The Limitations of Forward Guidance

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ABSTRACT

This paper examines the economic effects of Odyssean forward guidance—a promise by the central bank to keep future policy rates lower than its policy rule suggests. We focus on the impact of forward guidance when the policy rate is at its zero lower bound (ZLB), but also show its effects when the ZLB does not bind. Our analysis is conducted using a New Keynesian model where forward guidance enters as news shocks to the monetary policy rule. Three key findings emerge: (1) Conventional monetary policy is more stimulative than forward guidance away from the ZLB; (2) If the economy is in a deep recession or households expect a slow recovery, then the stimulative effect of forward guidance is minimal because the policy rate is likely to remain at its ZLB even without forward guidance; and (3) Longer forward guidance horizons do not reverse the stimulative effect or cause it to explode, but instead spread the effect across the horizon. We then compare the predictions of our model to survey data collected before and after recent FOMC announcements. Our analysis reveals that the FOMC's pessimism about near-term economic growth likely dampened household expectations, which limited the stimulative effect of forward guidance.

Keywords: Forward Guidance; News Shocks; Zero Lower Bound; Yield Curve
JEL Classifications: E43; E47; E58; E61

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1 INTRODUCTION

This article examines the effect of central bank forward guidance at and away from the zero lower bound (ZLB) on the nominal interest rate. Forward guidance refers to central bank communication about future policy, which takes many forms including announcements about objectives, contingencies, policy actions, and speeches. The decision of central banks to announce their expected policy rate path is a recent phenomenon that has accompanied the evolution of inflation targeting.

Campbell et al. (2012) introduce terms to differentiate the types of forward guidance: Delphic and Odyssean. Delphic forward guidance is a central bank's forecast of its own policy, which is based on its projections for inflation and real GDP growth as well as an established policy rule. One reason to use Delphic forward guidance together with economic projections is to clarify the central bank's policy strategy.[1] Odyssean forward guidance is a commitment to deviate from the established policy rule at some point in the future by promising to set the policy rate lower than the policy rule recommends. Central banks utilize Odyssean forward guidance when the short-term nominal interest rate is constrained at its ZLB because they can no longer lower the policy rate.

Several papers argue that Odyssean forward guidance can raise output when the policy rate is at its ZLB.[2] To stimulate demand, a central bank commits to a lower policy rate path than the interest rate path recommended by the policy rule. The promise to keep the future policy rate low reduces the expected policy rate and raises demand in the near-term. We also find that Odyssean forward guidance can stimulate the economy but caution that situations exist where its effects are limited.

We examine the economic effects of forward guidance using a New Keynesian model with a ZLB constraint on the nominal interest rate. Forward guidance enters our model as news shocks to the monetary policy rule similar to Laséen and Svensson (2011), so that it operates through an expectations channel in the same way that researchers have modeled the effect of news about future technology. Although many papers use nonlinear methods to examine the effects of a ZLB constraint,[3] this paper is the first to study forward guidance in a constrained nonlinear model.

Some researchers who study forward guidance do not impose the ZLB constraint [Laséen and Svensson (2011) and Del Negro et al. (2012)]. Although that assumption makes the model numerically tractable, it means current and expected nominal interest rates can be negative, which causes the model to overstate the stimulative effect of forward guidance. We focus on the impact of forward guidance when the nominal interest rate is at the ZLB, but also show its effects when the ZLB does not bind. Three key findings emerge from our analysis of Odyssean forward guidance:

1. Conventional monetary policy is more stimulative than forward guidance away from the ZLB.

2. If the economy is in a deep recession or households expect a slow recovery, then the stimulative effect of forward guidance is minimal because the short-term nominal interest rate is likely to remain at its ZLB even without forward guidance.

3. Longer forward guidance horizons do not reverse the stimulative effect or cause it to explode, but instead, spread the effect across the horizon by flattening the yield curve.

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[1]See the Bank of England (2013) for a discussion of how forward guidance helps the public form more accurate expectations about future central bank policy. See den Haan (2013) for a collection of essays about forward guidance. The International Monetary Fund (2013) provides a detailed account of recent unconventional monetary policies.

[2]See Krugman (1998), Reifschneider and Williams (2000), Eggertsson and Woodford (2003), Nakov (2008), Walsh (2009), Levin et al. (2010), Werning (2011), Campbell et al. (2012), Del Negro et al. (2012), and Woodford (2012).

[3]See, for example, Wolman (2005), Basu and Bundick (2012), Fernández-Villaverde et al. (2012), Nakata (2012), Aruoba and Schorfheide (2013), Gust et al. (2013), Mertens and Ravn (2014), and Gavin et al. (2014).

We then compare the predictions of our model to Blue Chip survey data collected immediately before and after recent episodes of date-based forward guidance by the Fed. Our analysis reveals that the Fed's pessimism about near-term economic growth expressed in their policy statements likely dampened household expectations, which limited the stimulative effect of forward guidance.

Many papers that study forward guidance assume the central bank pledges to maintain an interest rate peg for a specified period of time [Laséen and Svensson (2011), Carlstrom et al. (2012), and Del Negro et al. (2012)]. The effect of forward guidance on the nominal interest rate is measured as the difference between the interest rate peg and the interest rate that would occur without any forward guidance. To preserve the peg, the central bank must provide news over the entire forward guidance horizon to offset shocks that affect the interest rate. Thus, there are two dimensions to a forward guidance policy: its horizon and the intensity of the news (i.e., the variance of the news process). Del Negro et al. (2012) show that when they extend the forward guidance horizon, it leads to predictions that overstate the increase in output and inflation. Their experiment, however, relies on a stronger intensity of news to maintain the interest rate peg in the future. In our analysis, we weight the news shocks to keep the variance of the news process constant, which allows us to isolate the effects of a longer forward guidance horizon from a change in the intensity of the news.

Our analysis of forward guidance builds on the seminal work of Eggertsson and Woodford (2003), which characterizes optimal commitment policy when the nominal interest rate is at its ZLB. Some papers refer to these promises as forward guidance [Levin et al. (2010) and English et al. (2013)]. Their analysis is thorough, but it is based on a linearized version of a constrained New Keynesian model in which there is no expectation of returning to the ZLB after the nominal interest rate rises. The limitations of those features, which are discussed in Braun et al. (2012), Fernández-Villaverde et al. (2012), and Gavin et al. (2014), motivate us to solve a nonlinear model with endogenous ZLB events to more accurately account for the expectational effects of the ZLB.

The paper is organized as follows. Section 2 provides a post-financial crisis account of FOMC forward guidance in its policy statements. Section 3 describes the nonlinear model, including the specification of forward guidance, its calibration, and its solution method. Section 4 quantifies the effect of Odyssean forward guidance. Specifically, we describe the decision rules and the impulse responses to news shocks across alternative forward guidance horizons. Section 5 focuses on the Federal Open Market Committee's (FOMC) date-based forward guidance announcements in order to examine how our model's predictions compare with the data. Section 6 concludes.

2 FORWARD GUIDANCE AT THE FEDERAL RESERVE

The FOMC uses two primary methods to communicate information about the path of future policy rates. One, it releases the individual forecasts of its members four times per year, but that information can be diverse and reveals differences in opinions. Two, it provides forward guidance about the future federal funds rate in its policy statements and has consistently done so since 2008.[4]

At the December 16, 2008 meeting, the FOMC cut the federal funds rate to 0.25% and announced it would remain at that unusually low level for an *extended* period. The FOMC continued to use that vague language until its August 9, 2011 statement, which said the low range was likely

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[4]Various forms of qualitative and quantitative forward guidance have also been used by the Bank of Canada, Bank of England, European Central Bank, Bank of Japan, Reserve Bank of New Zealand, Norges Bank, and the Riksbank. See Andersson and Hofmann (2010), Filardo and Hofmann (2014), Kool and Thornton (2012), Moessner and Nelson (2008), and Svensson (2011) for an overview of their policies and econometric analysis of their economic impacts.

warranted "...*at least through mid-2013*." That announcement was the first time the FOMC used date-based forward guidance, and it had a major effect on current and expected future interest rates.

The next change in forward guidance occurred in the statement released after the January 25, 2012 FOMC meeting. That policy statement was different in two ways. One, the time that the federal funds rate was expected to remain at zero was updated to read "...*at least through late 2014*," which was an increase of six quarters. Two, the FOMC expressed a more pessimistic economic outlook and indicated the projected path for the federal funds rate was conditional on that outlook. Subsequent speeches by policymakers provided support for that assessment.

The forward guidance provided in the January 25, 2012 statement was likely viewed as Delphic for two reasons. One, the statement expressed more pessimism about the economy, which suggests the FOMC's policy rule was already projecting a later date for raising the federal funds rate. Two, the FOMC never stated the new projected interest rate path was different from the path implied by its policy rule. By late summer 2012, the economy continued to disappoint policymakers and the statement issued following the September 13, 2012 meeting was amended to read:

> To support continued progress toward maximum employment and price stability, ...*a highly accommodative stance of monetary policy will remain appropriate for a considerable time after the economic recovery strengthens*. ...the Committee also decided today to keep the target range for the federal funds rate at 0 to $1/4$ percent and currently anticipates that exceptionally low levels for the federal funds rate are likely to be *warranted at least through mid-2015*.

That statement included a 2-quarter extension to the time the FOMC promised to maintain a zero federal funds rate and a new pledge to add \$85 billion to the Fed's balance sheet every month until the labor market significantly improved. The language "...for a considerable time after the economic recovery strengthens" conveys Odyssean forward guidance. Without that language, it suggests the FOMC would raise its policy rate as the recovery strengthens. On the other hand, the FOMC likely dampened economic expectations because the statement expressed more pessimism about business spending, and FOMC participants lowered their GDP growth forecasts.

On December 12, 2012, the FOMC switched its forward guidance from the date-based language "at least through mid-2015" to threshold-based language. The policy statement said:

> ...this exceptionally low range for the federal funds rate will be appropriate *at least as long as the unemployment rate remains above* $6\text{-}1/2$ *percent*, inflation between one and two years ahead is projected to be no more than a half percentage point above the Committee's 2 percent longer-run goal, and longer-term inflation expectations continue to be well anchored.

FOMC participants' forecasts indicated the unemployment rate would likely hit 6.5% in mid-2015. Thus, the statement was not intended to change expectations about when the policy rate would rise, but rather to emphasize that the timing of the FOMC's decision to increase its policy rate is conditional on inflation expectations and labor market conditions. The phrase "at least as long as the unemployment rate remains above $6\text{-}1/2$ percent" was used to emphasize that the unemployment threshold was not a trigger for when the FOMC would automatically raise its policy rate.

Over the next year, the labor market continued to improve and it was evident the unemployment rate may cross the 6.5% threshold. On December 18, 2013, the FOMC decided to begin tapering their monthly asset purchases and reformulated their forward guidance communication to explain how they intended to react to future economic developments. Specifically, the committee said

> . . . it likely will be appropriate to maintain the current target range for the federal funds rate *well past the time that the unemployment rate declines below* 6-1/2 *percent*, especially if projected inflation continues to run below the Committee's 2 percent longer-run goal.

The change in language from "at least as long as" to "well past" may have been viewed as Odyssean in nature because the FOMC implied that the federal funds rate would likely remain near zero even though stronger economic conditions would normally cause the FOMC to raise its policy rate.

In 2014, the FOMC continued to reduce their asset purchases and focused on communicating state-contingent forward guidance. For example, the July 18, 2014 statement said

> . . . it likely will be appropriate to maintain the current target range for the federal funds rate for a *considerable time after the asset purchase program ends. . .*

FOMC participant forecasts of output and inflation for 2015 were near their long-run levels, but their federal funds rate forecasts remained about 2 percentage points below its long-run level. We believe this statement also conveys Odyssean forward guidance since the FOMC communicated that they expected to maintain a low policy rate well after concluding their asset purchase program.

Market reaction over the past few years to the FOMC's policy statements indicates the committee had limited success at stimulating the economy with forward guidance. We contend that the severity of the recession and an expectation of a slow economic recovery are the primary reasons.

3 ECONOMIC MODEL, CALIBRATION, AND SOLUTION METHOD

We use a textbook New Keynesian model in which the ZLB on the nominal interest rate occasionally binds due to persistent discount factor shocks. Our innovation is to introduce Odyssean forward guidance on the short-term nominal interest rate via monetary policy shocks that households observe before they impact the economy while also imposing the ZLB constraint.

3.1 HOUSEHOLDS A unit measure of households choose $\{c_t, n_t, b_t\}_{t=0}^{\infty}$ to maximize expected lifetime utility, $E_0 \sum_{t=0}^{\infty} \tilde{\beta}_t [\log c_t - \chi n_t^{1+\eta}/(1+\eta)]$, where c_t is consumption, n_t is labor hours, b_t is a 1-period real bond, $1/\eta$ is the Frisch elasticity of labor supply, E_0 is an expectations operator conditional on information in period 0, $\tilde{\beta}_0 \equiv 1$, and $\tilde{\beta}_t = \prod_{i=1}^{t} \beta_i$ for $t > 0$. Following Eggertsson and Woodford (2003), we model changes in demand with a time-varying discount factor, β_t, that evolves according to

$$\beta_t = \bar{\beta}(\beta_{t-1}/\bar{\beta})^{\rho_\beta} \exp(\upsilon_t), \tag{1}$$

where $\bar{\beta}$ is the steady-state discount factor, $0 \leq \rho_\beta < 1$, and $\upsilon_t \sim \mathbb{N}(0, \sigma_\upsilon^2)$. Those choices are constrained by $c_t + b_t = w_t n_t + r_{t-1} b_{t-1}/\pi_t + d_t$, where $\pi_t = p_t/p_{t-1}$ is the gross inflation rate, w_t is the real wage rate, r_t is the gross nominal interest rate, and d_t are profits from intermediate firms. The optimality conditions to each household's problem imply

$$w_t = \chi n_t^\eta c_t, \tag{2}$$
$$1 = r_t E_t[\beta_{t+1}(c_t/c_{t+1})/\pi_{t+1}]. \tag{3}$$

3.2 FIRMS The production sector consists of monopolistically competitive intermediate goods firms and a final goods firm. Intermediate firm $i \in [0, 1]$ produces a differentiated good, $y_t(i)$, according to $y_t(i) = n_t(i)$, where $n_t(i)$ is the labor used by firm i. Each intermediate firm chooses

its labor input to minimize operating costs, $w_t n_t(i)$, subject to its production function. The final goods firm purchases $y_t(i)$ units from each intermediate goods firm to produce the final good, $y_t \equiv [\int_0^1 y_t(i)^{(\theta-1)/\theta} di]^{\theta/(\theta-1)}$ according to a Dixit and Stiglitz (1977) aggregator, where $\theta > 1$ measures the elasticity of substitution between the intermediate goods. The optimality condition to the firm's profit maximization problem then yields the demand function for intermediate inputs given by $y_t(i) = (p_t(i)/p_t)^{-\theta} y_t$, where $p_t = [\int_0^1 p_t(i)^{1-\theta} di]^{1/(1-\theta)}$ is the price of the final good.

Following Rotemberg (1982), each firm faces a cost to adjusting its price, $adj_t(i)$, which emphasizes the negative effect that price changes can have on customer-firm relationships. Following Ireland (1997), $adj_t(i) = \varphi[p_t(i)/(\bar{\pi} p_{t-1}(i)) - 1]^2 y_t/2$, where $\varphi \geq 0$ scales the size of the adjustment cost and $\bar{\pi}$ is the steady-state gross inflation rate. Real profits are then given by $d_t(i) = [(p_t(i)/p_t)y_t(i) - w_t n_t(i) - adj_t(i)]$. Each firm i chooses its price, $p_t(i)$, to maximize the expected discounted present value of real profits $E_t \sum_{k=t}^{\infty} \lambda_{t,k} d_k(i)$, where $\lambda_{t,t} \equiv 1$, $\lambda_{t,t+1} = \beta_{t+1}(c_t/c_{t+1})$ is the stochastic pricing kernel between periods t and $t+1$ and $\lambda_{t,k} \equiv \prod_{j=t+1}^{k} \lambda_{j-1,j}$. In a symmetric equilibrium, where intermediate firms make identical decisions, the optimality condition implies

$$\varphi\left(\frac{\pi_t}{\bar{\pi}} - 1\right)\frac{\pi_t}{\bar{\pi}} = (1-\theta) + \theta w_t + \varphi E_t\left[\lambda_{t,t+1}\left(\frac{\pi_{t+1}}{\bar{\pi}} - 1\right)\frac{\pi_{t+1}}{\bar{\pi}}\frac{y_{t+1}}{y_t}\right]. \tag{4}$$

In the absence of price adjustment costs (i.e., $\varphi = 0$), the real wage rate equals $(\theta-1)/\theta$, which is the inverse of a firm's markup of price over marginal cost.

3.3 MONETARY POLICY
Households receive forward guidance (news) about future monetary policy through discretionary monetary policy shocks. Specifically, the central bank sets the gross nominal interest rate according to the following Taylor rule, subject to the ZLB constraint:

$$r_t = \max\{1, \bar{r}(\pi_t/\pi^*)^{\phi_\pi}\exp(x_t)\},$$

$$x_t \equiv \sum_{j=0}^{q} \alpha_j \varepsilon_{t-j}, \quad \sum_{j=0}^{q} \alpha_j^2 = 1, \tag{5}$$

where π^* is the gross inflation rate target, ϕ_π is the policy response to inflation, $\varepsilon_t \sim i.i.d.\ \mathbb{N}(0, \sigma_\varepsilon^2)$ is a monetary policy shock, α_j is the intensity of the news j periods in the future, and $q > 0$ is the forward guidance horizon. For example, when $(\alpha_0, \alpha_1, \ldots, \alpha_q) = (1, 0, \ldots, 0)$, the shock is unanticipated (no forward guidance) and when $(\alpha_0, \alpha_1, \ldots, \alpha_q) = (0, 0, \ldots, 1)$, households anticipate the shock q periods before it occurs (q-period forward guidance). The restriction on α_j guarantees the variance of the MA(q) process, x_t, is the same as ε_t. In other words, the distribution of the news does not affect the variance of the monetary policy shock process. That restriction is particularly important because it isolates the effect of lengthening the forward guidance horizon.[5]

Several papers that examine forward guidance assume the central bank commits to an explicit interest rate peg for a specific period of time. Such a rigid assumption does not allow households' expectations to account for the possibility that unanticipated future economic conditions will cause a shift in monetary policy. Our framework avoids this limitation by assuming the central bank provides forward guidance through news about monetary policy shocks rather than through a specific

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[5]The forward guidance provided in the FOMC's policy statements is usually conditional on a future state of the economy. The news shocks in this paper are unconditional. With threshold-based forward guidance, the central bank states that it plans to maintain a certain policy at least until the economy passes some economic threshold. One way to model threshold-based forward guidance is to make the news shocks dependent on the state of the economy.

interest rate peg. That difference enables households' expectations to account for the possibility of further news. For example, suppose the central bank provides information at time t that it plans to keep its policy rate lower than its policy rule suggests for the next 4 quarters. Households in our model know the central bank could reverse that news in any of the intervening periods.

3.4 EQUILIBRIUM, CALIBRATION, AND SOLUTION METHOD

The resource constraint is given by $c_t = [1 - \varphi(\pi_t/\bar{\pi} - 1)^2/2]y_t = \tilde{y}_t$, where $\tilde{y}_t$ includes the value added by intermediate firms, which is their output minus quadratic price adjustment costs. A competitive equilibrium consists of sequences of quantities $\{c_t, n_t, y_t, b_t\}_{t=0}^{\infty}$, prices $\{w_t, r_t, \pi_t\}_{t=0}^{\infty}$, and discount factors $\{\beta_t\}_{t=0}^{\infty}$ that satisfy the exogenous driving process for the discount factor, (1), each household's and each firm's optimality conditions, (2)-(4), the monetary policy rule, (5), the production function, $y_t = \int_0^1 n_t(i) = n_t$, the bond market clearing condition, $b_t = 0$, and the resource constraint, $c_t = \tilde{y}_t$.

We calibrate the model at a quarterly frequency using values commonly found in the monetary policy literature. The risk-free real interest rate is set to 4 percent annually, which implies a steady-state quarterly discount factor, $\bar{\beta}$, equal to 0.99. The Frisch elasticity of labor supply, $1/\eta$, is set to 1 and the leisure preference parameter, χ, is set so that steady-state labor equals $1/3$ of the available time. The price elasticity of demand between intermediate goods, θ, is calibrated to 6, which corresponds to an average markup of price over the wage rate equal to 20 percent. The costly price adjustment parameter, φ, is set to 58.25, which is similar to a Calvo (1983) price-setting specification in which prices change on average once every four quarters ($\omega = 0.75$). In the policy sector, the steady-state gross inflation rate, $\bar{\pi}$, is calibrated to 1.005 so that the annual inflation rate target is 2 percent. The monetary response to changes in inflation, ϕ_{π}, is set to 1.5.

Richter and Throckmorton (2014) show that as the persistence of a shock process increases, the standard deviation of that shock must decline, otherwise our numerical algorithm will not converge to an minimum state variable (MSV) solution. The failure to converge occurs because the economy either remains at the ZLB too long when the shocks are very persistent or falls to the ZLB too frequently when the processes are highly volatile. We set the persistence of the discount factor, ρ_{β}, equal to 0.8 and the standard deviation of the shock, σ_v, equal to 0.0025. Those values follow Fernández-Villaverde et al. (2012) who assume that a discount factor shock has a half life of about 3 quarters. The standard deviation of the monetary policy shock, σ_{ε}, is set to 0.0025.

We solve the model using the policy function iteration algorithm described in Richter et al. (2013), which is based on the theoretical work on monotone operators in Coleman (1991). This solution method discretizes the state space and uses time iteration to solve for the updated policy functions until the tolerance criterion is met. We use linear interpolation to approximate future variables, since this method accurately captures the kink in the policy functions, and Gauss-Hermite quadrature to numerically integrate. See Appendix A for a formal description of the algorithm.

4 DECISION RULES, IMPULSE RESPONSES, AND FORWARD GUIDANCE

4.1 ONE-QUARTER HORIZON ($q = 1$)

Figure 1 plots the decision rules for consumption, the inflation rate, and the current and expected future nominal interest rates as a function of the monetary policy shock, $\hat{\varepsilon}_t$, which ranges from $\pm 1\%$.[6] The subscript on the shock represents the period in which households learn about it and not necessarily the period the shock impacts the economy.

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[6]In all our results, a hat denotes percent deviation from the deterministic steady state (i.e., for some generic variable x in levels, $\hat{x}_t \equiv 100((x_t - \bar{x})/\bar{x})$ and a tilde denotes a net rate (i.e., for some gross rate x, $\tilde{x}_t = 100(x_t - 1)$).

If, for example, the central bank provides no forward guidance, then $\hat{\varepsilon}_t$ represents an unanticipated monetary policy shock, which is observed and impacts the economy in period t. When the central bank provides 1-quarter forward guidance, $\hat{\varepsilon}_t$ represents a news shock that households learn about in period t but does not impact the economy until period $t+1$.
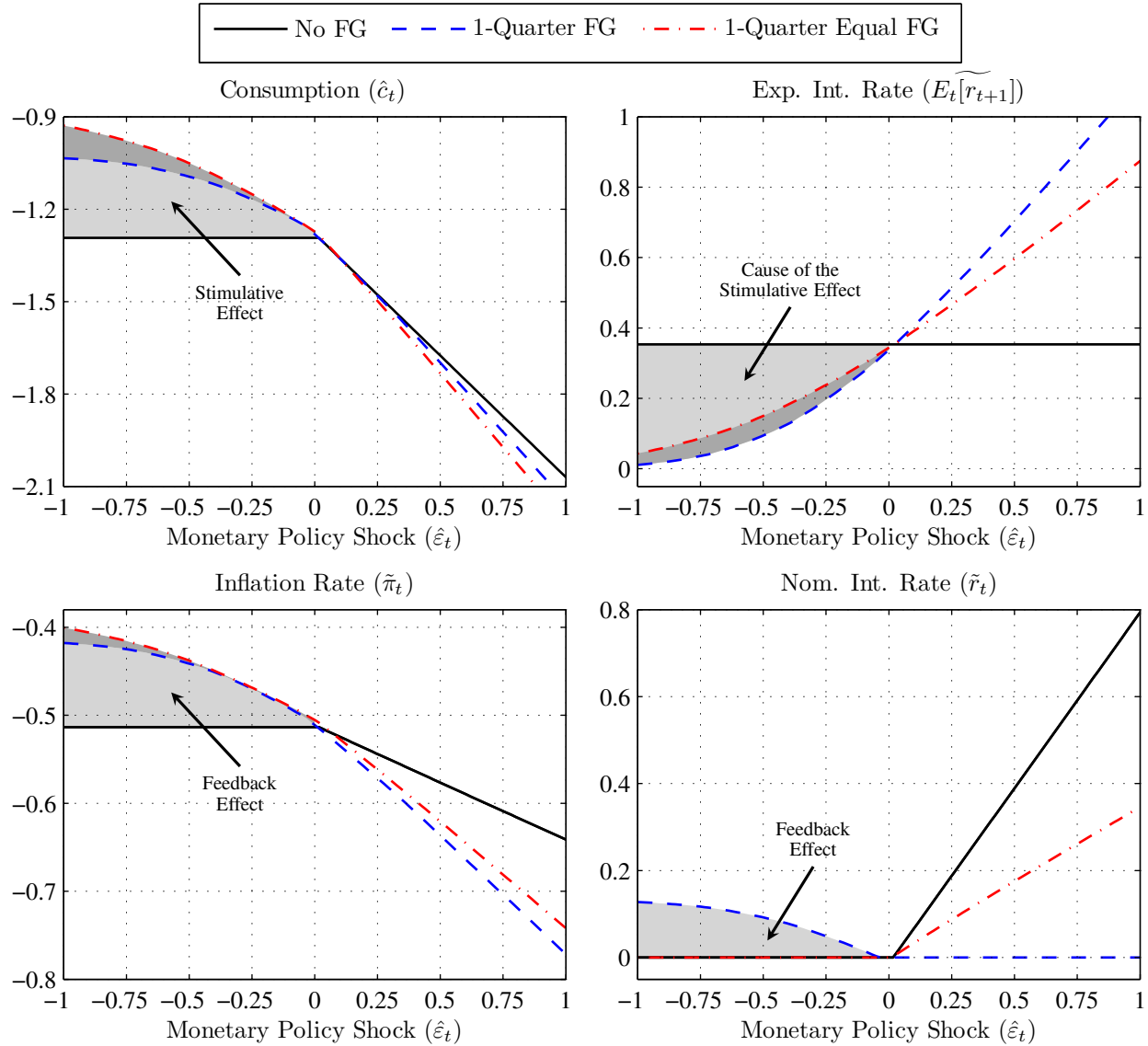

Figure 1: Decision rules as a function of the monetary policy shock with no forward guidance, $(\alpha_0, \alpha_1) = (1, 0)$ (solid line), 1-quarter forward guidance, $(\alpha_0, \alpha_1) = (0, 1)$ (dashed line), and 1-quarter equal forward guidance, $(\alpha_0, \alpha_1) = (\sqrt{1/2}, \sqrt{1/2})$ (dash-dotted line). The initial shadow rate equals zero in these decision rules.

We focus on decision rules where the initial shadow interest rate $(\tilde{r}_0^*)$—the nominal interest rate a central bank would select if there was no ZLB constraint—equals zero. When the initial shadow rate is zero, the discount factor is 1.15% above its steady state. The high discount factor depresses consumption by about 1.3%. Households expect the discount factor will follow its law of motion and revert to its steady state over time. If the central bank does not intervene, that belief raises the expected nominal interest rate in all future periods above the ZLB. For example, next period's

expected nominal interest rate increases by 0.35 percentage points. Our initial value for β was chosen because it enables forward guidance to produce the largest possible stimulative effect at the ZLB. Later in the paper, we will discuss reasons why the stimulative effect is typically smaller.

When $(\alpha_0, \alpha_1) = (1, 0)$ (solid line), the central bank provides no forward guidance, so $\hat{\varepsilon}_t$ represents an unanticipated policy shock. When $\hat{\varepsilon}_t > 0$, the monetary policy shock contracts economic activity by pushing up the current nominal interest rate and lowering current consumption. The expected nominal interest rate is unaffected since the shock is serially uncorrelated. If, on the other hand, $\hat{\varepsilon}_t < 0$, then monetary policy has no impact on the current nominal interest rate since it is already at its ZLB. Thus, the decision rules are flat when $\hat{\varepsilon}_t < 0$.

When $(\alpha_0, \alpha_1) = (0, 1)$ (dashed line), the central bank provides households with 1-quarter forward guidance. A central bank announcement this period about future monetary policy shocks can affect the current economy even though the shock has not yet happened. Suppose, for example, households receive information in period t that an expansionary monetary policy shock, $\hat{\varepsilon}_t < 0$, will occur in period $t + 1$. If the discount factor is *not* expected to revert to its mean (i.e., the economy is expected to remain stagnant), then forward guidance will have hardly any stimulative effect since households expect that next period's nominal interest rate will remain near its ZLB. In our model, the discount factor is above its mean and households expect it to decline in the future. That belief increases expected nominal interest rates. When households are informed this period about an expansionary monetary policy shock next period, they expect next period's nominal interest rate to increase less than if they received no forward guidance. That expectational effect stimulates consumption, which pushes up inflation and the current nominal interest rate—what we call *feedback effects*—even though the discount factor remains 1.15% above its steady state.

Another way to understand how forward guidance stimulates current consumption is through an intertemporal consumption smoothing motive. Households know an expansionary monetary policy shock will occur in period $t + 1$ and expect higher future consumption. Consequently, households raise current consumption to smooth their consumption across time. The increase in demand pushes up inflation, which feeds into the Taylor rule and drives up the current nominal interest rate. The light-shaded regions in figure 1 represent the effects of 1-quarter forward guidance.

The feedback effect of higher consumption on the nominal interest rate dampens the stimulative effect of 1-quarter forward guidance. Although the central bank could provide *additional* news in the current period that would mitigate the feedback effect (e.g., set $(\alpha_0, \alpha_1) = (1, 1)$), they could also *redistribute* the news so that the variance of the news process, x_t, remains constant while still enhancing the stimulative effect. An example of that policy is $(\alpha_0, \alpha_1) = (\sqrt{1/2}, \sqrt{1/2})$ (dash-dotted line), which we call 1-quarter equal forward guidance. That specification equally shocks the policy rule in periods t and $t + 1$ but keeps the variance of the news process constant.

The dark-shaded regions in figure 1 represent the marginal effects on the decision rules of switching from 1-quarter forward guidance to 1-quarter equal forward guidance. The effects of 1-quarter equal forward guidance differ in two ways. One, the current expansionary policy shock with 1-quarter equal forward guidance further stimulates the economy because it eliminates the feedback effect that causes the current nominal interest rate to rise with 1-quarter forward guidance. Without that feedback effect, current consumption increases more. Two, the smaller expansionary policy shock next period with 1-quarter equal forward guidance moderates both the decline in the expected nominal interest rate and the increase in current consumption. Overall, the stimulative effect from the lower current nominal interest rate dominates the negative effects of a smaller drop in the expected nominal rate so consumption rises more with 1-quarter equal forward guidance.
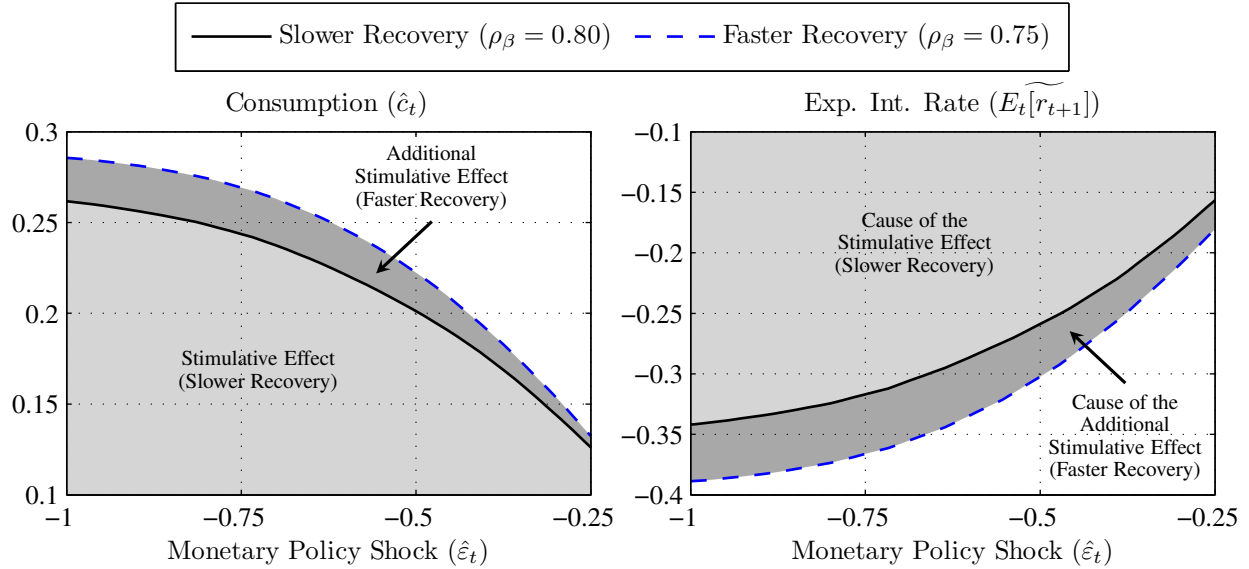
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Figure 2: The stimulative effect of forward guidance given a slower recovery (solid line) and a faster recovery (dashed line). The initial shadow rate equals zero in these decision rules. The vertical axis is the difference between 1-quarter forward guidance, $(\alpha_0, \alpha_1) = (0, 1)$, and no forward guidance, $(\alpha_0, \alpha_1) = (1, 0)$.

Figure 2 shows the stimulative effect of forward guidance is stronger when households expect a faster economic recovery.[7] The left panel displays the decision rules for consumption while the right panel shows the decision rules for next period's expected nominal interest rate. The vertical axis on the left (right) panel illustrates the percentage (percentage point) difference in the decision rules from the model with no forward guidance. The light-shaded region is same as the region shown in figure 1. The dark-shaded illustrates how the stimulative effect of forward guidance changes when households expect a faster recovery. Just like in figure 1, the discount factor is abnormally high in period t, which depresses current consumption. A lower ρ_β is a proxy for a quicker recovery because the discount rate is expected to return to its steady state faster. A more rapid decline in the discount factor encourages households to raise their expected consumption growth rate, which further drives up next period's expected nominal interest rate. The larger jump in the expected nominal rate suggests that a promise by the central bank to maintain a low policy rate in the future will have a greater stimulative effect on the economy in the current period. For example, an announcement this period by the central bank to change its policy rate by -0.5% next period causes current consumption to rise by 0.20% when $\rho_\beta = 0.80$ and by 0.22% when $\rho_\beta = 0.75$. Our finding suggests that if a central bank expresses a more pessimistic economic outlook when communicating information about future expansionary monetary policy, then the expectation of a slower recovery will dampen the stimulative effect of forward guidance.

The stimulative effect of forward guidance also depends on the initial state of the economy. Figure 3 compares the stimulative effect of forward guidance at four alternative cross sections of the decision rules that are based on different initial shadow interest rates ($\tilde{r}_0^*$): (1) The economy is in steady state ($\tilde{r}_0^* = 1.5$, solid line); (2) The economy is in a low state that is not severe enough for the ZLB to bind ($\tilde{r}_0^* = 0.25$, which corresponds to the Fed's policy rate in late 2003, dashed line); (3) The economy is in a recession just deep enough for the ZLB to bind ($\tilde{r}_0^* = 0$,

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[7]Levin et al. (2010) make a similar point in their study of optimal policy in a New Keynesian model with a ZLB.

circle markers); (4) The economy is in a severe recession where the central bank is constrained by the ZLB ($\tilde{r}_0^* = -0.5$, which is based on its estimated value during the Great Recession, triangle markers).[8] Each shadow interest rate is inversely related to an initial discount factor, which is a proxy for the current level of demand because it determines households' willingness to postpone consumption. When the discount factor is high, households prefer to save more, which indicates that they are less confident in the economy. As in figure 2, the vertical axis of the left (right) panel in figure 3 shows the percentage (percentage point) difference between the decision rules for 1-quarter forward guidance and no forward guidance. Forward guidance is more (less) stimulative than conventional monetary policy when the vertical axis values are positive (negative).
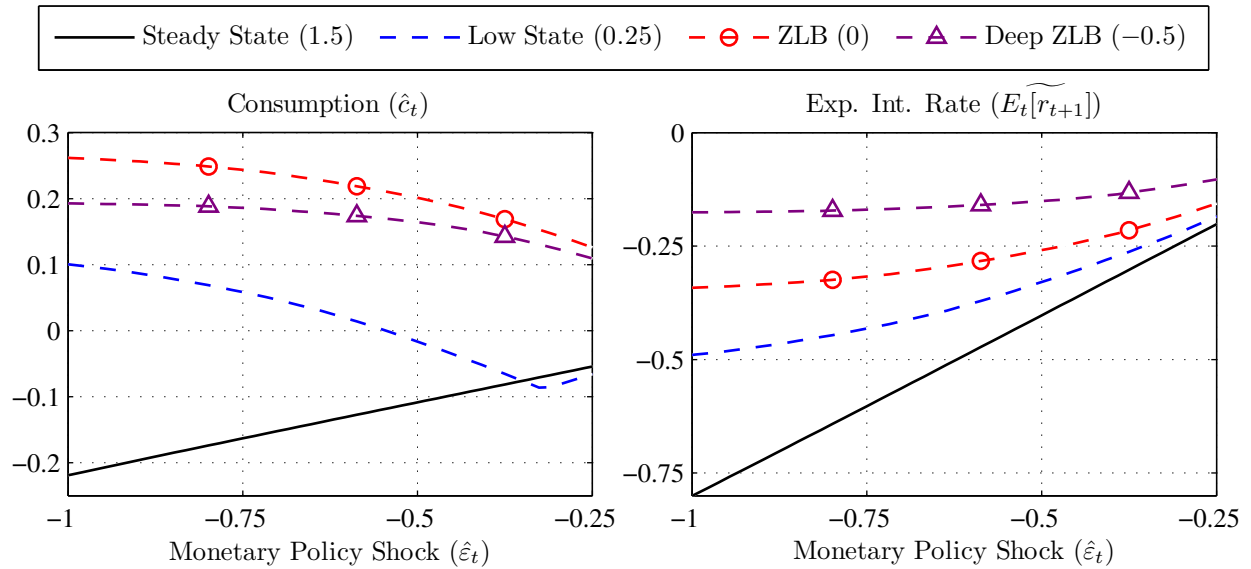

Figure 3: Forward Guidance at and away from the ZLB. Four alternative cross sections are considered: (1) $\tilde{r}_0^* = 1.5$, solid line; (2) $\tilde{r}_0^* = 0.25$, dashed line; (3) $\tilde{r}_0^* = 0$, circle markers; (4) $\tilde{r}_0^* = -0.5$, triangle markers. The vertical axis is the difference between 1-quarter forward guidance, $(\alpha_0, \alpha_1) = (0, 1)$, and no forward guidance, $(\alpha_0, \alpha_1) = (1, 0)$.

We begin our analysis by examining the case in which the economy is in steady state, $\tilde{r}_0 = 1.5$, which signifies that the current nominal interest rate is far enough from the ZLB that no plausible policy shock will push it to the ZLB. In that situation, an unanticipated expansionary monetary policy shock (i.e., no forward guidance) in the current period generates a larger increase in consumption than a promise to provide the same-sized expansionary monetary policy shock next period (i.e., 1-quarter forward guidance). That finding shows conventional monetary policy is more stimulative than forward guidance when the nominal interest rate is far from its ZLB.

The economic effects of an unanticipated expansionary monetary policy shock are more limited when the initial shadow interest rate is low enough that the policy shock causes the ZLB to bind. Since households believe the economy will improve, there are situations in which a promise to lower future nominal interest rates generates a larger increase in consumption than an equivalent unanticipated expansionary shock to the current nominal interest rate, which cannot fall below zero. Consider the case where $\tilde{r}_0 = 0.25$. A small expansionary policy shock, $\hat{\varepsilon}_t > -0.32$, does not drive the current nominal interest rate to its ZLB. A moderate-sized policy shock, $-0.54 < \hat{\varepsilon}_t < -0.32$,

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[8]Gust et al. (2013), Krippner (2013), Wu and Xia (2014), and Bauer and Rudebusch (2014) estimate the shadow federal funds rate when the ZLB binds. They find the shadow rate has been well below zero since the Great Recession.

reduces the current nominal interest rate to its ZLB, but the effects of that shock are still stronger than those produced by 1-quarter forward guidance. Finally, a large policy shock, $\hat{\varepsilon}_t < -0.54$, generates a smaller increase in consumption than an equivalent 1-quarter forward guidance shock.

When the economy is in a recession and the ZLB initially binds, 1-quarter forward guidance is always more stimulative because an unanticipated expansionary monetary policy shock cannot push the nominal rate any lower. As the shadow interest rate falls to $\tilde{r}_0 = -0.5$, households place a smaller probability on exiting the ZLB next period. That lower probability reduces how much households expect next period's nominal interest rate to rise, which limits the economic effects of forward guidance. In our specification, the promise to stimulate the economy next period reduces that period's expected nominal interest rate as $\tilde{r}_0$ becomes more negative. Those results reinforce our conclusion that a pessimistic economic forecast diminishes the stimulative effect of forward guidance. In fact, the possibility exists that forward guidance can have no stimulative effect if the discount factor is sufficiently high. That situation occurs in our model with 1-quarter forward guidance when $\tilde{r}_0 < -1.6$. Our finding of a limited stimulative effect of forward guidance at the ZLB offers an explanation for the Forward Guidance Puzzle described in Del Negro et al. (2012).[9]
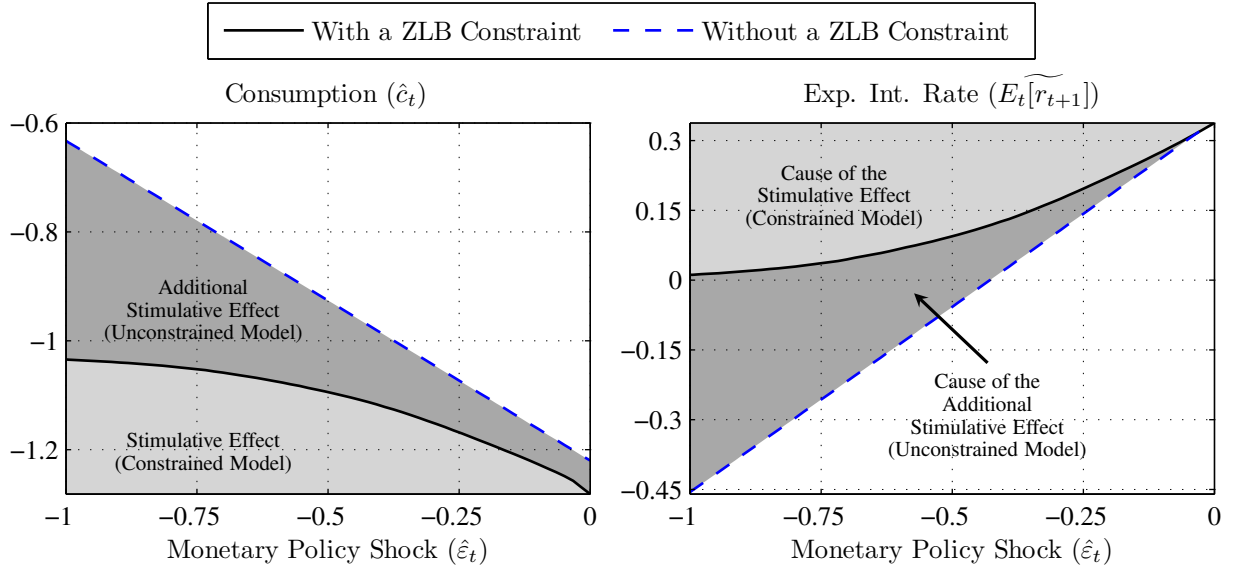

Figure 4: Comparison of decision rules with (solid line) and without a ZLB constraint (dashed line) given 1-quarter forward guidance, $(\alpha_0, \alpha_1) = (0, 1)$. The initial shadow rate equals zero in these decision rules.

Another plausible explanation for the Forward Guidance Puzzle is that some models do not impose a ZLB constraint. Suppose, for example, the central bank announces a plan this period to stimulate the economy using 1-quarter forward guidance, $(\alpha_0, \alpha_1) = (0, 1)$. Figure 4 compares the responses of key decision rules to that announcement for our model with a ZLB constraint (solid line) and without a ZLB constraint (dashed line). The ZLB constraint is critical because it truncates the expected nominal interest rate, which limits the stimulative effect of forward guidance. We utilize 1-quarter forward guidance rather than 1-quarter equal forward guidance so that the stimulative effect is entirely due to changes in the expected nominal interest rate. Without a ZLB constraint, the expected nominal interest rate can be negative, which leads to an overstatement

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[9]The Forward Guidance Puzzle is that New Keynesian models seem to overstate the effect of forward guidance.

of the stimulative effect of forward guidance. For example, a -0.5% (-1%) news shock in the constrained model reduces the expected nominal interest rate to 9 (1) basis points and increases consumption to 1.1% (1.04%) below its steady state. The same shock in the unconstrained model pushes down the expected nominal rate to -6 (-45) basis points and raises consumption to 0.92% (0.64%) below its steady state. In the unconstrained version of our model, a negative expected nominal interest rate occurs whenever $\hat{\varepsilon}_t < -0.43$. Any analysis that allows the nominal interest rate to fall below zero will significantly overstate the stimulative effect of forward guidance.
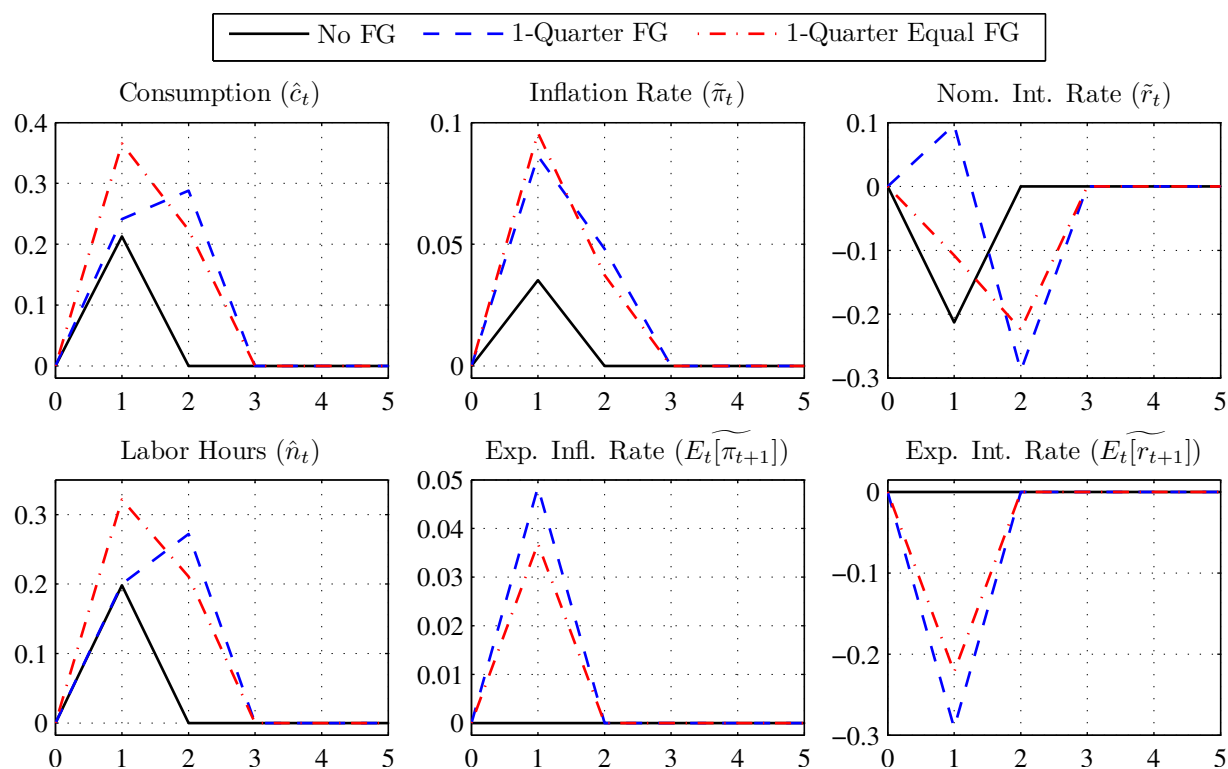

Figure 5: Generalized impulse responses to a -0.5% monetary policy shock. Three types of monetary policy are examined: No forward guidance, $(\alpha_0, \alpha_1) = (1, 0)$ (solid line); 1-quarter forward guidance, $(\alpha_0, \alpha_1) = (0, 1)$ (dashed line); and 1-quarter equal forward guidance, $(\alpha_0, \alpha_1) = (\sqrt{1/2}, \sqrt{1/2})$ (dash-dotted line). Each case is initialized at the average discount factor conditional on the ZLB binding in a 500,000 quarter simulation.

We also examine forward guidance by computing generalized impulse response functions (GIRFs) of a policy shock. GIRFs show the quantitative effects of forward guidance over time. They are based on an average of model simulations where the realization of shocks is consistent with households' expectations over time. Figure 5 plots the GIRFs to a -0.5% policy shock at the ZLB with no forward guidance (solid line), 1-quarter forward guidance (dashed line), and 1-quarter equal forward guidance (dash-dotted line). To compute the GIRFs, we calculate the mean of 10,000 Monte Carlo simulations of the model conditional on a random shock in the first quarter. We then calculate a second mean from another set of 10,000 simulations, but this time the random shock in the first quarter of each simulation is replaced with the -0.5% policy shock.[10] The GIRFs are the percentage change (or the difference in the rates) between the two means. To initialize the

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[10]The general procedure for computing GIRFs is outlined in Koop et al. (1996). See Appendix B for details.

economy so the ZLB binds in period 0, we calculate the average value of the discount factor conditional on the ZLB binding in a $500{,}000$ quarter simulation of the model. The shadow interest rate equals -0.24% at that mean value. Households believe the discount factor will gradually fall to its steady state, so the nominal interest rate will eventually rise above zero in all of the simulations.

In each GIRF, households learn about the monetary policy shock in period 1. With no forward guidance, the shock is unanticipated and occurs in period 1. With 1-quarter forward guidance, households are informed in period 1 about the policy shock that hits in period 2. An unanticipated expansionary shock without forward guidance $[(\alpha_0, \alpha_1) = (1, 0)$, solid line] is stimulative on average because 27% of the simulations exit the ZLB despite the cut in the nominal interest rate. The expected future nominal interest rate is unchanged since the shock is transitory. A policy shock announced with 1-quarter forward guidance $[(\alpha_0, \alpha_1) = (0, 1)$, dashed line], in contrast, lowers the expected nominal interest rate for next period, which pushes up current consumption, inflation, output, and labor hours. It also causes the current nominal rate to rise due to the feedback effect. Specifically, the period 1 announcement of a -0.5% monetary policy shock next period raises expected consumption and lowers the expected nominal interest rate for period 2. That change causes households to increase their current consumption and reduce their labor supply. Firms respond to the increase in demand by raising their prices, output, and labor demand. The additional labor demand dominates the decline in labor supply so the equilibrium level of labor and the real wage both increase. Our results demonstrate that 1-quarter forward guidance is stimulative if households expect the economy to recover and exit the ZLB over the forward guidance horizon.

The stimulative effect of 1-quarter forward guidance increases the current nominal interest rate by an average of 0.31% due to the feedback effect. To offset that effect, the central bank could provide an unanticipated expansionary monetary policy shock in period 1. With 1-quarter equal forward guidance $[(\alpha_0, \alpha_1) = (\sqrt{1/2}, \sqrt{1/2})$, dash-dotted line], households learn that identical monetary policy shocks will impact the economy in periods 1 and 2. The unanticipated policy shock in period 1 counteracts the feedback effect so the current nominal interest rate falls. As a result, consumption rises 50% more on impact than with 1-quarter forward guidance.

4.2 Two-Quarter Horizon ($q = 2$) This section analyzes the effect of 2-quarter forward guidance when the initial shadow interest rate is zero. Figure 6 plots the 2-quarter forward guidance decision rules $[(\alpha_0, \alpha_1, \alpha_2) = (0, 0, 1)$, dashed line] as a function of the current monetary policy shock, $\hat{\varepsilon}_t$. As a reference, the decision rules for no forward guidance $[(\alpha_0, \alpha_1, \alpha_2) = (1, 0, 0)$, solid line] are also displayed. In this paper, 2-quarter forward guidance occurs when households receive news about a future monetary policy shock two periods before that shock impacts the economy.

If households receive news in period t that an expansionary monetary policy shock will happen in period $t + 2$, the reaction of consumption is similar to its reaction to 1-quarter forward guidance that was shown in figure 1. Given households prefer a smooth consumption path, the expectation of monetary stimulus in period $t + 2$ encourages households to raise their consumption in not only period $t + 2$, but also in periods t and $t + 1$. The higher demand for consumption in periods t and $t + 1$ pushes up the inflation rate and the current and expected future nominal interest rates (i.e., the feedback effect). The rise in the nominal rate, however, is inconsistent with actual policy.

Central banks, in practice, offset increases in current and expected future nominal interest rates by promising to keep the nominal rate at zero over the entire forward guidance horizon. Figure 6 also shows the decision rules when households receive 2-quarter equal forward guidance $[(\alpha_0, \alpha_1, \alpha_2) = (\sqrt{1/3}, \sqrt{1/3}, \sqrt{1/3})$, dash-dotted line]. Substantial differences exist between the
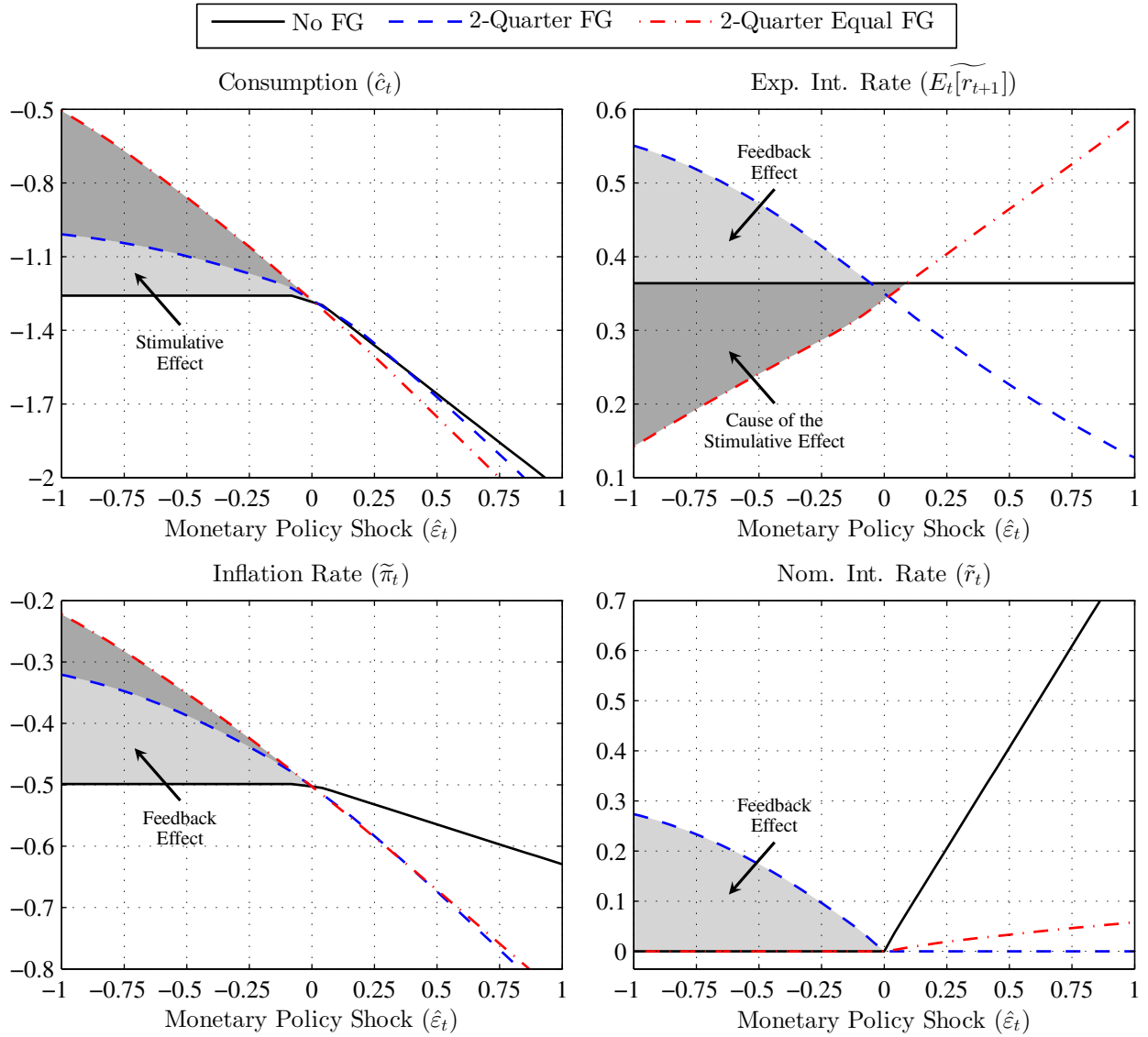
Figure 6: Decision rules as a function of the monetary policy shock with no forward guidance (solid line); 2-quarter forward guidance, $(\alpha_0, \alpha_1, \alpha_2) = (0, 0, 1)$ (dashed line); and 2-quarter equal forward guidance, $(\alpha_0, \alpha_1, \alpha_2) = (\sqrt{1/3}, \sqrt{1/3}, \sqrt{1/3})$ (dash-dotted line). The initial shadow rate equals zero in these decision rules.

two types of 2-quarter forward guidance. With 2-quarter equal forward guidance, the central bank announces in period t that an expansionary monetary policy shock will occur in periods t, $t+1$, and $t+2$. The shocks in periods t and $t+1$, which are not present with 2-quarter forward guidance, hold the current nominal interest rate at zero and lower next period's nominal interest rate. Thus, the period t and $t+1$ policy shocks eliminate the feedback effects from 2-quarter forward guidance. Those three policy shocks also more than compensate for the smaller period $t+2$ news shock, so that 2-quarter equal forward guidance produces a larger stimulative effect than a monetary stimulus with a bigger policy shock that only occurs in period $t+2$. For example, a -0.5% (-1%) shock announced in period t increases current consumption by 0.24 (0.50) percentage points *more* with 2-quarter equal forward guidance than with 2-quarter forward guidance.

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Extending the forward guidance horizon from 1 to 2 quarters does not double the size of its stimulative effect, unless the monetary policy shock is large ($\hat{\varepsilon}_t < -1$). For example, a -0.5% (-1%) policy shock increases current consumption by 0.24 (0.37) percentage points with 1-quarter equal forward guidance and by 0.40 (0.75) percentage points with 2-quarter equal forward guidance. Thus, the extra quarter of central bank forward guidance raises consumption by an additional 0.16 (0.38) percentage points. That result, however, differs from much of the literature because we assume the variance of the news process remains constant across alternative horizons.
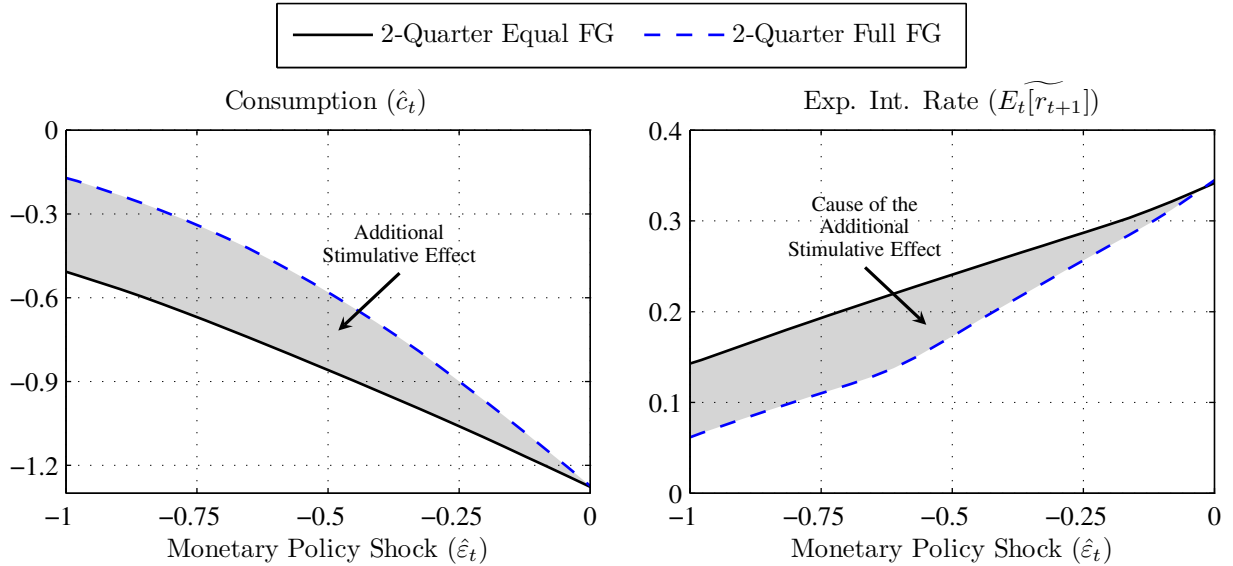

Figure 7: Decision rules as a function of the monetary policy shock with 2-quarter equal forward guidance, $(\alpha_0, \alpha_1, \alpha_2) = (\sqrt{1/3}, \sqrt{1/3}, \sqrt{1/3})$ (solid line), and 2-quarter full forward guidance, $(\alpha_0, \alpha_1, \alpha_2) = (1, 1, 1)$ (dashed line). The initial shadow rate equals zero in these decision rules.

Many researchers who examine forward guidance do not hold the variance of the news process constant [Laséen and Svensson (2011), Carlstrom et al. (2012), and Del Negro et al. (2012)]. Without that restriction, extending the forward guidance horizon by an additional quarter is the same as increasing the intensity of the news. Figure 7 contrasts key decision rules with 2-quarter equal forward guidance [$(\alpha_0, \alpha_1, \alpha_2) = (\sqrt{1/3}, \sqrt{1/3}, \sqrt{1/3})$, solid line] to 2-quarter *full* forward guidance [$(\alpha_0, \alpha_1, \alpha_2) = (1, 1, 1)$, dashed line]. Our comparison shows that a higher intensity of news with 2-quarter full forward guidance further stimulates consumption. That finding, however, could erroneously lead one to conclude that the model overstates the stimulative effect of forward guidance. For example, a -0.5% (-1%) monetary policy shock with 2-quarter full forward guidance increases consumption by 0.27 (0.33) percentage points *more* than 2-quarter equal forward guidance. That result is due to a 7 (8) basis point decline in the 1-quarter ahead expected nominal interest rate and an even larger reduction in the 2-quarter ahead expected rate. Thus, much of the additional stimulus is due to the higher intensity of news and not to the longer horizon.

It is important to reiterate that regardless of how the news is distributed the stimulative effect of forward guidance declines as the shadow interest rate become more negative. When the economy moves deeper into the ZLB region, the probability the nominal interest rate will rise above zero in the next period falls. The likelihood of leaving the ZLB next period also becomes smaller as households' expect a slower recovery (i.e., $\hat{\beta}_t$ is expected to slowly return to steady state). In both cases,

expected consumption is lower in the following period, which dampens any expected increase in next period's nominal interest rate. Without any meaningful rise in next period's expected nominal rate, the stimulative effect of forward guidance is extremely limited. Therefore, the stimulative effect of forward guidance is smaller than figures 1 and 6 indicate whenever the forward guidance announcement is interpreted as a downward revision in the central bank's economic outlook.

4.3 LONGER HORIZONS ($q > 2$) This section compares the stimulative effect of forward guidance policies over longer horizons. Our results in sections 4.1 and 4.2 rely on Gauss-Hermite quadrature to evaluate expectations. That approach enables us to obtain an accurate approximation of the decision rules and to quantify the stimulative effect of forward guidance for a *continuous range* of news shocks. The solution method, however, is numerically infeasible at longer forward guidance horizons because of the size of the state space. Therefore, we reduce the dimensionality of the problem when analyzing longer forward guidance horizons by discretizing the continuous distribution of the news shock process using the method described in Tauchen (1986). Specifically, we examine three values for each monetary policy shock, $(-0.5, 0, 0.5)$, and then calculate the probabilities of each transitional event. Tauchen's (1986) method is particularly useful for calculating GIRFs because it enables us to analyze the effects of *specific* shocks to the news process without solving the model across the complete distribution of shocks.[11]

Figure 8 shows the GIRFs to a -0.5% monetary policy shock that is equally distributed over a 1-quarter (solid line), 4-quarter (dashed line), 8-quarter (circle markers), and 10-quarter (triangle markers) horizon. Specifically, we set $\alpha_i = \sqrt{1/(q+1)}$ for $i \in \{0, \ldots, q\}$ to keep the variance of the shock process constant for different values of q. Each case is initialized at the average discount factor conditional on the ZLB binding in a $500{,}000$ quarter simulation, which corresponds to an average shadow interest rate of -0.24%. In every case, households know about the current and future monetary policy shocks in period 1, but their effects on the economy depend on the forward guidance horizon. An unanticipated expansionary monetary policy shock has, on average, a small stimulative effect due to the discount factor slowly falling to its steady state. When the central bank provides 1-quarter equal forward guidance, households expect a lower nominal interest rate in period 2, which stimulates the economy in period 1. Over that 1-quarter horizon, the stimulative effect of equal forward guidance is considerable and the unexpected policy shock in period 1 is usually big enough to reverse the feedback effect on the current nominal interest rate.

Additional periods of forward guidance generate more persistent increases in consumption, labor hours, inflation, and output because households expect a lower future nominal interest rate and higher future consumption in every period until the last shock hits the economy in period q. At longer horizons, the feedback effect from forward guidance dominates the smaller weight on the policy shocks, so the current and expected nominal interest rates increase. Those feedback effects, however, can be eliminated by increasing the size of the monetary policy shocks over the entire forward guidance horizon. Beyond period q, forward guidance has no additional effects on the economy. Our findings reveal that equal forward guidance for up to 10 quarters does not cause the stimulative effect to explode or reverse, but rather spreads the effect across the entire horizon.

Figure 9 compares the paths of the expected future nominal interest rate, $E_0[r_j]$, (left panel) and the yield curve (right panel) at various forward guidance horizons assuming the initial shadow interest rate is zero.[12] In this example, the central bank announces in period 0 that it will cut the nom-

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[11]See Appendix C for further details on how this solution procedure differs from the method used in earlier sections.

[12]Swanson and Williams (2013) empirically show how the ZLB affects intermediate- and longer-term yields.
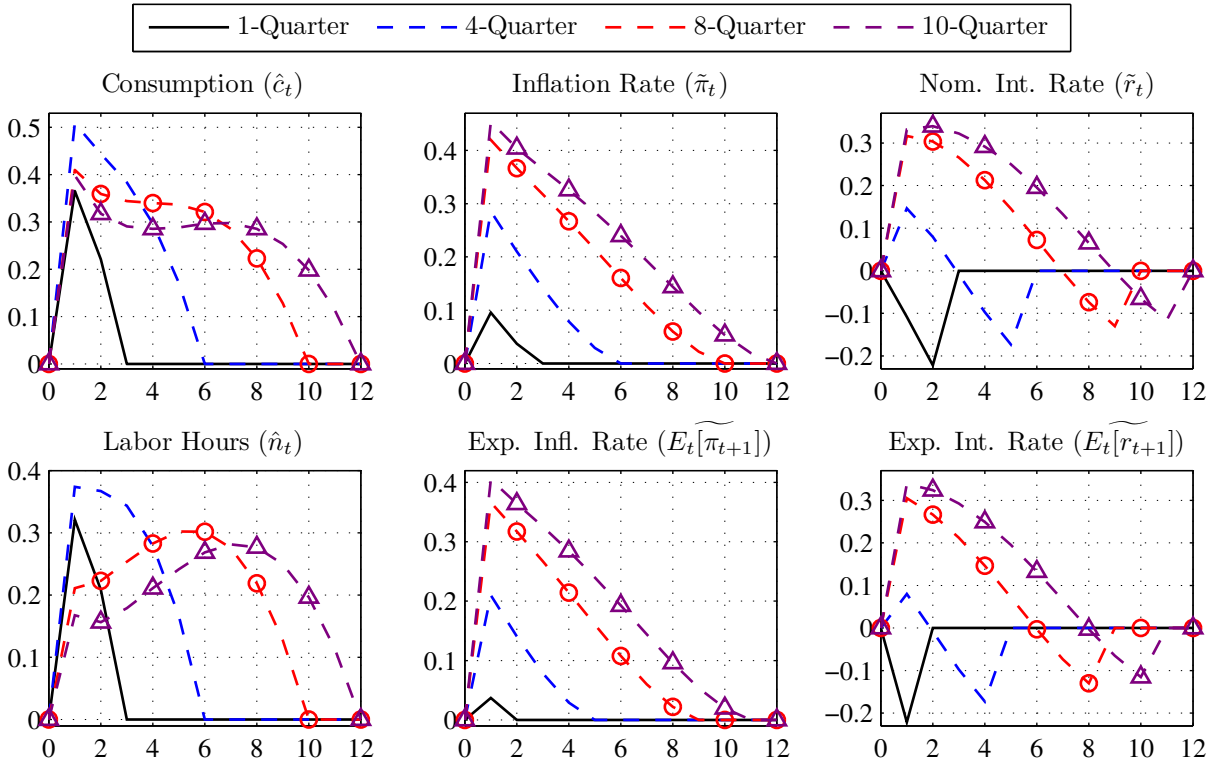
Figure 8: Generalized impulse responses to a -0.5% monetary policy shock with 1-quarter (solid line), 4-quarter (dashed line), 8-quarter (circle markers), and 10-quarter (triangle markers) equal forward guidance. Each case is initialized at the average discount factor conditional on the ZLB binding in a 500,000 quarter simulation.
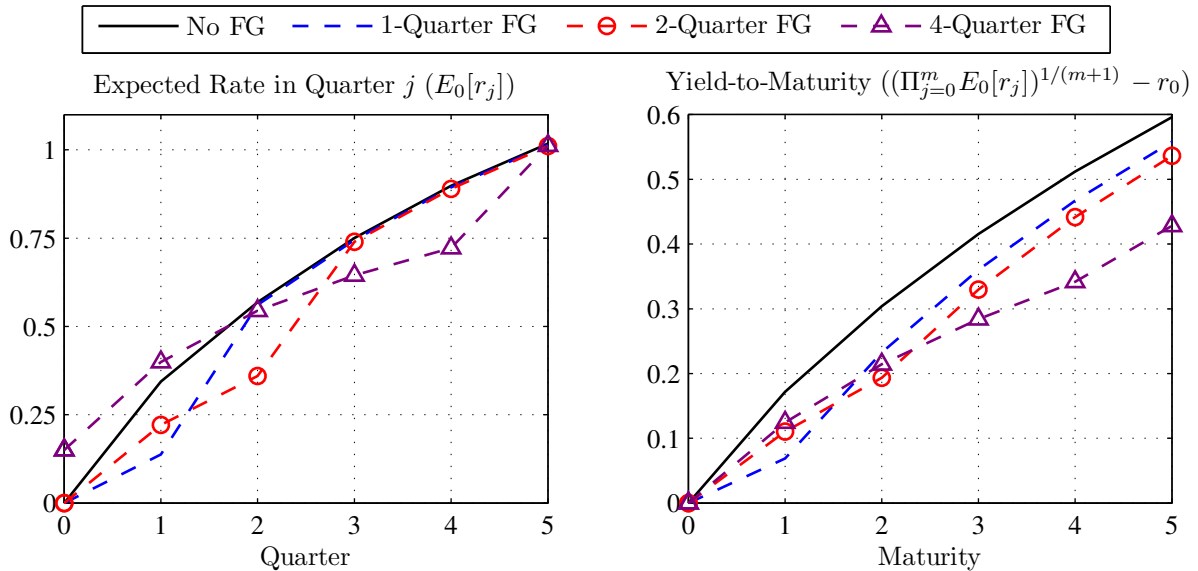
Figure 9: Comparison of expected rates and yields with no forward guidance (solid line) and 1-quarter (dashed line), 2-quarter (circle markers), and 4-quarter (triangle markers) equal forward guidance. The left panel shows the expected increase in the net nominal interest rate j periods in the future. The right panel shows the yield curves as a percentage point difference from the initial net nominal interest rate. In period 0, households receive news of a -0.5% monetary policy shock. The initial shadow rate equals zero in these decision rules.

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inal interest rate by -0.5% with no forward guidance (solid line), or with 1-quarter (dashed line), 2-quarter (circle markers), or 4-quarter (triangle markers) equal forward guidance. The intensity parameters with q-quarter equal forward guidance are set to $\alpha_i = \sqrt{1/(q+1)}$ for $q \in \{0, 1, 2, 4\}$ to keep the variance of the news process constant across the forward guidance horizon.

The left panel of figure 9 shows the impact of forward guidance on the expected nominal interest rate over six quarters. In most cases, forward guidance lowers the expected nominal rate during the forward guidance horizon, but in all cases, it returns to its baseline expected value in the periods beyond that horizon. The baseline expected nominal interest rate is the case with no forward guidance (solid line) because an unanticipated monetary policy shock in period 0 has no economic effects when the current nominal interest rate is already at zero. The expected increase in the nominal interest rate in the no forward guidance specification reflects the expected recovery implied by the belief that the discount factor will revert to its mean. The 1- and 2-quarter equal forward guidance cases lower the expected nominal rates during the forecast horizon but revert to their no forward guidance expected values beginning in periods 2 and 3, respectively. The 4-quarter equal forward guidance horizon behaves just like the 1- and 2-quarter cases, except the policy shock is not strong enough in period 0 to completely eliminate the feedback effect.

The right panel of figure 9 shows the yield curve for maturities from 0 to 5 quarters. The yield at each maturity, m, is calculated according to the expectations hypothesis as $(\Pi_{j=0}^{m} E_0[r_j])^{1/(m+1)}$. The stimulative effect of forward guidance is evident as the yield curve flattens with longer horizons. For example, 4-quarter equal forward guidance reduces the expected increase in the 1-year yield (i.e., $m = 3$) by 13 basis points. Beyond the forward guidance horizon, the yield curves with q-quarter forward guidance eventually converge to the yield curve with no forward guidance.

Figure 10 compares the effect of forward guidance on the yield curve for two different initial shadow interest rates: $\tilde{r}_0^* = 0$ (left panel) and $\tilde{r}_0^* = -0.5$ (right panel). The solid line is the yield curve when the central bank offers no forward guidance [$(\alpha_0, \alpha_1) = (1, 0)$], while the dashed line is the yield curve with 4-quarter equal forward guidance [$\alpha_i = \sqrt{1/5}$ for $i \in \{0, \ldots, 4\}$]. In the left panel, the economy is in a recession that causes the ZLB to bind, but households place a high probability on exiting the ZLB next period. If the central bank provides 4-quarter equal forward guidance, then the expected future nominal interest rates over the forward guidance horizon fall, which flattens the yield curve. In the right panel, the economy is in a deep recession and households do not expect it to recover in the near-term. An announcement of 4-quarter equal forward guidance has little stimulative effect because households expect the nominal interest rate will remain near zero over most of the forward guidance horizon. In that situation, the yield curve with 4-quarter equal forward guidance is much closer to the yield curve with no forward guidance.

The results in figure 10 provide an explanation for the Forward Guidance Puzzle. In particular, they show that models can easily overpredict the stimulative effect of forward guidance by assuming households are overly optimistic about future economic conditions. Suppose, for example, the central bank decides to provide a -0.5% news shock that is equally distributed via 4-quarter equal forward guidance. The response of the yield curve depends on how the nominal interest rate is expected to respond over the next 4 quarters. When households are highly confident that the nominal interest rate will leave its ZLB over the forward guidance horizon [$\tilde{r}_0^* = 0$], the 4-quarter equal forward guidance shock pushes down the annual yield on the 4-quarter bond by about 54 basis points. That response declines to 8 basis points when the economy is weak enough that households place a high probability on the nominal rate remaining at or near zero [$\tilde{r}_0^* = -0.5$]. Thus, any downward revision in the expected economic outlook will weaken the stimulative effect of forward guidance.
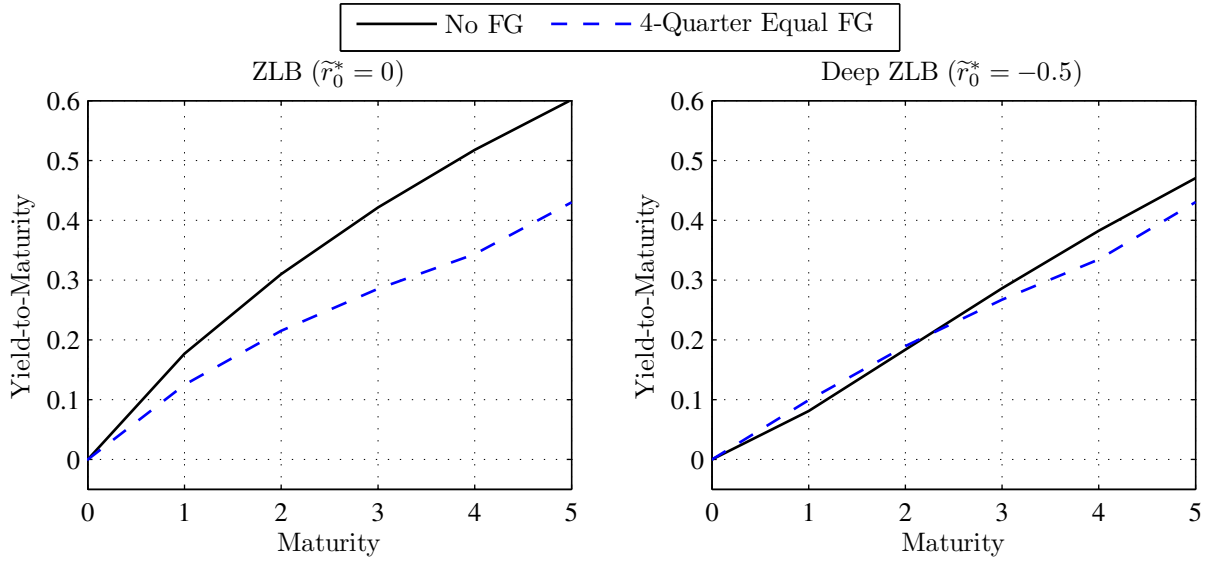
Figure 10: Comparison of yield curves with no forward guidance (solid line) and 4-quarter equal forward guidance (dashed line) at two different initial shadow rates: $\tilde{r}_0^* = 0$ (left panel) and $\tilde{r}_0^* = -0.5$ (right panel). The yield curves are given as a percentage point difference from the initial net nominal interest rate. In period 0, households receive news of a -0.5% monetary policy shock. The vertical axis values are net percentages.
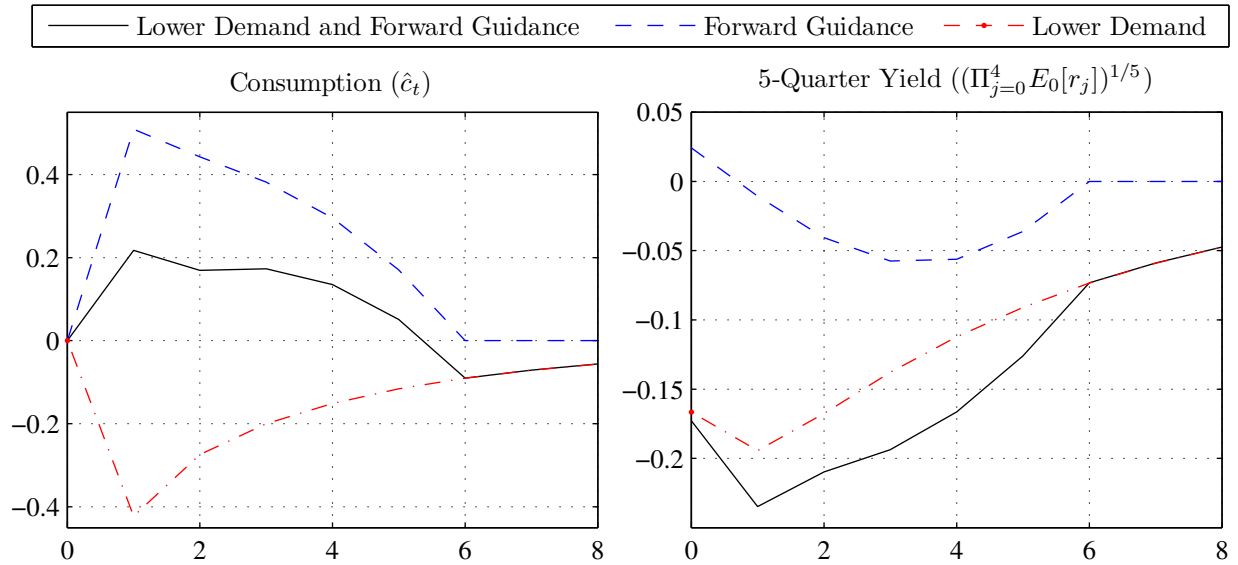
Figure 11: Generalized impulse responses to a 1 standard deviation positive discount factor shock and a -0.5% monetary policy shock with 4-quarter equal forward guidance (solid line). We compare those responses to cases with only the monetary policy shock (dashed line) and only the discount factor shock (dash-dotted line). Each case is initialized at the average discount factor conditional on the ZLB binding in a 500,000 quarter simulation.

Figure 11 compares the separate and combined effects from an increase in economic pessimism and 4-quarter forward guidance. Possible reasons for the increased pessimism include an exogenous event that dampens economic expectations or a response of households to a gloomier economic outlook by the central bank. To assess the combined effects of both shocks, we compute GIRFs to a simultaneous 1 standard deviation positive discount factor shock (more pessimism)

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and a -0.5% monetary policy shock with 4-quarter equal forward guidance (solid line). Those responses are then compared to the GIRFs with only the monetary policy shock (dashed line) and the GIRFs with only the discount factor shock (dash-dotted line). The simulations are initialized in the same way as our previous GIRFs. The distance between the dashed line and the solid line measures the effect of the increased pessimism, whereas the distance between the dash-dotted line and the solid line quantifies the marginal benefit of implementing 4-quarter equal forward guidance.

A announcement by the central bank that it intends to keep its policy rate low for the next 4 quarters reduces expected nominal interest rates and raises consumption over the forward guidance horizon. One problem that might prevent forward guidance from generating this result is that households may revise their economic outlook when the central bank's policy statement is released. If, for example, a forward guidance announcement is accompanied by a belief that economic growth will be slower than expected, then the stimulative effect of forward guidance will be smaller. Specifically, expectations of a slower recovery will reduce demand and flatten the yield curve, which leaves a smaller margin for the forward guidance policy to reduce expected future interest rates. If, however, the central bank decides not to provide forward guidance after a negative demand shock, then the fall in consumption is larger.[13] Our findings illustrate two key points. One, it is difficult to identify the source of changes in the yield curve observed in the data because central banks often simultaneously communicate information about the future state of the economy and the likely path of their policy rate. Two, forward guidance by itself is stimulative, but its net effect on the economy may be small or even negative if other information in the policy statement lowers demand. Given our theoretical results, the next section examines how well our model's predictions correspond with three recent episodes of date-based forward guidance by the Fed.

5 CASE STUDIES OF FEDERAL RESERVE FORWARD GUIDANCE

5.1 AUGUST 9, 2011 POLICY STATEMENT
The FOMC announced it "anticipates that economic conditions...are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013," which was their first use of date-based forward guidance. The statement also said, "The Committee now expects a somewhat slower pace of recovery over coming quarters." This statement could be considered Delphic in nature since the forward guidance is communicated with a more pessimistic economic outlook. If the forward guidance horizon exceeds the time the economy is expected to remain weak, then the statement conveys Odyssean forward guidance.[14]

Crump et al. (2013) find the August 9th FOMC statement changed Blue Chip forecasts of interest rates, GDP, and inflation. Estimating the effect of that announcement on economic forecasts is complicated by a downward revision of GDP just 11 days before the statement was released. To separate the impact of the two events, Crump et al. (2013) compare the consensus forecasts of the federal funds rate from the Blue Chip Financial Forecasts (BCFF) survey to the consensus forecasts of the 3-month Treasury bill rate from the Blue Chip Economic Indicators (BCEI) survey.

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[13]Christiano et al. (2014) find that if the Fed had not provided any forward guidance after 2011 then the policy rate would have started to rise in 2014 and output would have been 2% lower. Bernanke (2014) reiterates this viewpoint by saying "Skeptics have pointed out that the pace of recovery has been disappointingly slow...However,...economic growth might well have been considerably weaker, or even negative, without substantial monetary policy support."

[14]Woodford (2012) argues that information about the central bank's policy intentions is more likely to impact market expectations than knowledge about its economic forecasts because the central bank has undisclosed information about its own policy intentions, whereas it does not enjoy the same advantage when forecasting economic conditions.

Table 1 shows the term structure implied by the BCFF and BCEI forecasts. The BCFF forecasts were made on July 20-21 before the GDP revisions were released on July 29th, while the BCEI forecasts were made on August 4-5 just prior to the August 9th FOMC announcement. The difference between the late-July and early-August forecasts is an implicit measure of the impact the GDP revisions had on the economic forecasts. The next BCFF survey forecasts were made on August 24-25. The difference between the BCEI's August 4-5 forecasts and the BCFF's August 24-25 forecasts is an indirect measure of the effect of forward guidance on those forecasts.

	1Q	2Q	3Q	4Q	5Q	6Q
BCFF (7/20-21)	0.140	0.150	0.187	0.237	0.336	0.459
BCEI 3-mo. T-bill (8/4-5)	0.090	0.110	0.137	0.175	0.240	0.328
BCFF (8/24-25)	0.120	0.125	0.130	0.135	0.138	0.145
Total Change	-0.020	-0.025	-0.057	-0.102	-0.197	-0.313
Change following GDP	-0.050	-0.040	-0.050	-0.062	-0.096	-0.131
Change following FOMC	0.030	0.015	-0.007	-0.040	-0.102	-0.183

Table 1: Term structure implied by Blue Chip consensus forecasts. All values are annualized net interest rates.

Our calculations indicate the July 29th GDP revisions led to a 6 basis point decline in the 4-quarter rate and a 13 basis point decline in the 6-quarter rate. The flatter yield curve reflects a more pessimistic economic outlook. The August 9th FOMC announcement also pushed down the yield curve, with the 4-quarter and 6-quarter rates falling by an additional 4 and 18 basis points. In fact, the 6-quarter rate was only 2.5 basis points higher than the 1-quarter rate, which suggests that people believed the policy rate would not change over the next year and a half. That change in expectations is likely attributable to either the Fed's forward guidance or increased pessimism.

Using a distribution of expected future Eurodollar rates, Raskin (2013) estimates the August 9th statement generated a significant decline in 1- to 5-year ahead future Eurodollar rates and a substantial decline in the dispersion of those expected interest rates. For example, the 2-year ahead Eurodollar rate fell by about 25 basis points after the statement was released. Moessner (2013) also finds the 1-year ahead U.S. Treasury bill rate at 3- to 8-year horizons fell by a similar amount.

Table 2 shows the 2012 consensus forecasts of GDP and inflation as summarized by Crump et al. (2013). The forecast for 2012 GDP growth dropped 0.3 percentage points after the downward revisions to GDP on July 29th and another 0.3 percentage points following the release of the FOMC's policy statement on August 9th. The inflation forecast, on the other hand, dropped only 0.1 percentage points following the GDP revisions and remained unchanged after the statement was released. These forecasts suggest the economic pessimism expressed in the policy statement dampened output forecasts more than any stimulative effect from the change in forward guidance.[15]

Our model predicts Odyssean forward guidance will reduce longer-term interest rates and push up output given the appropriate economic conditions. An analysis of the data following the August 9th FOMC announcement indicates that longer-term interest rates and output declined. We contend that those outcomes occurred for two primary reasons. First, the implied yield curve from the 8/4-5

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[15]Bullard (2012) and Woodford (2012) contend that date-based forward guidance may cause people expect worse economic conditions over its horizon. Threshold-based forward guidance alleviates that problem by linking policy rate changes to economic conditions. Yellen (2013, 2014) refers to this type of communication as an "automatic stabilizer."

	Pre-GDP	Pre-FOMC	Post-FOMC
2012 real GDP growth rate	2.8	2.5	2.2
2012 CPI growth rate	2.3	2.2	2.2

Table 2: Blue Chip consensus forecasts reproduced from Crump et al. (2013). All values are annualized rates.

BCEI survey was fairly flat. That is, the yield curve observed prior to the FOMC statement was flatter than the yield curve in the right panel of figure 10, even though it is based on a shadow interest rate equal to -0.5%. This result suggests that households placed a low probability on exiting the ZLB before the date-based forward guidance was announced on August 9th, which decreased its effectiveness. Second, people's expectations about future economic conditions appear to have been dampened by the additional economic pessimism expressed in the policy statement. We show in figure 11 that an increase in pessimism can limit the stimulative effect of forward guidance by simultaneously flattening the yield curve and reducing consumption.

5.2 2012 POLICY STATEMENTS   Two other FOMC statements (January 25 and September 13) included extensions to the expected "lift-off" date for the federal funds rate. The January statement extended the lift-off date by 6 quarters (from mid-2013 to late-2014) similar to the August 2011 statement, but it was announced 5 quarters before the end of the August 2011 forward guidance horizon. The September statement then extended that horizon by an additional 2 quarters (from late-2014 to mid-2015), 6 quarters before the January forward guidance ended. That statement also contained Odyssean-like language that the FOMC "expects that a highly accommodative stance of monetary policy will remain appropriate for a considerable time after the economic recovery strengthens," whereas the January statement only included language related to the lift-off date.

Announcement	Expected Change in 1-Year Rate j-Years Ahead							
Date	0	1	2	3	4	5	6	8
8/9/2011	-0.10	-0.20	-0.27	-0.30	-0.31	-0.30	-0.27	-0.21
1/25/2012	-0.04	-0.10	-0.13	-0.14	-0.13	-0.11	-0.08	-0.02
9/13/2012	-0.01	-0.03	-0.05	-0.07	-0.07	-0.06	-0.04	0.02

Table 3: Expected changes in future 1-year rates on the date of the announcement. Values are annualized net rates.

Table 3 shows the expected changes in future 1-year interest rates following three FOMC statements. The rates are constructed from daily term structure data by Gürkaynak et al. (2007) and are updated regularly by the Board of Governors. Following the January statement, the decline in the 1-year rate 0 to 3 years ahead was about half the decline from the August 2011 statement. At longer horizons, the response is smaller and at 8-years ahead is almost zero. The effect of the September statement was even smaller. Both of these findings are consistent with Raskin (2013).

When analyzing the data, Raskin (2013) finds the 6-quarter extension of forward guidance only marginally reduced expected interest rates and was not statistically significant. He argues that the difference in the market's reaction to the August 2011 and the January 2012 statements is attributed to the beliefs of market participants. Specifically, the market was surprised by the first announcement of date-based forward guidance, but not by the second statement of a longer

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forward guidance horizon. He also suggests that the greater uncertainty about the longer horizon may have made the later announcement less relevant for market's expectations of future rates.

Table 4 includes Blue Chip consensus forecasts of real GDP growth and inflation. We focus on the 2012 (2013) forecasts from immediately before and after the January 2012 (September 2012) FOMC statement was released. The only meaningful revision in these forecasts was a reduction in 2013 real GDP growth by 0.2% following the September announcement. As was the case with the August 2011 statement, both 2012 forward guidance announcements were communicated with forecasts of a weaker economy in the coming months. Specifically, both statements state that "Strains in global financial markets continue to pose significant downside risks to economic growth." The FOMC, however, was generally more optimistic in its 2012 statements than in August 2011, which may have contributed to the smaller revisions in the real GDP forecasts.

| | 2012 Forecasts | | 2013 Forecasts | |
	Pre-Jan FOMC	Post-Jan FOMC	Pre-Sept FOMC	Post-Sept FOMC
Real GDP growth rate	2.2	2.2	2.2	2.0
CPI growth rate	2.1	2.1	2.0	2.0

Table 4: Blue Chip consensus forecasts. All values are annualized rates.

The evidence is inconclusive on whether the 2012 forward guidance announcements were successful at flattening the yield curve, but the economic pessimism expressed in those statements likely pushed down yields and slowed the recovery. In fact, a headline in *The New York Times* on the day the January FOMC statement was released read "Fed Signals That a Full Recovery Is Years Away." That reaction to the policy statement demonstrates the challenge central banks face in communicating future policy rates. Once again, these findings are consistent with figure 11. While the data does not separate out the effects of the forward guidance and the downward revision in GDP, our results suggest the economy would have recovered slower without forward guidance.

6 CONCLUSION

This paper examines the economic effects of Odyssean forward guidance at and away from the ZLB. The central bank conducts forward guidance by promising to keep the future nominal interest rate lower than its policy rule suggests. That policy can stimulate economic activity if households believe the economy will recover and exit the ZLB. If, on the other hand, households do not expect any meaningful recovery, then future nominal interest rates will remain at or near zero, which means forward guidance will have little effect on the economy. Thus, the ability of forward guidance to stimulate demand is limited when the economy is in a deep recession or people expect it to slowly recover. Those points offer plausible explanations for the Forward Guidance Puzzle.

Empirical estimates indicate that recent forward guidance announcements likely reduced long-term interest rates [Gürkaynak et al. (2005), Campbell et al. (2012), Moessner (2013), and Swanson and Williams (2013)]. It is unclear, however, how much of that decline was due to the Fed's policy and how much was because of the repeated assertion in the FOMC's policy statements that the economy was expected to remain weak in the near-term. Our results suggest that if a central bank expresses a more pessimistic economic outlook when communicating Odyssean forward guidance, then its stimulative effect will be much more limited.

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A NUMERICAL ALGORITHM

A formal description of the numerical algorithm begins by writing the model compactly as

$$\mathbb{E}[f(\mathbf{z}_{t+1}, \mathbf{w}_{t+1}, \mathbf{z}_t, \mathbf{w}_t)|\Omega_t] = 0,$$

where f is a vector-valued function that contains the equilibrium system, $\mathbf{z}$ is a vector of exogenous variables, $\mathbf{w}$ is a vector of endogenous variables, and $\Omega_t = \{M, P, \mathbf{z}_t\}$ is households' information set in period t, which contains the structural model, M, its parameters, P, and the state vector, $\mathbf{z}$. With 1-quarter forward guidance, $\mathbf{z}_t = (\varepsilon_{t-1}, \varepsilon_t, \beta_t)$ and with 2-quarter forward guidance, $\mathbf{z}_t = (\varepsilon_{t-2}, \varepsilon_{t-1}, \varepsilon_t, \beta_t)$. Independent of the forward guidance horizon, $\mathbf{w} = (c, \pi, y, n, w, r)$.

Policy function iteration approximates the vector of decision rules, Φ, as a function of the state vector, $\mathbf{z}$. The time-invariant decision rules for the exogenous model are

$$\underbrace{\Phi(\mathbf{z}_t)}_{\text{True RE Solution}} \approx \underbrace{\hat{\Phi}(\mathbf{z}_t)}_{\substack{\text{Approximating} \\ \text{Function}}} .$$

We choose to iterate on $\Phi = (c, \pi)$ so that we can easily solve for future variables that enter the households' expectations using f. Each state variable in $\mathbf{z}$ is discretized into N^d points, where $d \in \{1, \ldots, D\}$ and D is the dimension of the state space. Thus, the discretized state space contains $N = \Pi_{d=1}^D N^d$ nodes. We set the bounds of stochastic state variables to encompass 99.999 percent of the probability mass of the distribution. We specify 61 grid points for each continuous state variable and use 15 Gauss-Hermite nodes for each continuous shock (with 1-quarter forward guidance $N = 226{,}981$ and with 2-quarter forward guidance $N = 13{,}845{,}841$). Those techniques minimize extrapolation and ensure that the location of the kink in the decision rules is accurate.

The following outline summarizes the policy function algorithm we employ. Let $i \in \{0, \ldots, I\}$ index the iterations of the algorithm and $n \in \{1, \ldots, N\}$ index the nodes.

1. Obtain initial conjectures for the approximating functions, $\hat{c}_0$ and $\hat{\pi}_0$, on each node, from the log-linear model without the ZLB imposed. We use `gensys.m` to obtain those conjectures.

2. For $i \in \{1, \ldots, I\}$, implement the following steps:

   (a) On each node, solve for $\{r_t, w_t\}$ given $\hat{c}_{i-1}(\mathbf{z}_t^n)$ and $\hat{\pi}_{i-1}(\mathbf{z}_t^n)$ with the ZLB imposed.

   (b) Linearly interpolate $\{c_{t+1}, \pi_{t+1}\}$ given $\{\varepsilon_t, \varepsilon_{t+1}^m, \beta_{t+1}^m\}_{m=1}^M$ (1-quarter forward guidance) and $\{\varepsilon_{t-1}, \varepsilon_t, \varepsilon_{t+1}^m, \beta_{t+1}^m\}_{m=1}^M$ (2-quarter forward guidance). Each of the M pairs of $\{\varepsilon_{t+1}^m, \beta_{t+1}^m\}$ are Gauss-Hermite quadrature nodes. We use Gauss-Hermite quadrature to numerically integrate, since it is very accurate for normally distributed shocks. We use piecewise linear interpolation to approximate future variables that show up in expectation, since this approach more accurately captures the kink in the decision rules than continuous functions such as cubic splines or Chebyshev polynomials.[16]

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[16]Aruoba and Schorfheide (2013) use a linear combination of two Chebyshev polynomials—one that captures the dynamics when the ZLB binds and one that captures the dynamics when the Taylor principle holds. Although that approach is more accurate than using one Chebyshev polynomial, there is no guarantee that it will accurately locate the kink. Moreover, Richter et al. (2013) show that Chebyshev polynomials can lead to large approximation errors due to extrapolation. With a dense state space, linear interpolation more accurately locates the kink in the policy functions.

(c) On each node, solve for the remaining $t+1$ variables, $\{y_{t+1}^m, c_{t+1}^m\}_{m=1}^M$, which enter the expectation operators.

(d) We use the nonlinear solver, `csolve.m`, to minimize the Euler equation errors. On each node, numerically integrate to approximate the expectation operators,

$$\mathbb{E}\left[f(\mathbf{x}_{t+1}^m, \mathbf{x}_t^n)|\Omega_t\right] \approx \frac{1}{\sqrt{\pi}} \sum_{m=1}^M f(\mathbf{x}_{t+1}^m, \mathbf{x}_t^n)\phi(\varepsilon_{t+1}^m, \beta_{t+1}^m),$$

where $\mathbf{x} \equiv (\mathbf{z}, \mathbf{w})$, and ϕ are the respective Gauss-Hermite weights. The superscripts on $\mathbf{x}$ indicate which realizations of the state variables are used to compute expectations. The nonlinear solver searches for $\hat{c}_i(\mathbf{z}_t^n)$ and $\hat{\pi}_i(\mathbf{z}_t^n)$ so that the Euler equation errors are less than 1^{-4} on each node.

3. Define $\mathrm{maxdist}_i \equiv \max\{|\hat{c}_i - \hat{c}_{i-1}|, |\hat{\pi}_i - \hat{\pi}_{i-1}|\}$. Repeat the steps in item 2 until $\mathrm{maxdist}_i < 1^{-13}$ for all n and for 10 consecutive iterations.

To provide evidence that the solution is unique, we randomly perturb the converged decision rules and check that the algorithm converges back to the same solution.

B GENERALIZED IMPULSE RESPONSE FUNCTIONS

The generalized impulse response functions (GIRFs) are based on the average of 10,000 Monte Carlo simulations of the model. The advantage of this approach is that the realization of shocks are consistent with households' expectations that the stochastic processes will mean revert when the GIRF is initialized away from the model's stochastic steady state. The general procedure for calculating GIRFs is laid out in Koop et al. (1996). We apply the following steps to our models:

1. Find the state vector at which to initialize each Monte Carlo simulation by simulating the model for 500,000 quarters using random draws of discount factor shocks. The initial state vector is the average state vector conditional on the ZLB binding, $\mathbf{z}_0^{zlb}$.

2. Draw random policy and the discount factor shocks, $\{\varepsilon_t, \upsilon_t\}_{t=0}^N$, from their independent normal distributions. Simulate the model for R different draws of the sequence of shocks beginning at the initial state vector, $\mathbf{z}_0^{zlb}$. This yields R equilibrium paths, $\{\mathbf{x}_t^j(\mathbf{z}_0^{zlb})\}_{t=0}^N$, where $j \in \{1, 2, \ldots, R\}$. N corresponds to the number of quarters to plot and we set $R = 10{,}000$.

3. Using the same R draws of shocks from step 2, replace the policy rate shock in period one with a -0.5% shock (i.e., set $\varepsilon_1 = -0.5$ for all $j \in \{1, 2, \ldots, R\}$). Simulate the model with these alternate sequences of shocks. This yields R equilibrium paths, $\{\mathbf{x}_t^j(\mathbf{z}_0^{zlb}, \varepsilon_{z,1})\}_{t=0}^N$.

4. Average across the R simulations from step 2 and step 3 to obtain average paths, given by,

$$\bar{\mathbf{x}}_t(\mathbf{z}_0^{zlb}) = \frac{1}{R}\sum_{j=1}^R \mathbf{x}_t^j(\mathbf{z}_0^{zlb}), \qquad \bar{\mathbf{x}}_t(\mathbf{z}_0^{zlb}, \varepsilon_{z,1}) = \frac{1}{R}\sum_{j=1}^R \mathbf{x}_t^j(\mathbf{z}_0^{zlb}, \varepsilon_{z,1}).$$

5. The difference between the two average paths is a GIRF. In our figures, a variable with a hat is calculated as $100(\bar{\mathbf{x}}_t(\mathbf{z}_0^{zlb}, \varepsilon_{z,1})/\bar{\mathbf{x}}_t(\mathbf{z}_0^s) - 1)$, and a variable with a tilde is calculated as $100(\bar{\mathbf{x}}_t(\mathbf{z}_0^{zlb}, \varepsilon_{z,1}) - \bar{\mathbf{x}}_t(\mathbf{z}_0^{zlb}))$.

C COMPUTING LONGER HORIZONS

To make the numerical algorithm numerically tractable with longer forward guidance horizons (i.e., $q > 2$), we discretize each shock in the monetary policy news process with 3 points. Although this simplification prevents us from examining the complete distribution of each shock, it still allows us to compute generalized impulse response functions of a specific shock. We use the method outlined in Tauchen (1986) to obtain the 3 points and the corresponding weights for numerical integration. The state vector, $\mathbf{z}_t = (\beta_t, s_{0,t}, s_{1,t})$, is independent of the forward guidance horizon and determines the realization of the monetary policy shock, ε_t, according to

$$\varepsilon_t = \begin{cases} -0.005 & \text{for} \quad s_{0,t} = 1 \\ 0 & \text{for} \quad s_{0,t} = 2 \\ 0.005 & \text{for} \quad s_{0,t} = 3 \end{cases}.$$

When $s_{0,t} = 1$, there is a -0.5% monetary policy shock, which is the same size shock used to generate the generalized impulse response functions in the paper. $s_1 \in \{1, 2, \ldots, 3^q - 1, 3^q\}$ determines the realization of the lagged shocks in the news process, where q is the forward guidance horizon. For example, if $q = 1$, then $\varepsilon_{t-1} \in \{-0.005, 0, 0.005\}$, and if $q = 2$, then

$$(\varepsilon_{t-1}, \varepsilon_{t-2}) \in \{(-0.005, -0.005), (0, -0.005), (0.005, -0.005), \ldots, (0, 0.005), (0.005, 0.005)\}.$$

The transition matrix for $s_{0,t}$ is ergodic and is characterized by a single vector of probabilities,

$$P = (\lambda_1, \lambda_2, \lambda_3) = (0.1587, 0.6827, 0.1587),$$

where $\lambda_k = \Pr(s_{0,t+1} = k)$. We discretize β_t with 151 points so the state space contains $N = 151 \times 3 \times 3^q$ nodes. Expectations formation is given by

$$\mathbb{E}\left[f(\mathbf{x}_{t+1}^k, \mathbf{x}_t^n)|\Omega_t\right] \approx \sum_{k=1}^{3} \lambda_k \frac{1}{\pi} \sum_{m=1}^{M} f(\mathbf{x}_{t+1}^k, \mathbf{x}_t^n)\phi(\beta_{t+1}^m),$$

where $\mathbf{x} \equiv (\mathbf{z}, \mathbf{w})$, and ϕ are the Gauss-Hermite weights to numerically integrate across realizations of β_{t+1}. The superscript k on $\mathbf{x}$ indicates the realization of $s_{0,t+1}$. The policy function iteration algorithm is otherwise the same as outlined in Appendix A.