\frac{d\tau}{di}, \tag{65}$$

where $\frac{d\tau}{di} = \frac{\tilde{\pi}^p_A b}{A^2(1-2\tau)} > 0$ from (61). As $\Delta(A, i^\delta, \tilde{\pi}^*, \tau, \tilde{\pi}^p_A) = 0$ for all A, we get that for all A and all $i < i^\delta$, $\Delta(\cdot) < 0$ and for all $i > i^\delta$, $\Delta(\cdot) > 0$. Hence there is no nominal interest rate $i > 0$ that induces the fiscal authority to default on its debt in a state-contingent manner. Finally, from (64), we get $\tilde{\pi}^p_A > 0$ and $\frac{d\tilde{\pi}^p_A}{dA} > 0$. ∎

The lemma establishes two critical properties of $\tilde{\pi}(A, i, s^p)$. First, for all A, $\tilde{\pi}(A, i, s^p) > 0$, which rules out any issue of demonetization of the economy and state-contingent complete default via inflation. Second, the policy rule is countercyclical: $\tilde{\pi}(A, i, s^p)$ is increasing in A, i.e. the lower the technology realization, the higher is inflation. This policy distributes resource from seignorage across states, with high seignorage revenue $\nu^m \tilde{\pi}^*(1 - \tilde{\pi}^p_A)$ for low realizations of A. Hence, even if seignorage revenue is not essential to rule out equilibria with default, the policy further contributes to lower the fiscal burden in states where fiscal needs are the highest, i.e. the induced fiscal policy is also countercyclical.[32]

When the central bank commits to $\tilde{\pi}(A, i, s_{-1})$, there is a unique price for debt, namely the fundamental price under inflation targeting. That is, there is no sunspot equilibrium affecting the valuation of debt. Formally,

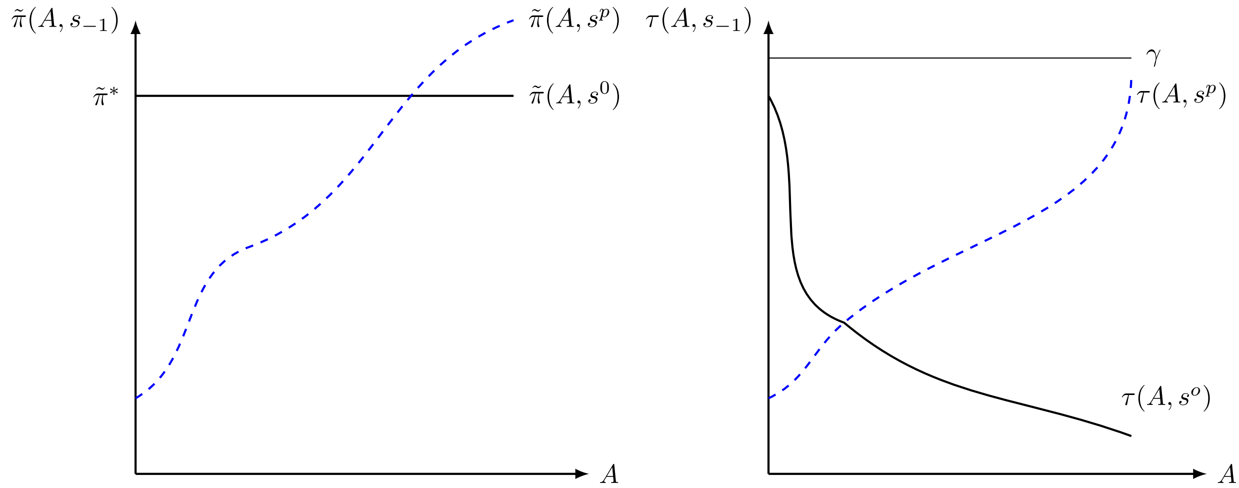
Proposition 6. *Under Assumptions 2, 3, when the monetary authority commits to $\tilde{\pi}(A, i, s_{-1})$, with $\tilde{\pi}(A, i, s^p)$ given in Lemma 6, debt is uniquely valued and risk-free. Debt fragility is eliminated.*

Proof. Under Assumption 3, there is a risk-free outcome under strict inflation target $0 < \tilde{\pi}^* \leq 1$. Hence, there is an equilibrium nominal interest rate $\underline{i}$ under optimism that satisfies $(1 + \underline{i})\tilde{\pi}^* = R$. Now under pessimism, the monetary authority commits to $\tilde{\pi}(A, i, s^p)$ as defined in Lemma 6. As seen in the proof of this lemma, this rule delivers inflation as a function of the technological shock A. It is noted $\{\tilde{\pi}^p_A\}$ in the following developments. We verify that under this rule, the best response of the treasury is to repay its debt for all A and that the equilibrium interest rate is $\underline{i}$.

A central property of $\{\tilde{\pi}^p_A\}$ is that it delivers the inflation target on average. By continuity and monotonicity of $\tilde{\pi}^p_A$ in A, there is a realization $\tilde{A}$ such that $\tilde{\pi}^p_{\tilde{A}} = \tilde{\pi}^*$. In this case, the best-response of the fiscal authority is to raise taxes and repay its debt. Second, $\{\tilde{\pi}^p_A\}$ is such that if the fiscal authority repays its

[32] In fact, the proof in the text focuses on the case of ν^m near zero, where seignorage resource is negligeable.

Figure 3: State Dependent Monetary and Fiscal Policy



(a) Monetary Policy (b) Fiscal Policy

The left panel represents the state dependent monetary policy to which the central bank commits. The right panel represents the induced fiscal policy. The dependence of the policies on the sunspot and realized productivity are displayed.

debt with positive probability, it repays its debt with probability 1. Hence, under $\{\tilde{\pi}_A^p\}$, the fiscal authority repays its debt for all A: debt is risk-free. Finally, the no-arbitrage condition under inflation target $\tilde{\pi}^*$ uniquely pins down the nominal interest rate. Hence, under $\{\tilde{\pi}_A^p\}$, the nominal interest rate is $\underline{i}$:

$$(1+\underline{i})\int_A \tilde{\pi}_A^p dF(A) = (1+\underline{i})\tilde{\pi}^* = R. \tag{66}$$

∎

This proposition makes clear that the commitment of the central bank rules out the effect of pessimism on the value of debt. The key to this result is the relaxation of the incentive to default by the fiscal authority through the provision of seignorage revenue and the erosion of the real return to debt in low productivity states.

Figure 3 displays the equilibrium monetary policy rule and the induced tax policy, as described in Proposition 6.[33] In the case $s_{-1} = s^p$, note the distribution of inflation over realization of A: for low A, high inflation, i.e. low real value of debt and high seignorage revenue. Hence, in case of pessimism, the monetary authority implements a countercylical policy that stabilizes the price of debt and provides fiscal relief for low values of A, compensated by lower inflation for higher realizations of A. A critical element of this policy is the commitment of the central bank so that inflation expectations are anchored and the real money tax base is not sensitive to variations in private agents sentiments. It illustrates how the central bank can alter the real value of debt, and incidentally distribute income from seignorage, so as to contain the fiscal pressure that weights on the fiscal authority. In this sense, the monetary authority leans against the winds of pessimism as well as those associated with low productivity.

[33]The dependence on i is not explicit as these are the policy functions along the equilibrium path.

As written, the monetary intervention depends jointly on the sunspot from the previous period as well as the interest on outstanding debt. Along the equilibrium path, from Proposition 6, only the fundamental price of debt will be observed. Though extraneous uncertainty may still exist, it will not be reflected in the equilibrium interest rates. With this in mind, it may be more natural to condition monetary interventions on interest rates so that along the equilibrium path, no actual intervention is needed. But, the monetary authority stands ready to intervene in response to higher interest rates that reflect investor pessimism. This is, in effect, a threat of the monetary authority off the equilibrium path to intervene either to support the fiscal authority or, if interest rates are too high, to allow default with probability one.

6 Conclusions

The goal of this paper was to determine whether monetary policy enhances or mitigates fiscal fragility. Cast as a real economy, the basic environment has fragile debt: there are multiple valuations of government debt depending on the beliefs of investors.

The effects of introducing monetary interventions depends on the commitment of the central bank. If the central bank is committed to an inflation target, then debt fragility remains. If the central bank is allowed full discretion, then the presence of an inelastic source of finance through seignorage is internalized by private agents. Any temptation to inflate the real value of debt is anticipated and debt fragility remains.

Finally, we analyze how a committed central bank can deter debt fragility, by designing a specific monetary policy rule. We devise a state contingent intervention that eliminates pessimistic evaluations of government debt. The policy requires the monetary authority to implement a countercyclical policy, that erodes the real value of debt and provides resources, through seignorage, in times of low productivity and thus low revenue. By supporting the fiscal authorities in these states, the incentive for default is eliminated. Sovereign debt is no longer subject to multiple valuations driven by investors' sentiments.

A number of extensions are worth consideration. First, the paper studies the extremes of commitment and discretion. An interesting middle case would be stochastic commitment. A government acting in period t would be allowed to adjust its policy in period $t+1$ with a probability less than one. This partial commitment would create a cost of high inflation and thus enrich the analysis.

Second, the model is dynamic but the fiscal policy is within a generation. Thus we have assumed away the possibilities of debt turnover and intertemporal punishments for default.

Third, our analysis has underlined that the capacity of the central bank to stabilize debt valuations rely on the issuance of non contingent nominal assets labelled in domestic currency. It does not apply to real, indexed debt or debt issued in foreign currency. Allowing governments this choice would be of interest.

Finally, as in many other studies, the outcome with discretion imposes an upper bound on inflation. Providing further micro foundations for this bound remains an open area. Perhaps a political economy model that stresses the redistribution aspects of labor income vs inflation tax would be a productive approach.

7 Appendix

7.1 Welfare under Repayment and under Default

As explained in section (2.2), the repayment vs. default decision in this environment is a discrete choice that affects only the welfare of old agents. Hence, the welfare criteria of interest for $D \in \{r, d\}$ is:

$$W^D(A, i, m_{-1}, \tau, \sigma, \tilde{\pi}) = \nu^m \Big[c_o^m(D) - \frac{n_o^m(D)^2}{2}\Big] + \nu^I \Big[c_o^I(D) - \frac{n_o^I(D)^2}{2}\Big]. \tag{67}$$

Using the labor supply policy functions from (4) and (10), we get the following consumption and labor supply vectors:

$$\begin{aligned}
c_o^m(r) &= An_o^m(r)(1-\tau) + m_{-1}\tilde{\pi}^r & \qquad c_o^m(d) &= An_o^m(d)(1-\gamma) + m_{-1}\tilde{\pi}^d + t \\
n_o^m(r) &= A(1-\tau) & n_o^m(d) &= A(1-\gamma) \\
c_o^I(r) &= An_o^I(r)(1-\tau) + (1+i)\tilde{\pi}^r b^I + Rk & c_o^I(d) &= An_o^I(d)(1-\gamma) + Rk + t \\
n_o^I(r) &= A(1-\tau) & n_o^I(d) &= A(1-\gamma).
\end{aligned}$$

Using $\nu^I b^I = \theta b$, one can solve for k, the risk-free component of individual portfolio of rich agents from their budget constraint:

$$zn_y^I = Rz^2 = b^I + k + \Gamma \Rightarrow \ \nu^I Rk = \nu^I R\big(Rz^2 - \Gamma\big) - R\theta b. \tag{68}$$

We derive the expressions for $W^r(\cdot)$ and $W^d(\cdot)$:

$$W^r(A, i, m_{-1}, \tau, \sigma, \tilde{\pi}^r) = \frac{\big[A(1-\tau)\big]^2}{2} + \nu^m m_{-1}\tilde{\pi}^r + \big((1+i)\tilde{\pi}^r - R\big)\theta b + \nu^I R(Rz^2 - \Gamma) \tag{69}$$

$$W^d(A, i, m_{-1}, \sigma, \tilde{\pi}^d) = \frac{\big[A(1-\gamma)\big]^2}{2} + \nu^m m_{-1}\tilde{\pi}^d - R\theta b + \nu^I R(Rz^2 - \Gamma) + T(\cdot), \tag{70}$$

where τ solves the government budget constraint under repayment and $T(\cdot) = \nu^m m_{-1}\sigma\tilde{\pi}^d$ is a lump sum transfer that implements $\tilde{\pi}^d$ under default.

Default is optimal whenever $\Delta(\cdot) = W^d(\cdot) - W^r(\cdot) \geq 0$.

7.2 Proof Lemma 5

As discussed in section 4.3.3, $\tilde{\pi}^e(s^o) \geq \underline{\tilde{\pi}}$ can be part of a stationary equilibrium if and only if it satisfies:

$$\tilde{\pi}^e(s^o) = p \max\Big\{\frac{\nu^m\tilde{\pi}^e(s^o)}{\nu^m\tilde{\pi}^e(s^o) + (1+i)b}; \underline{\tilde{\pi}}\Big\} + (1-p)\max\Big\{\frac{\nu^m\tilde{\pi}^e(s^p)}{\nu^m\tilde{\pi}^e(s^o) + (1+i)b}; \underline{\tilde{\pi}}\Big\}, \tag{71}$$

where the no-arbitrage condition gives: $(1+i)\tilde{\pi}^e(s^o) = R$, and p is the stationary probability of optimism.

First consider the situation in which seignorage alone is not sufficient to service principal and interest on debt. In this case, the government sets $\tilde{\pi}^r(s, \cdot) = \underline{\tilde{\pi}}$ for all s, and raises additional labor taxes. Agents form expectations accordingly and (71) writes:

$$\tilde{\pi}^e(s^o) = (1-p)\underline{\tilde{\pi}} + p\underline{\tilde{\pi}} = \underline{\tilde{\pi}}. \tag{72}$$

This case emerges whenever $\frac{\nu^m \underline{\tilde{\pi}}}{\nu^m \underline{\tilde{\pi}} + \frac{R}{\underline{\tilde{\pi}}} b} \leq \underline{\tilde{\pi}}$, which rewrites:

$$b \geq \hat{b} = \frac{\nu^m \underline{\tilde{\pi}}(1-\underline{\tilde{\pi}})}{R}. \tag{73}$$

Next, we show that whenever $0 < b < \hat{b}$, there is a level of inflation expectation under optimism, $\tilde{\pi}^e(s^o)$, such that seignorage alone is sufficient to service principal and interest on debt for all (A,s). (71) writes then:

$$\tilde{\pi}^e(s^o) = p\frac{\nu^m \tilde{\pi}^e(s^o)}{\nu^m \tilde{\pi}^e(s^o) + \frac{R}{\tilde{\pi}^e(s^o)} b} + (1-p)\frac{\nu^m \underline{\tilde{\pi}}}{\nu^m \tilde{\pi}^e(s^o) + \frac{R}{\tilde{\pi}^e(s^o)} b}. \tag{74}$$

Multiply both sides by $\nu^m \tilde{\pi}^e(s^o) + \frac{R}{\tilde{\pi}^e(s^o)} b$ and get:

$$\nu^m \tilde{\pi}^e(s^o)^2 - p\nu^m \tilde{\pi}^e(s^o) + Rb - (1-p)\nu^m \underline{\tilde{\pi}} = 0. \tag{75}$$

Hence, (74) has at least a positive solution if $b \leq b^\alpha$, where:

$$b^\alpha = \frac{p^2 \nu^m + 4(1-p)\nu^m \underline{\tilde{\pi}}}{4R}. \tag{76}$$

Under this condition, the solution to (75) that is necessarily positive[34] is given by:

$$\tilde{\pi}^e(s^o) = \frac{p + \sqrt{p^2 + 4(1-p)\underline{\tilde{\pi}} - 4\frac{Rb}{\nu^m}}}{2}. \tag{77}$$

This solution is compatible with (71) if it satisfies the following two conditions:

$$\frac{\nu^m \tilde{\pi}^e(s)}{\nu^m \tilde{\pi}^e(s^o) + \frac{R}{\tilde{\pi}^e(s^o)} b} \geq \underline{\tilde{\pi}} \qquad \forall s \in \{s^o, s^p\}, \tag{78}$$

We verify that $b^\alpha \geq \hat{b}$ and that for all $b < \hat{b}$, when $\tilde{\pi}^e(s^o)$ is given by (77), then the conditions (78) are satisfied.

Note $F(p) = 4R\big(b^\alpha - \hat{b}\big)$. Substituting and rearranging:

$$F(p) = p^2 \nu^m - p4\nu^m \underline{\tilde{\pi}} + 4\nu^m \underline{\tilde{\pi}}^2 = \nu^m (p - 2\underline{\tilde{\pi}})^2 \geq 0, \tag{79}$$

which gives $b^\alpha \geq \hat{b}$.

Next, note $G(p,b) \equiv \tilde{\pi}^e(s^o)$, where $\tilde{\pi}^e(s^o)$ is given by (77). The feasibility condition (78) for $s = s^p$ then

[34] The other solution to the polynomial can be both positive and feasible, hence there is possibly multiple stationary inflation regimes due to the Laffer curve property of seignorage.

reads:

$$G(b,p) = \frac{p + \sqrt{p^2 + 4(1-p)\underline{\tilde{\pi}} - 4\frac{Rb}{\nu^m}}}{2} \geq \sqrt{\frac{Rb\underline{\tilde{\pi}}}{\nu^m(1-\underline{\tilde{\pi}})}}. \tag{80}$$

In this expression, the left side $G(b,p)$ is decreasing in b, whereas the right side is increasing in b; $G(0,p) > 0$ and the right side is equal to 0 for $b = 0$; $G(\hat{b},p) \geq \underline{\tilde{\pi}}$ and the right side is equal to $\underline{\tilde{\pi}}$, for $b = \hat{b}$. Hence for all $b < \hat{b}$, (80) is satisfied.

Finally, the feasibility condition (78) for $s = s^o$ requires $b \leq b^\delta = \frac{\nu^m}{4R}$ and:

$$\frac{1 - \sqrt{1 - 4\frac{Rb}{\nu^m}}}{2} \leq G(b,p) \leq \frac{1 + \sqrt{1 - 4\frac{Rb}{\nu^m}}}{2}. \tag{81}$$

Since $\underline{\tilde{\pi}}(1-\underline{\tilde{\pi}}) \leq \frac{1}{4}$, we have $\hat{b} \leq b^\delta$. Note $\mathcal{B}_l(b)$ and $\mathcal{B}_u(b)$ the lower and upper bounds of this inequality.

$\mathcal{B}_l(b)$, is increasing in b, $\mathcal{B}_l(0) = 0$, $\mathcal{B}_l(\hat{b}) = \frac{1-\sqrt{1-4\underline{\tilde{\pi}}(1-\underline{\tilde{\pi}})}}{2} = \frac{1-\sqrt{(1-2\underline{\tilde{\pi}})^2}}{2} \leq \underline{\tilde{\pi}}$ for all $\underline{\tilde{\pi}} \in [0,1]$. As $G(b,p)$ is decreasing in b and $G(\hat{b},p) \geq \underline{\tilde{\pi}}$, we have that for all $b \in [0,\hat{b}]$, $G(b,p) \geq \mathcal{B}_l(b)$.

We finally verify that $G(b,p) \leq \mathcal{B}_u(b)$ for all $b < \hat{b}$. Taking the derivatives of $G(b,p)$ w.r.t. p:

$$\frac{dG(\cdot)}{dp} = \frac{1}{2}\Big(1 + \frac{p - 2\underline{\tilde{\pi}}}{\sqrt{p^2 + 4(1-p)\underline{\tilde{\pi}} - 4\frac{Rb}{\nu^m}}}\Big). \tag{82}$$

If $p - 2\underline{\tilde{\pi}} > 0$, then $\frac{dG(\cdot)}{dp} > 0$. If $p - 2\underline{\tilde{\pi}} < 0$, then verify that $-1 \leq \frac{p-2\underline{\tilde{\pi}}}{\sqrt{p^2+4(1-p)\underline{\tilde{\pi}}-4\frac{Rb}{\nu^m}}} \leq 0$, so that again $\frac{dG(\cdot)}{dp} > 0$. Hence, for all $p \in [0,1]$, all $\underline{\tilde{\pi}} \in [0,1]$, all $b \in [0,\hat{b}]$:

$$G(b,p) \leq G(b,1) = \frac{1 + \sqrt{1 - 4\frac{Rb}{\nu^m}}}{2} = \mathcal{B}_u(b). \tag{83}$$

Overall, we have shown that for all $b \leq \hat{b}$, there is $\tilde{\pi}^e(s^o)$ that satisfies (77) and solves (71).

7.3 Existence of the "Leaning Against the Winds" Policy

This section details the proof of Lemma 6 in the general case $\theta \in [0,1]$ and $\nu^m \geq 0$.

We adopt the following notations. Consider the central bank committing to a policy contingent on A, noted $\{\tilde{\pi}_A\}$, and such that $\int_A \tilde{\pi}_A dF(A) = \tilde{\pi}^*$. Given $m_{-1} = \tilde{\pi}^e(\cdot) = \tilde{\pi}^*$, where $\tilde{\pi}^*$ is the inflation target of the central bank, the discretionary default decision of the treasury is captured by:

$$\begin{aligned}\Delta(A,i,\tilde{\pi}^*,\tau,\tilde{\pi}_A) &= W^d(\cdot) - W^r(\cdot) \\ &= \frac{[A(1-\gamma)]^2}{2} - \frac{[A(1-\tau)]^2}{2} + \nu^m\tilde{\pi}^*(1-\tilde{\pi}_A) - (1+i)\tilde{\pi}_A\theta b,\end{aligned} \tag{84}$$

where τ solves the government budget constraint given $\tilde{\pi}_A$:

$$G(A,i,\tilde{\pi}^*,\tau,\tilde{\pi}_A) = A^2(1-\tau)\tau + \nu^m\tilde{\pi}^*(1-\tilde{\pi}_A) - (1+i)\tilde{\pi}_A b = 0. \tag{85}$$

Moreover, in the economy with $\theta > 0$, default occurs for two reasons: either it is the best response of the treasury: $\Delta(\cdot) > 0$, or the fiscal capacity of the country cannot service debt, since $\tau \leq \frac{1}{2}$.

We show that there is a unique state-dependent inflation policy $\{\tilde{\pi}_A^p\}$ and an induced interest rate cut-off i^δ such that the policy delivers the inflation target on average, and, if the central bank commits to $\{\tilde{\pi}_A^p\}$, then the fiscal authority services its obligation for all A if and only if $i < i^\delta$.

We proceed in two steps: first we show that for any i^t, there is a unique policy $\{\tilde{\pi}_A(i^t)\}$ such that the treasury reimburses its debt if and only if $i < i^t$. Second, we show that there is a unique i^δ such that $\{\tilde{\pi}_A(i^\delta)\}$ satisfies the inflation target. The desired policy is given by $\tilde{\pi}_A^p = \tilde{\pi}_A(i^\delta)$ for all A.

Part I. Consider a nominal interest rate i^t such that $1 + i^t > 0$ and a realization $A \in [A_l, A_h]$.

(i) The following elements establish that there is a unique inflation level $\tilde{\pi}_A(i^t)$ such that the fiscal authority is indifferent between repayment and default.

First, there is an inverse inflation rate $\tilde{\pi}_A^1(i^t)$ such that debt is serviced with no taxes on labor income.

$$G(A, i^t, \tilde{\pi}^*, \tau, \tilde{\pi}_A^1(i^t)) = 0 \Rightarrow \tau = 0. \tag{86}$$

In this case, using Assumption 2, $\Delta(\cdot) < 0$. Using the government budget constraint with $\tau = 0$, one gets:

$$\tilde{\pi}_A^1(i^t) = \frac{\nu^m \tilde{\pi}^*}{\nu^m \tilde{\pi}^* + (1 + i^t)b} > 0. \tag{87}$$

Similarly, the central bank can set the inverse inflation rate to $\tilde{\pi}_A^2(i^t)$ so that if the treasury desires to service its debt, it has to set $\tau = \frac{1}{2}$. Formally:

$$\tilde{\pi}_A^2(i^t) = \frac{\frac{A^2}{4} + \nu^m \tilde{\pi}^*}{\nu^m \tilde{\pi}^* + (1 + i^t)b}. \tag{88}$$

Importantly, for any inflation rate between these two cases, the lower the inflation, i.e. the higher $\tilde{\pi}_A$, the higher the tax rate to service debt. Formally, differentiating the government budget constraint w.r.t. τ and $\tilde{\pi}_A$:

$$\forall \tilde{\pi}_A \in [\tilde{\pi}_A^1(i^t), \tilde{\pi}_A^2(i^t)],\ \frac{d\tau}{d\tilde{\pi}_A} = \frac{\nu^m \tilde{\pi}^* + (1 + i^t)b}{A^2(1 - 2\tau)} > 0. \tag{89}$$

Moreover, the lower the inflation, i.e. the higher $\tilde{\pi}_A$, the higher the value of $\Delta(\cdot) = W^d(\cdot) - W^r(\cdot)$:

$$\frac{d\Delta(\cdot)}{d\tilde{\pi}_A} = \frac{1 - \tau}{1 - 2\tau}(\nu^m \tilde{\pi}^* + (1 + i^t)b) - (\nu^m \tilde{\pi}^* + (1 + i^t)\theta b) > 0, \tag{90}$$

since $\frac{1-\tau}{1-2\tau} > 1$ for $\tau \in [0, \frac{1}{2})$.

Hence, there is a unique $\tilde{\pi}_A(i^t)$ that has the desired property to make the treasury indifferent between repayment and default. Especially,

- if $\Delta\big(A, i^t, \tilde{\pi}^*, \frac{1}{2}, \tilde{\pi}_A^2(i^t)\big) > 0$, then $\tilde{\pi}_A^1(i^t) < \tilde{\pi}_A(i^t) < \tilde{\pi}_A^2(i^t)$,
- if $\Delta\big(A, i^t, \tilde{\pi}^*, \frac{1}{2}, \tilde{\pi}_A^2(i^t)\big) \leq 0$, then $\tilde{\pi}_A(i^t) = \tilde{\pi}_A^2(i^t)$.

(ii) Next, we verify that for any $i < i^t$, the fiscal authority services its debt, otherwise for any $i > i^t$, it defaults. Given $\tilde{\pi}_A(i^t)$, we have:

$$\frac{d\Delta(\cdot)}{di} = A^2(1-\tau)\frac{d\tau}{di} - \tilde{\pi}_A(i^t)\theta b = \frac{1-\tau}{1-2\tau}\tilde{\pi}_A(i^t)b - \tilde{\pi}_A(i^t)\theta b > 0. \tag{91}$$

(iii) Also, we establish the following properties of $\tilde{\pi}_A(i^t)$:

$$\frac{d\tilde{\pi}_A(i^t)}{dA} > 0 \qquad\qquad \frac{d\tilde{\pi}^p_A(i^t)}{di^t} < 0. \tag{92}$$

If $\tilde{\pi}_A(i^t) = \tilde{\pi}^2_A(i^t)$, these properties are straightforward. In the case $\tilde{\pi}_A(i^t) < \tilde{\pi}^2_A(i^t)$, first differentiate the government budget constraint w.r.t. $(A, i, \tau, \tilde{\pi}_A)$ to get:

$$\frac{d\tau}{dA} = -\frac{2(1-\tau)\tau}{A(1-2\tau)} \qquad \frac{d\tau}{di} = \frac{\tilde{\pi}_A b}{A^2(1-2\tau)} \qquad \frac{d\tau}{d\tilde{\pi}_A} = \frac{\nu^m\tilde{\pi}^* + (1+i)b}{A^2(1-2\tau)} \tag{93}$$

Then differentiate $\Delta(A, i, \tilde{\pi}^*, \tau, \tilde{\pi}_A)$ w.r.t to its arguments and using the derivative of τ w.r.t $(A, i, \tilde{\pi}_A)$, one gets:

$$\Big[A(1-\gamma)^2 - A\frac{(1-\tau)^2}{1-2\tau}\Big]dA + \Big[\frac{1-\tau}{1-2\tau}\big(\nu^m\tilde{\pi}^* + (1+i)b\big) - \big(\nu^m\tilde{\pi}^* + (1+i)\theta b\big)\Big]d\tilde{\pi}_A = 0 \tag{94}$$

$$\Big[\frac{1-\tau}{1-2\tau}\tilde{\pi}_A b - \tilde{\pi}_A\theta b\Big]di + \Big[\frac{1-\tau}{1-2\tau}\big(\nu^m\tilde{\pi}^* + (1+i)b\big) - \big(\nu^m\tilde{\pi}^* + (1+i)\theta b\big)\Big]d\tilde{\pi}_A = 0. \tag{95}$$

Since $\frac{1-\tau}{1-2\tau} > \frac{(1-\tau)^2}{1-2\tau} > 1$ for all $0 \le \tau \le \frac{1}{2}$ and $0 \le \theta \le 1$, we get the desired results.

(iv) Finally, the limits behavior of $\tilde{\pi}_A(i^t)$ are derived from the inequality

$$\tilde{\pi}^1_A(i^t) < \tilde{\pi}_A(i^t) \le \tilde{\pi}^2_A(i^t), \tag{96}$$

which gives $\lim_{i^t\to+\infty} \tilde{\pi}_A(i^t) = 0$ and $\lim_{i^t\to-1} \tilde{\pi}_A(i^t) > 1$.

Part II. By applying the inflation target requirement (57), we show that there is a unique $i^\delta > 0$ such that:

$$\int_A \tilde{\pi}_A(i^\delta)dF(A) = \tilde{\pi}^*. \tag{97}$$

Note $H(i) = \int_A \tilde{\pi}_A(i)dF(A)$, which is defined for all i such that $1 + i > 0$. The properties of $\tilde{\pi}_A(i)$ naturally convey to $H(i)$: $H(i)$ is strictly decreasing in i; $\lim_{i\to+\infty} H(i) = 0$; $\lim_{i\to-1} H(i) > 1$.

Hence there is a unique i^δ such that $H(i^\delta) = \tilde{\pi}^*$.

Overall, the monetary policy rule $\{\tilde{\pi}^p_A\}$ that meets the inflation target and deters state contingent default, exists, and satisfies:

$$\forall\ A\ \tilde{\pi}^p_A = \tilde{\pi}^p_A(i^\delta). \tag{98}$$

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