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Introducing ECB-BASE: The blueprint
of the new ECB semi-structural model
for the euro area

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Abstract

This paper presents the blueprint of a new ECB multi-country model. The version documented in the following pages is estimated on euro area data. As a prelude to the country models, this version is meant to enhance the understanding of the main model mechanisms, enlarge the suite of area wide tools, and provide a tool for a top down approach between euro area and country modelling. The model converges to a well-defined steady state and its properties are in line with macroeconomic theory and standard empirical benchmarks. The design is aligned to its role as workhorse model in the context of the forecasting and policy simulation exercises at the ECB.

Key words: Semi-structural model, euro area, simulations, forecasting, monetary policy

JEL Classification: C3, C5, E1, E2, E5

Non-technical summary

This is the first of a series of papers documenting a broad and ambitious project whose main goal is to enlarge the current suite of models used for the model-based policy advice at the ECB with a new version of the semi-structural multi-country model of the euro area and of its five largest countries. The new model will replace the current multi-country model (NMCM, documented in Dieppe, Pandiella, Hall, and Willman, 2011) as one of the main models in the ECB projection process. The model presented in this paper is estimated on euro area data. The planned structure of the individual countries in the country versions will be identical to the one of the ECB-BASE. Country estimates and properties will be reported in future papers.

The design of the new model is aligned to its ultimate purposes, in the context of the forecasting and policy simulation exercises at the ECB, namely to (i) account for the relationships among key macroeconomic variables in a systematic manner; (ii) provide input to the complex process of macroeconomic forecasting; and (iii) conduct scenario analyses and policy simulations.

ECB-BASE is a large-scale, estimated, semi-structural model inspired by the experience of other policy institutions. The main sources of inspiration for the new model are the FRB/US model – developed and maintained at the Federal Reserve Board – and the LENS model – developed and maintained at the Bank of Canada. As for those predecessors, this model is based on optimizing behaviour of economic agents subject to generalized adjustment costs. Moreover, the model has been designed to have a good empirical fit and to ensure that dynamic responses to a wide variety of (standard) shocks are in line with observed evidence. With a right balance between theoretical consistency and empirical fit the model is able to provide a credible narrative for observed economic developments. In this class of models expectations play a crucial role and are based on a VAR or are determined in a model consistent manner. In the version reported in the following pages the expectation formation is VAR-based.

As the model is devised to become a core model in the forecasting process, it is required to provide details at a granular level. In addition to the rich set of reporting variables, the model also needs to have in place several transmission channels and features that can inform the discussions in the policy and forecasting process. Nevertheless, a higher degree of granularity implies some inevitable trade-offs between the data-fit and the theoretical coherence. The chosen modelling strategy relies on a block-by-block representation of the economy allowing for an additional degree of flexibility necessary to adapt to a changing economic and policy environment and to

speak to current policy questions.

ECB-BASE also allows for an explicit role of the financial sector and accounts for a richer articulation and a more realistic magnitude of the monetary policy transmission mechanism than the current NMCM. In particular, the model features a risk free term structure determined by the expectations hypothesis. This provides the basis for the construction of the lending rates, affecting various parts of the economy. In addition to interest rates, a special role is given to housing and financial wealth. The inclusion of the interest rates and wealth channel makes the model suitable to analyze the effects of unconventional monetary policy measures such as asset purchases via the long-term rate.

The model has been scrutinized under the lens of a number of diagnostic checks to ensure that its blocks can have a meaningful use even in a partial or sectoral setting, and that the overall system's properties are consistent with prior expectations and benchmarks. In particular, the paper shows that the model has a well-defined steady state and converges to its balanced growth path in the long run. Moreover, impulse response functions computed at the steady state illustrate that the dynamic responses of the system to macroeconomic shocks are in line with those of standard macroeconometric models and with conventional empirical benchmarks. Also, the model has reasonable forecasting abilities in a RMSE metric. In fact, the model can both produce purely model-based out-of-sample predictions and allow for the possibility of including off-model information via exogenous add-factors especially when the underlying structure of the economy is changing too fast for a quick update of the model to account for such changes. Finally, stochastic simulations can be used to characterize uncertainty around baseline projections and scenarios analyses.

All models are wrong; some models
are useful.
GEORGE BOX

Preamble

This is the first of a series of papers documenting an ambitious endeavour which is the tangible result of a collective work and interactions of several people who have developed, implemented, supported, managed, and sponsored the project at different stages and in different forms. They all share the merit of this venture.

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We are aware that this model is wrong, as all other models. We hope it is going to be useful.

1 Introduction

This paper introduces ECB-BASE, the blueprint of a new ECB multi-country (ECB-MC) semi-structural model.¹ It is part of a broader project whose main goal is to enlarge the current suite of models used for the model-based policy advice at the ECB with a new version of the semi-structural multi-country model of the euro area and of its five largest countries. The new model will replace the current multi-country model (NMCM, documented in Dieppe et al., 2011) as one of the main macro models in the ECB projection process. The version documented in the following pages is estimated on euro area data that cover the sample 1995Q1-2016Q4. Other papers will follow describing the country versions and their properties. Regular updates of model versions and simulation results will also be posted on the ECB website.

The design of the new model is aligned to its ultimate purposes, in the context of the forecasting and policy simulation exercises at the ECB, namely to (i) account for the relationships among key macroeconomic variables in a systematic manner; (ii) provide input to the complex process of macroeconomic forecasting; and (iii) conduct scenario analyses and policy simulations (Constâncio (2017)).

ECB-BASE is a large-scale, estimated, semi-structural model inspired by the experience of other policy institutions. The main sources of inspiration for the new model are the FRB/US model – developed and maintained at the Federal Reserve Board – and the LENS model – developed and maintained at the Bank of Canada.²

Mimicking the main functions and characteristics of these models, ECB-BASE is designed to satisfy a number of desirable properties which will be important to achieve its final objectives.

Theoretically sound and empirically consistent...

The ECB-MC is based on optimizing behaviour of economic agents subject to generalized adjustment costs. The model adheres to economic theory, albeit in a less stringent way than a DSGE model. Moreover, the model has a good empirical fit ensuring that dynamic responses to a wide variety of (standard) shocks are in line with observed evidence. With a right balance

¹ECB-BASE refers to the estimated euro area version, that serves as the blueprint for the country models (ECB-MC). Since the structure of the models are almost identical, we will refer to the model interchangeably as ECB-BASE and ECB-MC unless stated otherwise.

²The FRB/US model has been in use at the Federal Reserve Board since 1996. Its documentation can be found in Brayton and Tinsley (1996) and at: <https://www.federalreserve.gov/econres/us-models-about.htm>. The LENS model has been in use at the Bank of Canada since 2014. Its documentation can be found in Gervais and Gosselin (2014)

between theoretical consistency and empirical fit the model is able to provide a credible narrative for observed economic developments. Expectations also play an important role and are based on a VAR or are determined in a model consistent manner. Finally, the model features convergence to an economically derived steady state along a balanced growth path, allowing for a structural interpretation of deviations from steady state.

...with a flexible and rich structure...

The ECB-MC is devised to become a core model in the forecasting process. Therefore, the model is required to provide details at a granular level, such as a decomposition of the demand components of GDP, including the respective deflators, a rich supply side to understand drivers of potential growth, a detailed financial, fiscal and trade sectors, as well as the inclusion of HICP and its subcomponents. In addition to the rich set of reporting variables, the model also needs to provide many transmission channels and features to support the discussions in the policy and forecasting process. Nevertheless, a higher degree of granularity implies some inevitable trade-offs between the data-fit and the theoretical coherence. The chosen modelling strategy relies on a block-by-block representation of the economy allowing for an additional degree of flexibility necessary to adapt to a changing economic and policy environment and to speak to current policy questions.

...and a realistic financial sector and monetary transmission mechanism

The new model provides an explicit role for the financial sector, which accounts for a richer articulation and a more realistic magnitude of the monetary policy transmission mechanism. In particular, the model features a risk free term structure determined by the expectations hypothesis. This provides the basis for the construction of the lending rates which affect wealth, property income and the demand components. In addition to interest rates, a special role is given to housing and financial wealth. The inclusion of the interest rates and wealth channel makes the model suitable to analyse the effects of unconventional monetary policy measures such as asset purchases via the long-term rate.

Fit for model-based forecast and structural change

The ECB-MC model is able to produce purely model based out-of-sample predictions, allowing also for the possibility of including off-model information via exogenous add-factors. When

the underlying structure of the economy is changing quickly one needs to update the model or judgmentally manage the model to account for such change. Important consideration is given to drawing patterns from the model residuals in order to exploit past prediction errors to increase the accuracy of forecasts.

This document describes the main technical features of the model and illustrates its properties with (i) a selective shock analysis based on a version that only contains backward-looking (as opposed to rational) expectation formation and (ii) some illustrative forecasting exercises.

The following pages contain: A bird's eye-view of the model (Section 2); A technical description of the main sectors and features (Section 3); A selective analysis of the main model properties (Section 4); An appendix with additional technical details on specification and estimation (Appendix A and B); A proposal for the use of the residuals for inference (Appendix C); Tables and charts (Appendix D).

2 A brief overview of the model

The ECB-BASE is an estimated large-scale model that features a high level of detail and an elevated number of endogenous variables following its main inspirational source, the FRB/US model. This section provides a bird-eye view of the main features of the model before illustrating with more details its structure and properties in the next sections.

2.1 Basic structure

The model can be broadly sketched into a demand, a supply and a financial block.

Demand

In the demand block we group households, firms, government and foreign sector behaviours. Consumption is modelled differentiating between two groups of agents. On the one hand, optimising agents with a high discount factor (reflecting uninsurable individual income risk) consume optimally subject to adjustment costs. For these households, the long-term target depends on permanent income. The latter is constructed from different sources of income, where the propensities to consume differ among the income components. On the other hand, the liquidity constrained households behave in a hand-to-mouth way and spend all their total disposable income in each period.

Private investment behaviour is determined by forward-looking firms optimizing their investment plans based on user costs and the expected price (value) of investment and output growth, in line with standard neoclassical investment theory (Jorgenson (1967)). In the short-run, agents deviate from the long term target due to adjustment costs. Moreover, private investment is also affected by lagged output directly, to capture either the effects of sales on liquidity-constrained firms' ability to invest, or sentiment effects.

In contrast to private investment, government investment does not follow an optimizing approach. The block for the government sector provides a detailed accounting of the main fiscal variables. In particular, the revenue side is modelled through implicit tax rates whose dynamics are specified in terms of past deviations from the trend, whereas the spending side is modelled using an error correction mechanism around specified trend expenditures. In addition to modelling the revenue and spending sides, a specific equation is introduced for interest payments. The equations for fiscal variables presented in this paper do not include a debt deviation term, which is necessary to ensure government debt stabilisation in the long run.³

The foreign sector affects domestic prices and real activity through equations for imports and exports of goods and services and their deflators. The net trade component of GDP is disentangled into intra- and extra-euro area imports and exports. Determinants for the export dynamics are relative prices and world demand for euro area products. On the import side, relative prices and domestic absorption determine overall import demand. The trade-weighted euro exchange rate is modelled by assuming an uncovered interest rate parity (UIP), which links the expected real return on safe long-run assets abroad to those in the euro area. Foreign nominal interest rates are exogenous and captured by the US long rate.

Supply

The supply block models the production factors, capital and labour, within a Cobb-Douglas production function. Labour augmenting technological progress is added to come up with a notion of potential output, based on the trend components of these factors. The labour market is centered around a labour demand equation for heads based on a derivation of target employment from the marginal product of labour. In addition, self-employed are also modelled by ensuring that the ratio of self-employed to employees is constant in steady state. This part of the model

³More specifically, the current model does provide a stationary debt level in steady state, but the level itself is not determined. In an extended version of the fiscal block we include measures of debt targeting and counter-cyclical fiscal policy.

also includes the modelling of prices and wages. The core inflation measures (GDP deflator and wages per head) are based on forward-looking Phillips curves inside a New Keynesian model with involuntary unemployment (Galí, 2011 and Galí, Smets, and Wouters, 2012), estimated in a small system with unemployment and output gap defining a satellite two-country model including the euro area and the rest of the world. The auxiliary model is called WAPRO, from Wage-Price-Output gap. The import price deflator, modelled in the trade block, together with the GDP deflator, are the determinants of the other demand deflators and the HICP and its subcomponents.

Financial

Finally, the financial block models wealth, monetary policy and a number of interest rates. Modelling of interest rates starts by evaluating a risk-free term structure in line with the expectation theory. The risk-free rates are then combined with endogenously modelled risk spreads to estimate specific lending rates. The wealth components are derived by modelling the stock of financial and housing wealth as well as the flows in terms of property income from financial assets and housing.

2.2 Expectation formation

Similarly to the FRB/US model, the ECB-BASE can technically allow for two alternative ways of forming the expectations of the different agents. Specifically, expectations can be either based on projections from an estimated small-scale auxiliary VAR model (VAR or limited information expectations) or consistent with a full knowledge of the dynamics of the model (model-consistent or rational expectations). The latter case assumes that agents are fully rational and their expectations are based on the solution of the model under the assumption that also the expected variables follow the internal logic of the model. Rational expectations are sometimes criticized as being overly optimistic on the assumption that agents have a complete understanding of the economy and base their expectation on this understanding. Hence, the VAR expectation case assumes only limited knowledge of the joint dynamics of the variables and corresponds to the same restricted information set used in the estimation of the model. Specifically, the VARs share a core set of macro variables: the policy rate, the GDP deflator, and the output gap. This design can be interpreted as a limited form of rational expectations. The system of the core VAR variables is augmented by the specific variable for which expectations are being formed.

In principle, there exists the technical possibility of switching between these two approaches or even to explore the implications of a combination of the two alternative characterizations of the expectations formation process. The latter might be useful given that various economic agents might differ in their knowledge about the current economy and its outlook. The current version of the ECB-BASE combines a rational expectation mechanism embedded in the building block of the supply side (WAPRO) while households and firms in the other model blocks base their expectations on the limited information and average historical relationships embedded in the VAR models. A version with a fully consistent expectation formation is under construction.

2.3 Parametrization and estimation

The ECB-BASE is a large-scale model and this makes it hard to estimate the full set of equations simultaneously. With the exception of the WAPRO block, which is estimated as a system, the rest of the model equations are estimated in a single-equation mode.

Our estimation strategy follows four principles. First, some of the parameters governing the model's long- and short-run relationships, such as factor elasticities in the production function or the determinants of the discount factor, are calibrated based on macro and micro empirical evidence. Most long- and short-run relationships are then estimated using co-integration and iterative OLS techniques.

Second, the estimation of the equations containing expectation terms is performed in two steps: (i) a separate estimation of a VAR that contains a “condensed” model of the overall economy and the sector-specific variables to be forecasted; (ii) use the forecasted variable as a proxy for the expectation term in a specific equation.

Third, following the FRB/US approach, the rigidities that apply to consumption, investment, a part of financial and labour markets are specified as a generalized form of adjustment costs, polynomial adjustment costs, or PAC (Tinsley, 2002). The main idea of the PAC approach is that agents cannot costlessly adapt their behaviour to be instantaneously in line with the optimality condition. Instead they choose an optimal path of their decision variable subject to minimizing the associated adjustment costs. The order of these adjustment costs in each PAC equation is then determined empirically as part of the estimation process. In the behavioral equations where the PAC approach is used, therefore, there is no external source of serial persistence.

Fourth, our general estimation strategy combines classical and Bayesian methods and the choice between the two approaches is dictated more by pragmatic usefulness and empirical

implementation than by philosophical orientation. Sometimes, especially in those cases where a pure calibration is challenged by the available data, a Bayesian approach reduces the arbitrariness of a dogmatic parameter elicitation. In other occasions, on the contrary, the relatively small sample on which the euro area data is collected makes an estimation based on the combination of data and prior information (typically based on micro evidence or on US analogous data) not only practical but also more realistic.

2.4 Model properties and validation

After the estimation, the model (in partial and general equilibrium) is scrutinized under the lens of a number of diagnostic checks with the twofold idea of ensuring that (i) the model's blocks can have a meaningful use even in a partial setting (e.g. by sector), and (ii) the overall system properties are consistent with either the empirical evidence or the dynamics of other (structural and non-structural) euro area models available at the ECB. In this paper, as remarked above, we focus on selected system's properties (a few impulse response functions to selected shocks and a pseudo-recursive forecast evaluation).

On a final note, it is worth mentioning here that the model has a well-defined steady state and converges to its balanced growth path in the long run. The main model properties in terms of impulse response functions are in fact computed at the steady state and the responses to a given shock are perceived as deviations from this baseline after the shock.

Notice that a full description of the equilibrium conditions in the stationarized steady state can be obtained by imposing rather uncontroversial assumptions such as a common growth rate for nominal (or real) endogenous variables. A full dynamic convergence of the model can then be reached by simply simulating the model from the last data point and setting all residuals consistently with this data point while assuming that they are zero from that point onward.

3 Model structure

Following the scheme outlined above, this section describes in more detail the model and its main blocks (demand, supply, and financial). As a general principle, for a clear exposition we structure individual blocks in three parts: first, we provide theoretical foundations when available or a literature-based motivation and setup; then we translate the theoretical model

into an econometric or empirical counterpart, which also specifies data, observable and derived variables; and finally we briefly present calibration and estimation results. We deviate from this systematic structure for the presentation of the government sector and the financial wealth where the setup follows formal accounting frameworks and does not necessarily rely on a specific theoretical background or follow from an optimization problem.

For the technical description of the model we adopt the standard notation convention, with uppercase letters representing variables in levels, lowercase letters represent log transformations, while Greek letters are reserved for parameters and coefficients. Variables with the hat accent represent gap measures expressed as log differences.

Regarding the estimation procedure, an iterative OLS is employed whenever a stochastic behaviour of variables is represented by the PAC equation, a 2-step framework with specified long-run target and short-run error-correction procedure is approached by a 2-step Engle-Granger procedure, whereas single empirical equations are estimated using simple OLS. In some cases, a quasi calibration is performed by using tight parameter priors. The system part of the model is estimated using Bayesian techniques.

3.1 The demand block

In this section we describe the demand behaviours of households, firms, government and foreign sector.

3.1.1 Household consumption

Theory and Setup. The consumption block is built around the permanent income hypothesis used in most macro-economic models. Our specification, based on the FRB/US model, enriches the standard approach by three additional features.⁴ First, the optimal consumption decision is a decision under uncertainty on future income streams. In this setting, future income is discounted with a standard time discount factor and by a risk adjustment factor. In combination these factors are leading to higher discounting. Second, persistence is introduced into the system by the general form of polynomial adjustment costs (PAC), rather than modelling specific rigidities, such as habit formation. Finally, two variants of heterogeneous consumption behaviour are introduced. In the first variant, we model two types of households: optimizing households, who

⁴See Brayton and Tinsley (1996) and Laubach and Reifschneider (2003)

are maximizing their expected lifetime utility subject to the resource constraint, and hand-to-mouth consumers whose consumption changes in line with the change of current income.⁵ In the second variant, heterogeneity is introduced by assuming that the economy is populated by different age cohorts with different propensities to consume and different income compositions depending on their life-cycle position. At the aggregate level, this implies different propensities to consume out of different income sources.

The optimization problem is built around risk averse households and uncertain future income streams, implying a discount factor significantly above the real interest rate. The life-time optimization problem can be written as follows:

$$\max \left\{ V = \mathbb{E}_t \sum_{j=0}^D \beta^j \frac{C_{t+j}^\gamma}{\gamma} \right\} \quad (1)$$

subject to

$$\mathbb{E}_t \sum_{j=0}^D \frac{C_{t+j}}{(1+R)^j} = HW_t + \mathbb{E}_t \sum_{j=0}^D \frac{Y_{t+j}}{(1+R)^j}, \quad (2)$$

where D is the number of remaining periods of life, C_t is consumption, γ is the risk aversion parameter, β is the rate of time preference, Y_t is labour income, R is the rate of return on savings, and HW_t is the value of capital assets at the start of the period t . \mathbb{E}_t denotes the expected value conditional on information up to time t .

Based on the first order condition and the resource constraint it is possible to derive an approximate solution for consumption as a function of risk adjusted lifetime income:

$$C_t \approx \eta_c \left[HW_t + \sum_{j=0}^D \frac{\phi_{t+j} \mathbb{E}_t Y_{t+j}}{(1+R)^j} \right] \quad (3)$$

with

$$\eta_c = \left[\sum_{j=0}^D \frac{[\beta^j (1+R)^j]^{-\frac{1}{\gamma-1}}}{(1+R)^j} \right]^{-1} \quad (4)$$

This expression shows that future income is discounted not only by the real interest rate, but also by a risk factor ϕ_t .

⁵The terms rule-of-thumb, hand-to-mouth and liquidity constrained households are used interchangeably in the paper.

Empirical Specification. For the empirical implementation of the consumption behaviour we use the PAC approach. In such a setting we take a version of equation (3) as target, but assume that agents' adjustment costs delay reaching the target.⁶

The introduction of heterogeneous age cohorts implies that we can write the (log) target as the sum of the different permanent income and wealth components with different propensities to consume:

$$c_t^* = \eta_0 + \eta_T eyh_t^T + \eta_P eyh_t^P + \eta_D hw_t^D + \eta_L eyh_t^L \quad (5)$$

where, by construction, $\eta_T + \eta_P + \eta_D + \eta_L = 1$.⁷ The consumption target (c^*) depends on expected permanent labour, transfer and property incomes (eyh^L , eyh^T , eyh^P respectively), as well as on financial and housing wealth of households (hw^D). Notice that, when constructing the target, the permanent income variables are not directly observed. Their construction is described in details in the appendix B.1.

The second group of households faces liquidity constraints, and follows a rule-of-thumb (ROT) behaviour, implying that consumption moves in line with their labour and transfer income:

$$\Delta c_t = \Delta(yh_t^L + yh_t^T) \quad (6)$$

Combining the behaviour of both types of households, it is possible to derive the following dynamic behaviour of aggregate consumption:

$$\begin{aligned} \Delta c_t = (1 - \theta) & \left(a_0 (c_{t-1}^* - c_{t-1}) + \sum_{i=1}^{m-1} a_i \Delta c_{t-i} + \beta_1 x_t + \mathbb{E}_{t-1} \sum_{j=0}^{\infty} d_j \Delta c_{t+j}^* \right) \\ & + \theta \Delta(yh_t^L + yh_t^T) + \epsilon_t^C \end{aligned} \quad (7)$$

where θ is the share of rule-of-thumb consumers, a_0 is the coefficient on the deviation of consumption from its target and a_i gives the weights on the backward looking terms. The term $\mathbb{E}_{t-1} \sum_{j=0}^{\infty} d_j \Delta c_{t+j}^*$ represents the expectations of future targets. Finally, notice that the dynamic equation has been arbitrarily augmented with an additional explanatory variable, x_t , which denotes the spread between the lending rate on consumption and the risk-free rate to account for direct effects of financial factors on durable consumption (a component that, unlike

⁶The appendix A.1 provides the general derivation of the PAC approach and its econometric implementation. Appendix B.1 derives a detailed description of the consumption problem and the necessary steps to derive the PAC consumption equation.

⁷See appendix B.1 for further details.

in the original FRB/US model, is not modelled separately due to data limitations).

Estimation and Results. The estimates of target and dynamic equations are reported in table D.1. All expectation terms in the consumption block are estimated under VAR-based expectations as described in section 2 and in the appendix A.2, where technical details on the VAR estimation and the construction of the expected targets are provided. The upper part of the table shows the estimation results for the consumption target (equation (5)).⁸ The propensities to consume out of the subcomponents of income are given by the elasticities in table D.1 weighted by the inverse share of the respective subcomponent to total income. Notice that transfer income is estimated to provide the highest propensity to consume while property income shows the lowest propensity. The consumption elasticity to financial and housing wealth, η_D , is estimated to be rather low. The lower part of table D.1 displays the estimation results of the dynamic equation. The relatively high estimated coefficient on the lagged deviation from target consumption implies a strong tendency for mean reversion. The share of liquidity constrained households is estimated to be slightly above a third of the total number of households. The additional variable on the spread on the lending rate is entering with a small negative coefficient and points towards a significant but relatively low direct effect of interest rates on consumption.⁹

3.1.2 Investment

Business Investment

Theory and Setup: The investment behavior is derived from a standard optimization problem, where firms maximize their profits subject to the capital accumulation equation. With respect to the latter, we adopt a time-to-build assumption according to which current investments enter into the capital stock in the next period only. The profit optimization problem can be written as:

$$\max_{\{K_t, I_t\}} \sum_{j=0}^{\infty} \left(\frac{1}{1 + R_{t+j}} \right)^j \{Y_{t+j} - W_{t+j}N_{t+j} - RP_{t+j}I_{t+j}\}$$

⁸A time trend complete the empirical specification of the target to ensure a stationary gap between target and actual consumption. This trend – which is modelled to fit the observed data – is assumed to fade out when simulating the model, to ensure convergence of the system. See also the section 4.1 for a description of the implementation of the long run in the model.

⁹The term “direct effect” of interest rates on consumption is used here as opposed to the “indirect” effects of interest rates on consumption via the VAR and the expectations, as well as the endogenous reactions of the full system. See also section 3.3 and the appendix B.5 for a complete description of the financial channels.

subject to

$$K_{t+1} = (1 - \delta)K_t + I_t \quad (8)$$

and

$$Y_t = F(N_t, K_t) = N_t^\alpha K_t^{1-\alpha} \quad (9)$$

where Y_t is the output of a firm given by the Cobb-Douglas production function with constant returns to scale and two production inputs, capital K_t and labour N_t , whose costs are given by the relative price of investment good, RP_t , and wages, W_t .¹⁰ The depreciation rate of capital is given by δ .

The solution to the first order condition of the optimization problem yields an expression for the user costs of capital, UC , which can be expressed in terms of investment costs (determined by the depreciation rate and financing cost for business investments R_{t+1}^{ib}) and net capital gains given by the relative price growth:

$$(1 - \alpha) \frac{Y_{t+1}}{K_{t+1}} = RP_t \left\{ R_{t+1}^{ib} + \delta - (1 - \delta) \left(\frac{RP_{t+1} - RP_t}{RP_t} \right) \right\} \equiv UC_{t+1} \quad (10)$$

Empirical Specification: In order to exactly compute the user cost we need to obtain series for the relative investment prices, the financing costs of business investment and the depreciation rate. Relative investment prices are expressed as a ratio between the investment deflator and the GDP deflator, both observed within the national accounts data. The financing cost, R_{t+1}^{ib} , is a constructed series and is defined as a composite average of the real lending rate for non-financial corporations (NFC), the real corporate bond yields, and the real cost of equity, with weights for each particular rate resembling the structure of liabilities of the NFC sector in the sector accounts statistics. The derivation of individual rates is detailed in Section 3.3. Finally, the depreciation rate, δ , comes from the ECB's projection database as a time series and, for the calculation of the user costs, it is averaged over the available time span.

From the optimality condition in (10) we can derive an expression for the target capital stock

¹⁰To ease the description and without loss of generality we have dropped the technology progress term from the production function.

as:

$$K_t^* = \frac{S_t^K Y_t}{UC_t} \quad (11)$$

where S_t^K denotes the capital to output share. While constant in the optimization problem, this ratio is allowed to be time-varying in the empirical implementation, in line with the trend that it exhibits in the data.¹¹

Using (11) and the law of motion for capital, we can then derive the target for business investment:

$$IB_t^* = (G_{t+1}^{K^*} + \delta) K_t^* \quad (12)$$

where IB^* denotes the target for business investment and $G_{t+1}^{K^*}$ is the growth rate of the (target) capital stock, which is approximated by the real GDP growth.

Combining equations (11) and (12) we can rewrite the target for business investment in terms of output and the user costs of capital:

$$IB_t^* = (G_{t+1}^{K^*} + \delta) \frac{S_t^K Y_t}{UC_t} \quad (13)$$

Frictions associated with the target investment are modelled using the PAC approach. In the short-run, not all agents adjust their investment behavior according to a polynomial cost as some agents base their decisions on cash-flow considerations. The behaviour of the latter enters the short-run specification in an additive way and can be interpreted as the accelerator effect of output growth on investment growth. It can be shown that the short-run investment dynamics (in log) is given by the following equation:

$$\Delta ib_t = (1 - \theta^{ib}) \left(a_0^{ib} (ib_{t-1}^* - ib_{t-1}) + \sum_{k=1}^{m-1} a_k^{ib} \Delta ib_{t-k} + \mathbb{E}_{t-1} \sum_{j=0}^{\infty} d_j^{ib} \Delta ib_{t+j}^* \right) + \theta^{ib} \Delta y_{t-1} + \epsilon_t^{ib} \quad (14)$$

where ib_t is the log of business investment, a_0^{ib} is the mean reversion parameter associated with previous period deviations from the target investment, a_k^{ib} is an autoregressive coefficient associated with k quarters lagged business investment, and d_j reflects the effect of today's adjustment of investment decisions due to expected changes in the investment target by $\mathbb{E}_{t-1} \Delta ib_{t+j}$. Finally θ^{ib} represents the share of cash-flow constrained agents.

¹¹In particular, the capital to output share, s_t , is an HP filtered series of the ratio: $(IB_t/Y_t(\frac{\bar{Y}_t - Y_{t-1}}{Y_{t-1}} + \delta))UC_t$, where \bar{Y}_t is a measure of potential output.

Estimation and Results: For estimation purposes the share of cash-flow constrained agents has been set to 0.5 following the FRB/US specification.¹² The target has been computed as in (12). Estimates related to the adjustment dynamics associated with the target are presented in Table D.3. The results point towards a relatively costly adjustment process for business investment, as approximately 2/3 of past dynamics is carried over into the current period. On average, approximately 16% of past deviations from target the investment are corrected within a period.

Residential Investment

Motivation and Setup. In the ECB-BASE, residential investment is modelled from a firm's side. Solving a variant of the optimization problem described in (8) for the case of housing capital allows us to derive the expression for the desired level (target) of residential investment. In particular, the optimality condition becomes:

$$(1 - \alpha) \frac{Y_{t+1}^H}{KH_{t+1}} = RP_t^H \left\{ R_{t+1}^{NFC} + \delta^H - (1 - \delta^H) \left(\frac{RP_{t+1}^H - RP_t^H}{RP_t^H} \right) \right\} \equiv UC_{t+1}^H \quad (15)$$

where UC^H is the user cost of housing capital, Y^H represents output in the residential sector, KH is the housing capital stock, RP^H are relative prices (house prices over the residential investment deflator), δ^H is a depreciation rate of housing capital and R_{t+1}^{NFC} is the lending rate for non-financial corporations.

We assume that the target for residential investment can be expressed as a function of output, relative prices and pure user costs of housing capital (user costs excluding relative house prices):

$$IH_t^* = Y_t (UC_{-RP_t^H}^H)^{\beta_1^{ih}} (RP_t^H)^{\beta_2^{ih}} \quad (16)$$

where a clear distinction between relative house prices and user costs of housing capital is made to allow the examination of different elasticities of investment to the two respective components.

Empirical Specification. For the empirical estimation, priors associated with the implied Cobb-Douglas elasticities are set to $\beta_1^{ih} = -1$ and $\beta_2^{ih} = 1$. The empirical specification of (16) then

¹² Alternative calibrations of the share of cash-flow-constrained agents have been tested but led to non-significant changes in the dynamic behaviour.

takes the following log-linear form:

$$ih_t^* = \beta_0^{ih} + y_t + \beta_1^{ih} uc_t^H + \beta_2^{ih} rp_t^H + \gamma^{ih} T \quad (17)$$

where ih_t^* is the log of the target for residential investment, β_0^{ih} is a constant, coefficients β_1 and β_2 are Cobb-Douglass implied elasticities, and γ^{ih} accounts for the effect of a linear time trend T .

In the estimation of the target, the relative price series is defined as a ratio of the residential property price index (encompassing used and new dwellings) and the residential investment deflator observed within the national accounts data. The log of the user cost uc^H is not observed and its calculation follows closely the theoretical counterpart:

$$uc_t^H = \alpha^{uc,H} + \delta^H + r_t^{NFC} - \mathbb{E}_{t-1} \Delta rp_t^H \quad (18)$$

where $\alpha^{uc,H}$ represents other costs of housing capital (e.g. administrative and notary costs) and is calibrated to 0.034 following the empirical results of Muellbauer (2012); the lending rate for non-financial corporations, r_{t+1}^{NFC} , is defined within the financial sector in Section 3.3; and $\mathbb{E}_{t-1} \Delta rp_t^H$ indicates the expected growth of relative house prices calculated as a simple moving average of past prices.

The short-run equation for residential investment is derived using the PAC approach and additionally including the de-trended real GDP growth rate as an accelerator effect:

$$\Delta ih_t = a_0^{ih} (ih_{t-1}^* - ih_{t-1}) + \sum_{i=1}^2 a_i^{ih} \Delta ih_{t-i} + \mathbb{E}_{t-1} \sum_{j=0}^{\infty} d_j^{ih} \Delta ih_{t+j}^* + \theta^{ih} (\Delta y_t - \Delta \bar{y}_t) + e_t^{ih}.$$

Here a_0^{ih} is a mean reversion parameter for residential investment, a_i^{ih} are autoregressive coefficients, θ^{ih} measures the accelerator effect, and the coefficients d_j are loadings on expected future target dynamics.

Estimation and Results. The estimation results for the target investment are reported in Table D.4. Both the user cost of capital and the relative price growth are statistically significant and economically important to explain the long-run residential investment. An increase of the user cost by 1 % decreases residential investment by 0.16 %. The effect of relative prices is relatively

strong and significant with residential investment dynamics in line with the change in relative house prices.

The adjustment dynamics described by the PAC equation is presented in Table D.4. Results suggest that the short-run investment dynamics moves in line with changes in output growth, whose corresponding coefficient is relatively high and statistically significant. On average, 10 % of the deviation from the target is closed on a quarterly basis.

3.1.3 Government

The modelling of the government sector takes into account the trending nature of fiscal variables. In particular, the revenue side is modelled through implicit tax rates whose dynamics are specified in terms of past trend deviations, whereas the spending side is modelled using error correction mechanisms around specified trends for expenditures. In addition to modelling the revenue and spending sides, a specific equation is introduced for interest payments. The main text here specifies technical details of modelling government revenues and spending. Meanwhile Appendix B.3 provides a description of the overall structure of the government sector, a specification of the dynamics for the modelling of interest payments and a list of identities. The equations for fiscal variables specified below do not include a debt deviation term ensuring government debt stabilisation in the long run. Such a term can easily be added but the selection of affected fiscal variables should depend on a conducted model analysis.

Revenue Side

Due to the unavailability of data on actual tax bases, we express government revenues of a particular category i in terms of the associated macro basis and an implicit tax rate. In particular, government revenues can be expressed with the following identity:

$$REV_{i,t} = \tau_{i,t} BASE_{i,t} \quad (19)$$

where $REV_{i,t}$ denotes i revenue category which are reported in the government finance statistics, $\tau_{i,t}$ is the corresponding implicit tax rate, and $BASE_{i,t}$, the corresponding macro base (e.g. the relevant macro base for taxes on production and imports consists of final private consumption, observed in the national accounts data, and government purchases as a part of the government

finance statistics).¹³

The implicit tax rates are therefore constructed series, expressed as ratios between government revenues and the relevant macro basis, and are for modelling purposes assumed to oscillate around their trends according to the following dynamics:

$$\tilde{\tau}_{i,t} = \beta_1^\tau \tilde{\tau}_{i,t-1} + \beta_2^\tau \tilde{\tau}_{i,t-2} + \alpha^\tau \hat{y}_t + e_t^\tau \quad (20)$$

where $\tilde{\tau}$ represents deviations of the implicit tax rates from their own trend, $\tau - \tau^T$, β_j^τ is an autoregressive coefficient of order j , and α^τ represents oscillations of implicit tax rates in relation to the business cycle as captured by the output gap, \hat{y} .

The trend associated with the implicit tax rates is, for simulation purposes, specified in terms of its own autoregressive term and a target for the implicit tax rates:

$$\tau_t^T = 0.9\tau_t^T + 0.1\tau^* + e_t^{\tau^T} \quad (21)$$

where τ^* is the target implicit rate given by the average implicit rate observed during the period 2014Q1-2017Q4. We select this horizon as a reference for a long-term fiscal policy because this period exhibited a relatively high degree of stability (i.e. no sizable moves related to fiscal expansions and post-crisis consolidations episodes). Note that the specification implies a gradual convergence of the trend to the target implicit rate.

Spending Side

Government expenditures evolve around their trends which are anchored to a long-run target for government spending. Specifically, the target government spending, G^* , is expressed as a constant share, S_g , of the nominal potential output, \bar{Y}^N :

$$G_t^* = S_g \bar{Y}_t^N \quad (22)$$

The ratio S_g is a constructed value computed as the average share of government spending relative to potential output over the period 2014Q1 through 2017Q4 (i.e. analogously to the revenue side). The government expenditures are observed within the government finance statistics, while the nominal potential output series is taken from the supply block in Section 3.2.

¹³For a detailed decomposition of tax revenues and corresponding macro bases refer to Appendix B.3.

Trend expenditures, G^T , are expected to mean revert to the specified target in the short-run and are assumed to follow a smoothed growth of potential output in the long run:¹⁴

$$\Delta g_t^T = -0.1(g_{t-1}^* - g_{t-1}^T) + \frac{1}{4} \sum_{k=0}^3 \Delta \bar{y}_{t-k}^N + e_t^{g^T} \quad (23)$$

Finally, the short-run dynamics of government variables is described by the error correction around the trend variable and its own autoregressive term to capture the persistence of government spending. In the long-run, dynamic homogeneity between the government spending and the specified trend expenditures is imposed:

$$\Delta g_t = \alpha^g(g_{t-1}^T - g_{t-1}) + \sum_{k=1}^2 \beta_k^g \Delta g_{t-k} + (1 - \sum_{k=1}^2 \beta_k^g) \Delta g_t^T + e_t^g \quad (24)$$

To derive nominal values of expenditures categories modelled in real terms fiscal deflators are necessary. Most notably, this applies to government consumption and the government investment. In the case of the former, the government consumption deflator effectively becomes a function of private consumption deflator, average public wage and productivity in the government sector (see the identities in Appendix B.3). Government investment deflator (p^{GI}) is modelled using the following error correction specification:

$$p_t^{GI,*} = (1 - \nu)p_t^y + \nu p_t^m$$

$$\Delta p_t^{GI} = \alpha^{pg} (p_{t-1}^{GI,*} - p_{t-1}^{GI}) + \sum_{k=1}^4 \beta_k^{pg} \Delta p_{t-k}^{GI} + \left(1 - \sum_{k=1}^4 \beta_k^{pg}\right) \Delta p_t^y + e_t^{p^{GI}} \quad (25)$$

where p^y and p^m denote GDP and import deflators, and the parameter ν is calibrated based on input-output tables and takes up a value of 0.16 for the euro area.

3.1.4 Foreign trade

Motivation and Setup. The trade block is modelled based on the traditional analytical framework of Goldstein and Khan (1985) according to which trade volumes can be expressed as functions of demand determinants and relative prices.¹⁵ A special focus is given to modelling

¹⁴The equation for trend expenditure applicable to variables modelled in real terms takes a slightly different form, $\Delta g_t^T = -0.1(g_{t-1}^* - (g_{t-1}^T + p_{t-1}^g)) + \frac{1}{4} \sum_{k=0}^3 \Delta \bar{y}_{t-k} + e_t^{g^T}$, where p_t^g is the relevant fiscal deflator and \bar{y}_t is the real potential output.

¹⁵See also Sawyer and Sprinkle (1997), Fagan, Henry, and Mestre (2001).

the intra/extra-euro area trade split, where three specific volume components are explicitly modelled: total euro area imports, extra-euro area imports, and extra-euro area exports. The respective intra-trade volumes and total exports are derived using accounting identities. The corresponding trade deflators are modelled as functions of competitors prices and domestic and external cost pressures. Modelling the intra/extra split is motivated by the attempt of better understanding the effects of external developments on the euro area economy. In particular, the empirical evidence suggests that employing a bottom-up approach in modelling trade related prices provides a better account of oil, exchange-rate pass-through and pricing-to-market effects that may remain concealed if approached by models related to total deflators data.¹⁶ Consistent with the rest of the model, the long-run behaviour of the modelled components is separated from their short-term dynamics with frictions associated to the long-run being modelled using the error-correction approach with imposed dynamic homogeneity. The latter is ensured by filtering volume series around the long-run output growth. Trade volumes are modelled in real terms.

Empirical Specification. Modelling the trade block relies on total trade data and the data related to the intra/extra trade split. The total trade data are observed within the national accounts statistics, while intra and extra trade data are obtained from the ECB's projection database, where the series are constructed based on the National Central Banks reporting.

Modelling the **trade volumes** starts by setting behavioural equations for the total euro area imports. In the long-run, total euro area import is determined by the import demand and the differential between import and domestic prices:

$$m_t^* = \alpha_0^m + wer_t + \alpha_1^m(mtdno_t - p_t^y) \quad (26)$$

where m^* is the target (long-run) total euro area import, α_0^m includes deterministic terms, $mtdno$ is the non-energy total import deflator¹⁷, p^y is the GDP deflator, and wer is the import demand proxied by the weighted average of import content of domestic final demand components, and can be considered as an indicator of import absorption, $WER = \kappa_C C + \kappa_I I + \kappa_G G + \kappa_X X$,

¹⁶See Dieppe and Warmedinger (2007).

¹⁷In our empirical setup: $mtdno = (mtd - 0.095med - exr)/(1 - 0.095)$, with MTD being the total import price deflator, MED is the energy price deflator expressed in USD , and EXR is the USD-EUR exchange rate.

with weights, κ , calculated using the input-output tables¹⁸. Note that in the long-run, a unit elasticity to import demand is assumed.

The short-run dynamics of total imports is governed by the speed of the mean reversion to the long-run optimal level, change in import demand and change in nominal effective exchange rate (*EENX*):

$$\Delta m_t = \rho^m(m_{t-1} - m_{t-1}^*) + \beta_1^m \Delta wer_t + \beta_2^m \Delta eenx_t + e_t^m \quad (27)$$

The behavioural equations for the extra-euro area imports, *mx*, mimics the specification for the total import with the only exception being the use of extra-area import deflator *mxdno* instead of the total import deflator *mtdno*. Note that for the sake of simplicity, the extra-area import demand is assumed to move in parallel with the total import demand *wer*.

On the exports side, only extra-euro area exports are explicitly modelled with the long-run behaviour determined by the extra-euro area world demand and relative export prices:

$$xx_t^* = \alpha_0^{xx} + wdex_t + \alpha_1^{xx}(xxd_t - cxdex_t) \quad (28)$$

where xx_t^* is the target (long-run) extra-euro area exports, *wdex* is external euro area world demand, *xxd* is the external euro area exports deflator, *cxdex* measures extra-area competitors' export prices, and *wdex* is computed as a weighted average of imports of trading partners outside the euro area. Competitors' export prices, *cxdex*, are calculated as the weighted average of extra-area partners' export prices. In both cases, weights are constructed as export shares from the euro area to respective trading partner countries.¹⁹ Similarly as in the case of imports, the long-run elasticity of export to world demand is assumed to be one.

The short-run dynamics of exports is governed by the error correction term, extra-area world demand and the effective nominal exchange rate, *eenx* expressed as EUR against a bundle of

¹⁸Eurostat data. See also IAD (for Import-intensity-Adjusted Demand) approach by Bussière, Callegari, Ghironi, Sestieri, and Yamano (2013).

¹⁹See Hubrich and Karlsson (2010) for details.

foreign currencies:

$$\Delta xx_t = \rho^{xx}(xx_{t-1} - xx_{t-1}^*) + \beta_1^{xx} \Delta wdex_t + \beta_2^{xx} \Delta eenx_t + e_t^{xx} \quad (29)$$

The remaining non-modelled trade volumes, intra-euro area imports, MN , intra-euro area exports, XN , and total exports, X , are derived by accounting identities. The intra-euro area imports are defined as a residual of total and extra-euro area imports, $MN = M - MX$. We assume that intra-imports and intra-exports are equal in the long-run but allow for a constant discrepancy in the short-run due to a measurement error, $\Delta xn_t = a^{xn} + \Delta mn_t$.²⁰ Total exports are just the sum of intra and extra exports, $X = XN + XX$.

Trade deflators are modelled in a bottom up fashion, by considering first the extra/intra-area split where deflators are used to derive nominal trade volumes, from which, in turn, the total deflators are obtained. The extra-euro area export deflator is modelled as a function of competitive export prices and domestic and import costs of export production. The target expression for the extra-euro area deflator is given by:

$$xxd_t^* = \alpha_0^{xd} + cxdex_t + \beta^{xd}(\kappa_x mxd + (1 - \kappa_x)p^y - cxdex) \quad (30)$$

The corresponding short-term dynamics are defined as:

$$\Delta xxd_t = \rho^{xd}(xxd_{t-1} - xxd_{t-1}^*) + \beta_1^{xd} \Delta cxdex_t + \beta_2^{xd} \Delta mxd + \beta_3^{xd} \Delta p^y + e_t^{xd}$$

The long-run extra-euro area import deflator is modelled as a function of competitor prices on the import side, commodity prices, and domestic prices:

$$mxd_t^* = \alpha_0^{md} + cmdex + \beta_1^{md}(med - exr - cmdex) + \beta_2^{md}(p^y - cmdex)$$

where $cmdex$ represents a weighted average of trading partners' export prices with weights expressed according to euro area import shares, med are energy prices, and exr is the USD-EUR exchange rate.

²⁰It is easy to show that by assuming inequality in intra trade volumes, we implicitly assume also inequality in intra trade deflators. Therefore, for the purpose of deriving the intra-export deflator, the nominal intra-extra exports, XN^n , is additionally expressed in terms of growth of nominal intra-import growth: $\Delta xn_t^n = a^{xn} + \Delta(mn_t + mnd_t)$, where mnd is log of intra-import deflator.

The associated short-term dynamics is given by:

$$\begin{aligned}\Delta mxd_t = & \rho^{mxd}(mxd_{t-1} - mxd_{t-1}^*) + \beta_1^{mxd}\Delta(med_t - exr_t) \\ & + \beta_2^{mxd}\Delta(med_{t-1} - exr_{t-1}) + \beta_3^{mxd}\Delta cmdex_t + \beta_4^{mxd}\Delta xxd_{t-1} + e_t^{mxd}\end{aligned}\quad (31)$$

The intra-euro area imports deflator is considered to be a function of extra-euro area import prices and domestic prices:

$$mnd_t^* = \alpha_0^{mnd} + mxd_t + \beta_1^{mnd}(p^y - mxd) \quad (32)$$

In the short-run the intra-euro area import deflator depends on the error correction term, its own autoregressive dynamics and imported energy prices:

$$\begin{aligned}\Delta mnd_t = & \rho^{mnd}(mnd_{t-1} - mnd_{t-1}^*) + \beta_1^{mnd}\Delta(mnd_{t-1}) + \beta_2^{mnd}\Delta(cmdex_t) \\ & + \beta_3^{mn}\Delta(med_t - exr_t) + \beta_4^{mnd}\Delta(med_{t-1} - exr_{t-1}) + e_t^{mn}\end{aligned}\quad (33)$$

The remaining deflators associated with intra-euro area exports, XND , total exports, XTD and total imports, XMD are derived using simple accounting identities.²¹

Estimation and Results. Tables D.5 through D.10 present the results for the behaviour of the stochastically specified trade related variables in the error-correction framework, where long-run equations are estimated by OLS and the short-run dynamics are estimated by 2SLS.

Long-run equations for trade volumes assume aligned movement with demand components and a proportionate response to changes in relative prices, where the effect of relative prices is notably stronger for import quantities compared to the exports. In the short-run extra-euro area exports are closely aligned with the changes in world demand along with smaller but statistically significant effects of exchange rate movements. On the import side, the most significant demand component is exports, while the effect of investment demand is smaller and more uncertain.

The dynamics of import prices is mainly driven by its own autoregressive component and changes in competitors' prices, while the effect of imported energy inflation is relatively smaller. Export prices are largely driven by production costs reflected in changes of prices of domestic

²¹These are: $XND = XN_n/XN$; $XTD = (XXR \cdot XXD + XNR \cdot XND)/(XXR + XNR)$; $MTD = (MXR \cdot MXD + MNR \cdot MND)/(MXR + MNR)$;

and imported goods, while responsiveness to changes in competitors' export prices is smaller but statistically significant.

Exchange Rate and Price/Interest Parity

The long-run relationship between nominal exchange rate, prices and interest rates follows Baxter (1994) and is derived assuming sticky prices and by combining two arbitrage conditions, the *ex-ante* purchasing power parity (PPP) and the uncovered interest parity (UIP).

With fully flexible prices, the common assumption is that *ex-ante* PPP would hold:

$$E_t(s_{t+k} + p_{t+k} - p_{t+k}^f) = s_t + p_t - p_t^f \quad (34)$$

where s_t represents the log nominal exchange rate expressed as domestic currency over a basket of foreign currencies, p_t is the log domestic price level and p_t^f is the log foreign price level.

UIP condition can be expressed as follows:

$$E_t(s_{t+1} - s_t) = i_t - i_t^f \quad (35)$$

where i_t represents the domestic nominal interest rate and i_t^f is the foreign one. We define the real exchange rate as $q_t = s_t + p_t^f - p_t$ and *ex-ante* (anticipated) real interest rates as $r_t = i_t - E_t(p_{t+k} - p_t)$ and $r_t^f = i_t^f - E_t(p_{t+k}^f - p_t^f)$. Under the assumption of flexible prices one would obtain that $q_{t+1} = q_t$, i.e. the real exchange rate is constant or follows a random walk. This might not be the case when prices are sticky, as an actual real exchange rate can deviate from a real exchange rate under flexible prices. Following Baxter (1994), we assume that real exchange rate follows the following equation:

$$E_t(q_{t+k} - \bar{q}_{t+k}) = \theta^k(q_t - \bar{q}_t) \quad (36)$$

where \bar{q}_t is a real exchange rate under the assumption of flexible prices. The setting implies that the actual real exchange rate equals the real exchange rate under flexible prices only in the long run, while in the short run it can deviate from it. Combining equations (34), (35) and (36) we get:

$$q_t = \bar{q}_t + \alpha(r_t^f - r_t) \quad (37)$$

where $\alpha = 1/(1 - \theta^k)$. According to (37) the real exchange rate depends on the gap between foreign *ex-ante* and domestic real interest rates, $(r_t^f - r_t)$, and the term \bar{q}_t which is a constant or a random walk component.

Empirically we estimate the following equation:

$$s_t = \beta_1^s p_t + \beta_2^s p_t^f + \beta_3^s r_t + \beta_4^s r_t^f + \beta_5^s + \varepsilon_t \quad (38)$$

where we assume $\bar{q}_t = \beta_5 + \varepsilon_t$ with ε_t following an autoregressive process.²² In the estimation, domestic prices are captured by the GDP deflator, foreign prices are proxied by competitors' export prices denominated in USD, while *ex-ante* foreign real interest rates are calculated as a difference between the 10-year US nominal rate and the expected US inflation rate. In the estimation we impose the following constraints: $\beta_1 = 1$, $\beta_2 = -1$ and $\beta_4 = -\beta_3$. Results are provided in Table D.11.

3.2 The supply block

3.2.1 Production function

The supply side of the model is centered around a standard Cobb-Douglas production function.

$$Y_t(i) = F^i(N_t(i), K_t(i)) = A_t (K_t)^\alpha (\zeta^t N_t)^{(1-\alpha)}$$

where K_t denotes aggregate capital stock and N_t denotes aggregate employment. A_t is total factor productivity and ζ_t is labour augmenting technology.

Besides its role as integral part of the economic set-up of the model, the production function is used to calculate potential output. To determine potential output aggregate employment is replaced by aggregate hours worked:²³

$$H_t = \frac{H_t}{N_t} * \underbrace{\frac{N_t + U_t}{WAP_t}}_{part.rate} * WAP_t * \underbrace{\frac{N_t}{LF_t}}_{empl.rate} \quad (39)$$

where N_t denotes employment headcount, WAP_t denotes working age population and LF_t is the labour force. Let us denote the total hours per employee/employment headcount $\frac{H_t}{N_t}$ as

²²The residual, ε_t , is defined as $\varepsilon_t = \beta\varepsilon_{t-1} + \nu_t$ with estimated β being 0.95.

²³Note that the production in other part is written with employment rather than total hours. The two representations can be unified by assuming that hours worked per person are exogenous and technical progress is γ .

HPE_t , the participation rate as $LFPR$ and the employment rate as $(1 - U_t)$ where U_t is the unemployment rate, and rewrite the production function as:

$$Y_t = A_t K_t^\alpha (HPE_t * LFPR_t * WAP_t * (1 - U_t))^{1-\alpha} \quad (40)$$

Taking logs and denoting variables in logs with small case letter, we can write

$$y_t^* = a_t^* + \alpha * k_t + (1 - \alpha)(hpe_t^* + lfpr_t^* + wap_t^* + \log(1 - u_t^*)) \quad (41)$$

where $*$ indicates the trend value of the corresponding variable.

3.2.2 Labour market

The Labour market block of the ECB-BASE is centered around an equation for total employees. Furthermore the block features an equation for hours worked, a measure for unemployment and a separate modelling of self-employed and the labour force participation rate.²⁴

Employees

The setup is closely linked to other parts of the supply block, such as the definition of labour market trends in the Wage-Price-Output gap (WAPRO) block and the production function. The equation for total employees is central to the labour market block. Starting from this equation we model self-employed to come up with a measure of total employment.²⁵

Firms choose optimal capital (K_t) and labour input (N_t) to minimize total input costs $R_{K,t}K_t + w_tN_t$ subject to a technology constraint given by the production function $Y_t = A_t K_t^\alpha (\gamma^t N_t)^{1-\alpha}$. Consistent with the situation in the European labour markets, we assume that firms decide on an optimal employment target (N_t^*) which is reached only gradually. Technically this translates into using a PAC approach, the same used for the demand components and described above. Hours worked per worker are then chosen to meet aggregate demand.

²⁴The current version does not feature a disaggregation into private and public labour. This simplification helps reduce the complexity to compute the steady state and guarantee a long-run convergence of the model.

²⁵Strictly speaking, this part could very well be included in the demand block together with the firm's problem for investment. We prefer to conventionally group it here with the wage-price-output gap block as part of the supply side of the economy.

This setup yields the following familiar first order conditions:

$$(1 - \alpha) \frac{Y_t}{N_t} MC_t = W_t$$

where MC is the Lagrange multiplier on the technology constraint.

The PAC equation for employees follows the generic logic explained in A.1, and can be written as:

$$\Delta n_t = a_0^n (n_{t-1}^* - n_{t-1}) + a_1^n \Delta n_{t-1} + \mathbb{E}_{t-1} \sum_{j=0}^{\infty} d_j \Delta n_{t+j}^* + e^n \quad (42)$$

where small letter denotes the logarithm of the respective variable and the target variable is determined by:

$$n_t^* = mc + \log(1 - \alpha) + y_t - w_t$$

The specification given in equation (42) is very parsimonious. Extensions of this specification are possible and will be evaluated case-by case in the country version of the model to account for likely heterogeneity in the country labour markets. For the main dynamics of the model the core equation suffices. The empirical estimates are reported in Table D.12. They show that the

Labour force participation rate

The change in the labour force participation rate (lfp_t) is modelled as a function of an error correction term and the change in the unemployment rate:

$$\Delta lfp_t - \Delta lfp_t^T = -0.5(lfp_{t-1} - lfp_{t-1}^T) - \beta_u \Delta u_t$$

Unemployment rate

Finally, we compute the unemployment rate as

$$U_t = 100 \left(1 - \frac{Employment}{Labour\ force} \right) \quad (43)$$

where the labour force is calculated as the working age population multiplied by the labour market participation rate.

3.2.3 Prices and wages

The setup of the price block consists of two steps. The first step focuses on the key measures of domestic inflation: the GDP deflator and wages. In the second step, domestic inflation and imported inflation are combined to model the demand deflators and the subcomponents of HICP inflation. With this approach it is possible to distinguish and model separately domestic and foreign price determinants.

A system approach for domestic prices and wages

To ensure plausible system properties, especially in the nexus of prices, wages, output gap and interest rates, a small general equilibrium model is estimated. Key features of the approach such as a Kalman filter based, Bayesian system estimation, some microfoundations and the explicit modelling of expectations are borrowed from the dynamic stochastic general equilibrium literature. However, several of the economic restrictions are relaxed to improve the data fit. While the structure of the equations is guided by optimizing behaviour of the economic agents, the cross-equation restrictions of the structural parameters are ignored and instead reduced-form estimates of the loading coefficients are estimated. The advantages of this approach relate to an explicit focus on system properties and the possibility to estimate unobserved concepts such as the output gap or measures of natural unemployment measures such as the NAIRU.

The approach is similar to the IMF approach as described in the QPM series (See Carabenciov, Ermolaev, Freedman, Juillard, Kamenik, Korshunov, Laxton, and Laxton (2008)).

The model is operational under two different forms of expectation formation, rational and VAR-based expectations.²⁶ In what follows the underlying VAR is the same as the Base VAR estimated for the PAC specification of the demand and labour equations.

The model

The euro area version of the wage, price and output gap block is modelled as a two country model. The first country is defined as the euro area (EA), the second country is defined as the rest of the world (ROW). The cross country relations between these two entities are kept simple and comprise the determination of (i) the exchange rate based on an uncovered interest rate parity (UIP) condition, (ii) the implicit trade as a function of foreign demand, and (iii) the transmission of foreign prices to consumer prices.

The structural part of the EA model consists of an IS equation, a price Phillips curve, a wage Phillips curve, an import price Phillips curve and a Taylor rule. It is augmented with an IS curve, Phillips curve and Taylor rule for the ROW and with non-behavioural processes driving oil prices and underlying trends. To connect this part to the setting of the remaining model with unemployment, the underlying structure follows the one described in Galí et al. (2012) (GSW), but deviates in some respects to serve as the wage and price block in the ECB-BASE.²⁷ The full model including the definition of the database is described in the appendix B.4

Output, Price and Wage Inflation

Price inflation is modelled via a reduced form NK Phillips Curve around a time-varying inflation attractor ($\bar{\pi}$). The equation shows that actual inflation depends both on a measure of expected inflation and on past inflation, the output gap (\hat{y}_t) and the wage gap (\hat{w}_t). Algebraically:

$$\pi_t = \frac{1}{1 + \beta_\pi \delta_\pi} \left\{ (1 - \delta_\pi + \delta_\pi \beta_\pi - \beta_\pi) \bar{\pi}_t + \beta_\pi (\mathbb{E}_t \pi_{t+1}) + \delta_\pi \pi_{t-1} + \beta_y^\pi \left(\hat{w}_t + \left(\frac{\alpha}{1-\alpha} \right) \hat{y}_t \right) \right\} + e_t^\pi \quad (44)$$

²⁶See section 2 for a brief discussion of the two expectation formation mechanisms.

²⁷The model also contains a measure of the private consumption deflator. To capture the effect of exchange rates and oil prices on headline inflation an import price Phillips curve is introduced.

where α is the capital share, β_π the discount factor, δ_π is the inflation indexation parameter and β_y^π is the loading on marginal cost. Note that the expression for marginal costs in GSW starts from the deviation of the average price markup from its natural value. This expression can be rewritten as $-\frac{\alpha}{1-\alpha}\hat{y}_t - \hat{w}_t^w + \alpha\hat{k}_t$, and under the assumption that the capital gap is always closed, as shown in equation (44).²⁸

Real wage inflation is modelled via a reduced form NK Phillips wage curve. The equation shows that actual real wage inflation depends on future wage inflation and past consumer price inflation, the wage gap (\hat{w}_t) and the unemployment gap (\hat{u}_t):

$$\begin{aligned} \pi_t^w + \pi_t^C = & \frac{1}{1 + \beta_w \delta_w} \left\{ (1 - \beta_\pi^w) (1 - \delta_\pi^w) (\bar{\pi}_t + \Delta \bar{y}_t) \right. \\ & + \beta_\pi^w \mathbb{E}_t(\pi_{t+1}^w + \pi_{t+1}^C) + \delta_\pi^w (\pi_{t-1}^w + \pi_{t-1}^C) - \beta_u^{\pi^w} \hat{u}_t \Big\} \\ & + \xi_w \hat{w}_t + e_t^{\pi^w} \end{aligned} \quad (45)$$

In line with the notation of the price Phillips curve, β_π^w is the discount factor, δ_π^w is the inflation indexation parameter and $\beta_u^{\pi^w}$ is the loading on the unemployment gap.

In the equation above the GSW specification for the markup term has been augmented to include not only the unemployment gap but also the wage gap, with loading ξ_w . This improves the empirical fit and the convergence behaviour in the main model.²⁹

The wage gap is defined as the discrepancy between actual wages and the wage target in line with the first order condition for labour demand. Furthermore lagged wage growth has been added to improve the fit and system properties of the model. Wages are indexed to the private consumption deflator. To come up with a deflator of private consumption it is necessary to include foreign price determinants in the form of an import price Phillips curve with an impact of foreign inflation, the exchange rate and oil prices. The model is closed by a monetary policy rule and an IS curve, both described in Appendix B.64.

²⁸Note that to improve the dynamic properties in the ECB-BASE the original specification of the price curve in GSW has been replaced by a variant of Cogley and Sbordone (2008), who introduce indexation to a time-varying inflation trend to increase inflation persistence.

²⁹As in the case of prices we have applied a variant of the Cogley and Sbordone (2008) approach to model the wage setting process.

Demand deflators and HICP

The modelling of the demand deflators and HICP (and subaggregates) essentially relies on two inputs, the **GDP deflator** and the **import price deflator**. Moreover, the long-run convergence related to main prices and the GDP deflator is ensured through imposed dynamic homogeneity and error-correction behaviour, where the price attractor is set as a weighted sum of the GDP deflator and import prices. The GDP deflator determined in equation (44) is the central domestic price deflator in the model. As described above it is closely linked to the output gap, unemployment and wages. Furthermore, it defines the price level in line with production possibilities. The import price deflator is reflecting international factors such as foreign inflation, the exchange rate and oil prices.

The general long-run expression for domestic demand deflators and HICP core prices takes the following form:

$$p^* = (1 - \omega)p^y + \omega p^m \quad (46)$$

where p^* represents a long-run level of a particular price measure (domestic deflators and core HICP), p^y is the GDP deflator, p^m is log of import deflator, and ω is the import content in the final use of respective sector in the economy determined by 'Input/Output' tables. The short-run price dynamics follows the error correction model with the mean reversion constructed around the long-run price level and short run coefficient constrained in line with an imposed dynamic homogeneity:

$$\Delta p = \beta_0^p(p_{t-1}^* - p_{t-1}) + \beta_1^p \Delta p_t^* + \beta_2^p \Delta p_{t-1}^* + (1 - \beta_1^p - \beta_2^p) \Delta p_{t-1} + e \quad (47)$$

The estimated reduced form equations provide the basis for system-wide simulations and projection exercises.

From a system wide perspective, the price block is linked to WAPRO while the foreign block supplies the import price measure. Domestic demand deflators provide an essential feedback to the demand part of the model. HICP is in turn modelled as a standalone object and does not add any feedback loop to the model. Core HICP measures follow the framework set above, while HICP energy is allowed to follow its own dynamics based on energy consumption deflator and prices of oil (see Appendix B.4). HICP food is extracted as a linear combination of HICP excluding energy and HICP excluding food and energy. HICP headline is given by the weighted

average of HICP excluding energy and HICP energy.

The estimation results for the whole wage-price block are reported in Tables D.13 to D.16. While the estimated parameters cannot directly be compared to estimates of DSGE model, we can still observe some differences. On domestic inflation the coefficient on inflation is lower and the loading on marginal cost is higher than in typical DSGE models.

3.3 The financial block

The financial block of the ECB-BASE contains two parts. In the first part, we model the interest rates that are used as reference financing measures in the consumption, investment (business and residential), and housing prices equations. The second part of the financial block models household property income, wealth and the net foreign position, which are an essential part of the consumption block.

3.3.1 Policy rule and interest rates

Motivation and Setup. The financial block of the ECB-BASE is constructed around the reference short-rate that is assumed to follow a simple monetary policy rule. In line with the term-structure expectation theory, we model long-term market rates as the average of current and expected short-rates, over a particular maturity horizon, and a term-premium. Expectations about the future short-term rates are derived from the Base VAR setting,³⁰ where the long-run properties of the model are driven by two attractor variables, the market based inflation expectations and the expectations about the short-rate 10 years ahead.³¹ The term-premium of a long-run market rate is modeled separately as a function of expected macroeconomic conditions and external market developments. Finally, the short-term rate and the long-term market rate, alongside the risk premiums associated with particular debt and equity instruments, provide the basis for the construction of several lending rates and financing cost measures used in other parts of the ECB-BASE model. Figure D.1 provides a schematic representation of the interest rate modelling in the financial block. The next subsections elaborate on each part of this scheme.

³⁰For details on the Base VAR specification see Appendix A.2.

³¹Note the difference between the average expected short-rate and the short-rate expectations 10-years ahead. The former represents the average of a short-term interest rate over the entire horizon span, whereas the latter relate to expectations about the short-term interest rate at the end point of the relevant horizon. Short-rate expectations 10-years ahead are obtained using a simple term-structure modelling techniques.

Policy rule

The reference rate used in the model is the 3-month EURIBOR, which is assumed to follow a simple log-linear interest-rate rule:

$$i_t = \Phi_i i_{t-1} + (1 - \Phi_i)(\bar{\pi} + \bar{r}) + (1 - \Phi_i)\Phi_{\Pi}\hat{\pi}_t + \Phi_{\Delta\Pi}\Delta\pi + \Phi_y\Delta\hat{y}_t + e_t \quad (48)$$

where $\bar{\pi}$ is the inflation target, \bar{r} is the real natural rate, and $\hat{\pi}$ and \hat{y} are inflation and output gaps. Parameters and the real natural rate are calibrated in line with the New Area-Wide Model. The output gap is observed implicitly within the WAPRO block in section 3.2.3.

Interest Rates

By adopting the term-structure expectation hypothesis we can use the reference rate to derive the term-structure of market interest rates. Namely, an interest rate of a particular maturity m can be modelled as the sum of two components: an average of the current and expected short-term rate R_0 over the period spanned by the maturity horizon (the average expected short rate), and a term premium:

$$R_t(m) = \frac{1}{m} \sum_{i=0}^{m-1} R_{0,t+i} + TP_t(m) \quad (49)$$

Empirical Specification: The average expected short rate in (49) is obtained by a simple average of m -period ahead forecasts of the 3-month EURIBOR rate using the Base VAR:

$$\frac{1}{m} \mathbb{E}_t \sum_{i=0}^{m-1} R_{0,t+i} = \frac{1}{m} \sum_{i=0}^{m-1} H^i Z_t \quad (50)$$

where H is a matrix of the Base VAR coefficients and Z is a stacked vector of the Base VAR variables.

Once the average expected path of the short rate is obtained we can construct the term premium series as implied by (49). In particular, the term-premium is extracted as a difference between the observed 10-year EONIA rate and the average expected short rate estimated by

(50).³² The empirical model for the term-premium is specified by:

$$TP_t(m) = \alpha^{TP} + \rho^{TP} TP_{t-1}(m) + \beta_1^{TP} \frac{1}{m} \mathbb{E}_t \sum_{i=0}^m \hat{y}_{t+i} + \beta_2^{TP} TP_t^{US} + e_t^{TP} \quad (51)$$

where α^{TP} is a constant, ρ^{TP} is the autoregressive coefficient, β_1^{TP} reflects dependency on expected macroeconomic conditions, and β_2^{TP} accounts for external financial spillovers. The US market term premium is not directly observed and is constructed using a term-structure model employed by the ECB for internal analyses.

Lending Rates

The above setup provides a modelling basis for the construction of several lending rates and financing cost measures that are used in other parts of the ECB-BASE. Specifically, we construct lending rates as a composite of the short rate and the long-term market rate, plus a residual risk spread corresponding to each particular lending or financing rate i :

$$LR_t^i = v_s^i R_t(1) + v_l^i R_t(40) + RPR_t^i \quad (52)$$

where v_s^i and v_l^i are weights assigned to the short-term rate (3-month) and the long-run rate (10-year), and are obtained using the bank balance sheet statistics data. In particular, the data reflects normalized shares of short-term (3 month) and long-term (10 years) new and outstanding loan amounts associated with a specific lending rate i . RPR denotes a risk spread associated with each particular lending rate. The risk spread series is constructed as a difference between the observed lending rate and weighted average of short and long-term risk-free rate. Empirically, lending rate risk spreads are modelled in the following way:

$$RPR_t^i = \alpha^{RPR} + \rho^{RPR} RPR_{t-1} + \beta^{RPR} \frac{1}{m} \mathbb{E}_t \sum_{i=0}^m \hat{y}_{t+i} + e_t^{RPR} \quad (53)$$

where we model six particular lending rates ($i = 1 : 6$): (i) the consumer loan rate, one of the determinants of consumption; (ii) the mortgage rate, a determinant of house prices and property

³²Note that different types of market rates are used as observables for short- and long-term reference rates. The 3-month EURIBOR is used as a reference short rate due to the forecasting application of the model, which should include the 3-month EURIBOR as a forecasted category. However, the purpose of (49) is to derive a risk-free term-structure for which the Eonia rate is more appropriate, since the corresponding contracts do not entail a credit risk.

income; (iii) the deposit rate, a determinant of property income; and three interest rates that determine the user cost of capital in the business and residential investment block, namely: (iv) the lending rate for Non financial Corporations (NFC), (v) the corporate bond yields, and (vi) the cost of equity (see section B.5 for the construction of the cost of equity).

Estimation and results: Table D.18 reports the coefficient estimates for term-premium and risk spreads equations. The term premium of a 10-year risk-free bond largely depends on the domestic macroeconomic environment and financial developments in the US. The effects of the output gap and 10-year US term premium are in absolute quantitative terms almost the same with both relevant coefficients resembling a strong statistical significance. Estimation results for risk spreads reveal a substantial heterogeneity in the responsiveness of particular lending rates to general macroeconomic conditions. In particular, the effect of the changes on risk spreads ranges from 0.6 in case of consumer loans rates to a negligible effect in case of corporate bond yields.

3.3.2 Property Income and Wealth

Property income and wealth enter into the determination of target consumption. The explicit modelling of property income and wealth in the ECB-BASE results in a rich set of transmission channels from changes in yields and prices of different asset classes on consumption and therefore on the whole economy.

Property income

The household nominal disposable property income in the ECB-BASE consists of the gross operating surplus of households and the net property income as defined in non-financial sector accounts, minus taxes and social contributions. Specifically, nominal disposable household property income, $DINP$, is expressed by the following identity:³³

$$\begin{aligned} DINP_t &= GOS_t + IRN_t + DDN_t + RIN_t + OIN_t + RTN_t + OCT_t - WTP_t - SCP_t \\ DINP_t &= GOS_t + IRN_t + DDN_t + \chi_t^{PI} - WTP_t - SCP_t \end{aligned} \tag{54}$$

³³ All variables are expressed in net terms (credit - debit) and relate to the household sector.

where we explicitly model household gross operating surplus (GOS_t), the net interest income (IRN_t), and the dividend income (DDR_t).³⁴ χ_t^{PI} represents an exogenous term and groups levies and other property income components such as reinvested earnings (RIN_t), other investment income (OIN_t), non-residential property rents (RTN_t), and other capital transfers (OCT_t). Taxes (WTP_t) and social contributions (SCP_t) associated with the property income are modelled in the fiscal block. As implied above, all modelled variables, whose specification is provided in following subsections, are directly observable within the sector accounts statistics.

Gross operating surplus

The gross operating surplus of households is modelled as a share of nominal GDP (Y^N) and is assumed to be primarily determined by income from the residential housing capital:

$$\frac{GOS_t}{Y_t^N} = \alpha^{GS} + \beta_1^{GS} \frac{KH_t * IHD_t}{Y_t^N} + \beta_2^{GS} \frac{HPI_t}{P_t^c} + e_t^{GS} \quad (55)$$

where the first term represent the nominal housing stock, whereas the second term reflects house prices relative to the private consumption deflator. Real housing capital, KH , is specified in the Residential Investment block in Section 3.1.2, while the residential investment deflator (IHD), the private consumption deflator (P^c) and the residential property price index ($RPPI$) are directly observed variables. When modelling GOS , β_1^{GS} is determined by the average of the ratio between gross operating surplus and nominal housing stock and can be interpreted as the average return on the residential housing stock, while β_2^{GS} is estimated and captures time-variations in return to residential assets. Table D.19 shows that the annual average rate of return is set to approximately 2.5%, while the estimate of β_2^{GS} suggests that roughly 2% of the variation in relative house prices is captured by the household gross operating surplus to GDP ratio on a quarterly basis.

Net interest income

The net interest income of households is modelled as a share of GDP and depends on its own autoregressive term, previous period's net foreign assets (NFA) position relative to GDP, the general level of interest rates captured by STN , and a spread between mortgage rate (R^m) and

³⁴Household gross operating surplus is not including mixed income, which we assume to be a part of the labour income.

deposit rate (R^d) that captures the net concept given by interest payable and receivable:

$$\begin{aligned} \frac{IRN_t}{Y_{t-1}^N} = & \alpha^{IRN} + \rho^{IRN} \frac{IRN_{HH,t-1}}{Y_{t-2}^N} + \beta_1^{IRN} \frac{NFA_{t-1}}{Y_{t-1}^N} \\ & + \beta_2^{IRN} STN_t + \beta_3^{IRN} (R_t^m - R_t^d) + e^{IRN} \end{aligned} \quad (56)$$

In the estimation, the NFA series is observed in the sector accounts statistics, while mortgage and deposit rates are constructed as suggested by equation (52).

Table D.20 reports the estimation results. As expected, the ratio is highly autocorrelated, an increase in NFA gives rise to an increase of net interest income, a higher general level of interest rates implies higher net interest income, while a higher interest rate spread between payables and receivables reduces the net interest rate income.

Dividends

The dividends are modelled within the PAC framework, implying a dependency of the real dividends growth on: (i) the degree to which dividends were out of equilibrium in the previous period, (ii) lagged dividend growth, and (iii) expected growth of desired dividend income. The desired level of dividend income is assumed to be a constant fraction of the (real) gross operating surplus and mixed income:

$$ddr_t^* = \alpha^D + gosmir_t + t \quad (57)$$

where we also include a linear trend to improve the empirical fit. The corresponding short-run PAC equation is then:

$$\Delta ddr_t = a_0^d (ddr_{t-1}^* - ddr_{t-1}) + \sum_{i=1}^3 a_i^d \Delta ddr_{t-i} + \mathbb{E}_t \sum d_i^d \Delta ddr_{t+i}^* + e_t^{ddr} \quad (58)$$

Table D.21 reports the estimation results. The mean reversion towards the desired level of dividend income is relatively fast, as 25 percent of the gap is closed on a quarterly basis. The nominal dividends are obtained by multiplying real dividends by the private consumption deflator.

Wealth

The wealth of the household sector is defined as:

$$HHW_t = GHW_t + NFW_t \quad (59)$$

where GHW_t is the gross housing wealth and NFW_t is the net financial wealth, where both series are observed within the sector accounts statistics.³⁵

The stock of nominal housing wealth is revalued in line with changes in residential house prices:

$$GHW_t = IH_t + (1 - \delta^h)GHW_{t-1}(RPPI_t/RPPI_{t-1}) \quad (60)$$

where IH_t are nominal residential investments, where $RPPI_t$ is a house prices index (new and existing dwellings), and δ^h is the depreciation rate associated with the housing stock. Land and other non-modeled components are ignored. In the estimation, IH and $RPPI$ are observed variables produced by official statistics, while the depreciation represents an average of housing depreciation rates reported by NCBs within BMP exercises.

Household net financial wealth (NFW) at time t is a function of the nominal net disposable income and a revaluation term:

$$NFW_t = (Y_t - C_t - IH_t) + NFW_{t-1}REV_t \quad (61)$$

where Y_t is the Nominal Disposable Income, C_t is the Nominal (total) Consumption, IH_t is the Nominal Residential Investment (gross fixed capital formation, dwellings) and REV_t is the Revaluation Term.

Net financial wealth includes several types of assets (treasury bonds, equities, deposits, foreign assets, other debt securities) that need to be re-valuated in line with changes in corresponding asset prices. The revaluation term is defined as a weighted average of changes in respective asset prices (equity prices and debt securities) with weights reflecting the relative importance of the various components in the asset holdings of households:

$$REV_t = s_0 + s^{GB}(R_{t-1}^{GB}/R_t^{GB}) + s^{CB}(R_{t-1}^{CB}/R_t^{CB}) + s^{COE}(COE_{t-1}/COE_t) \quad (62)$$

³⁵Other parts of the household wealth consist of inventories, other investments, non produced assets, capital transfers and adjustments for pensions. These components are assumed to be exogenous and are not modeled explicitly.

where s_0 is the fraction of assets not subject to revaluation (mostly deposits with banks), s^{GB} is the fraction of government debt securities (held directly and indirectly by household) in total financial assets, s^{CB} is the fraction of corporate debt securities (held directly and indirectly by household) in total financial assets and s^{EQ} is the fraction of equities (listed and unlisted equity held by households) in total financial assets. R_t^{OIS10Y} is a long term interest rate (OIS 10YR), R_t^{CB} is the domestic corporate bond yield and COE_t is the cost of equity.

House Prices

The target of house prices is derived from an inverted demand curve resulting from the optimization problem of the consumer who chooses between different consumption subcomponents. The resulting equilibrium condition is the following:

$$RP_t^{H,*} = C_t^* \frac{1}{KH_t} \frac{1}{UC_t^{HP}} \chi \quad (63)$$

where RP are relative prices (House Prices over Consumption Deflator), C^* is the target consumption and UC^{HP} is user cost of housing ownership.³⁶ The condition in (63) is the basis for the following empirical specification:

$$rp_t^{H*} = \alpha^{H*} + \beta_1^{H*} y h_t - \beta_1^{H*} k h_t + \beta_3^{H*} uc_t^{HP} \quad (64)$$

where the user costs are defined as:

$$uc_t^{HP} = \alpha^{UC_{hp}} + \delta_t^H + r_t^{mt} - \pi_t^e + \tau_t - 0.4 \Delta^e rp_t^H \quad (65)$$

with r_t^{mt} being the mortgage rate, π_t^e the inflation expectations, τ the taxes on housing and $\Delta^e rp_t^H$ the expectations on relative price growth.

We deviate from the theoretical specification by replacing the target consumption with disposable income of households, $y h_t$.³⁷ Moreover, the elasticities are estimated without dogmati-

³⁶Note that the user costs of housing ownership differ from the user costs of housing capital defined in the residential investment section.

³⁷Empirical results with target consumption are very similar. The replacement was done for practical purposes, as the derivation of target consumption is quite convoluted and the use of target consumption reduces the pragmatism of the equation in a partial equilibrium analysis, as one would need the whole system of equations to determine the target consumption.

cally imposing the theoretical assumption that the elasticity of substitution is equal to 1. This constraint is used as a prior in an informative Bayesian estimation. Results are reported in table D.24 and show that the elasticity with respect to disposable income is close to 1 and the elasticity with respect to user costs is -0.6.

The short-run equation for relative prices is the usual PAC equation:

$$\Delta r p_t^H = a_0^{rph} (r p_{t-1}^{H*} - r p_{t-1}^H) + \sum_{i=1}^1 a_i^{rph} \Delta r p_{t-i}^H + \mathbb{E}_{t-1} \sum_{j=0}^{\infty} d_j^{rph} \Delta r p_{t+j}^{H*} + e_t^{rph} \quad (66)$$

The results shown in table D.24 suggest that house prices return sluggishly to the target variable as only 3.4 percent of the gap is closed each quarter with the dynamics being predominantly driven by the relatively high autoregressive coefficient.

Net Foreign Asset

Accumulation of the net foreign asset (NFA) is assumed to depend on the trade balance and a residual component capturing the net interest income and a revaluation term:

$$NFA_t - NFA_{t-1} = TB_t + REST_t \quad (67)$$

The residual term, $REST$, is modelled relative to nominal GDP and is assumed to be driven by changes in the spread between interest rates on foreign liabilities (IR^{FL}) and interest rates on foreign assets (IR^{FA}), and by changes in domestic and world prices (P^y and P^W):

$$\frac{REST_t}{Y_{t-1}^N} = \alpha^{nfa} + \beta_1^{nfa} \Delta(IR^{FL} - IR^{FA}) + \beta_2^{nfa} \Delta(P_t^y) + \beta_3^{nfa} \Delta(P_t^W) + e_t^{nfa} \quad (68)$$

The empirical interest rates used in (68) are observed implicitly as a ratio between property income of foreign assets/liabilities and total foreign assets/liabilities:

$$IR_t^{FA} = \frac{IPN_t^{FA}}{FA_t} \cdot 400; R_t^{FL} = \frac{IPN_t^{FL}}{FL_t} \cdot 400.$$

For modelling purposes the interest rate on foreign assets is expressed in terms of a foreign

long-term rate, whereas foreign liabilities are determined by the domestic long-term rate:

$$IR_t^{FA} = c_t^{FA} + \beta^{ifa} R_t^{US_{10Y}} + e_t^{ifa} \quad (69)$$

$$IR_t^{FL} = c_t^{FL} + \beta^{ifl} R_t^{EA_{10Y}} + e_t^{ifl} \quad (70)$$

All interest rate variables are demeaned and c is set to follow an autoregressive process, implying the interest rate parity in the long run.³⁸

Estimation results of (68) are presented in table D.25. The estimated coefficients exhibit the expected signs. Specifically: (i) a positive change in the spread between interest rates on foreign liabilities and foreign assets affects negatively the net interest income and subsequently the NFA; (ii) a depreciation of nominal effective exchange rate positively affects foreign assets; and (iii) a domestic price increase induces a positive revaluation in foreign liabilities and implies a decrease in the NFA position.

4 Model properties

In this section we validate the model against a limited set of diagnostic tests to ensure that the overall system's properties are internally consistent and in line with macroeconomic theory and standard empirical benchmarks. In particular, (i) we emphasize the long-run properties of the model and its convergence to a well-defined steady state; (ii) we report selected impulse response functions computed at the steady state to shed light on the dynamic effects of given shocks and the model's transmission channels; and (iii) we show how the model can be used for and performs in forecasting, both unconditionally and conditional on a set of assumptions about fiscal, financial and external variables.

4.1 Long-run convergence to steady state

The core of the ECB-BASE resembles a standard open-economy New Keynesian model and shares the economic underpinnings with this class of models. This implies that the model has a well-defined steady-state to which it converges in the long-run.

A balanced growth path can be computed in various alternative and consistent ways: (i) from a full description of the microfounded equilibrium conditions that define the steady state

³⁸Namely: $c_t^{FA} = 0.99c_{t-1}^{FA}$; $c_t^{FL} = 0.99c_{t-1}^{FL}$

in the stationarised version of the model. The balanced growth path is then derived by imposing the same growth rates to all endogenous variables (with, e.g., the nominal ones growing at the sum of the long-run growth rates of productivity, working age population and inflation); (ii) from the plain simulation of the full dynamic model starting at the last data point, setting the residuals to zero out of sample.³⁹

In all cases, the values of the main ratios of the economy are plausibly close to their historical averages and therefore they represent a good baseline to be shocked to analyze the response of the economy in the steady state. Table D.26 gives an overview on selected ratios in steady state and the balanced growth path. The table is also benchmarking the model ratios to their counterpart in the data, using two samples. The longer sample ranges from 1970Q1 to 2017Q4, while the estimation sample is restricted to 2000Q1 to 2017Q4. The table shows that the model ratios are roughly in line with data, where some deviations from the sample means are tolerated. This is due to the sample being dominated by two crises and not fully representative for the calibration of the balanced growth path.

As it typically occurs in these models, the convergence may take several years after the start of the out-of-sample simulation. Figure D.2, for instance, shows the convergence path of selected variables of the model. The transition phase from the last in-sample observation to the steady state must not be interpreted as a sensible macroeconomic projection, but rather as the result of setting the residuals to zero and letting the exogenous variables grow at their balanced growth path rate.⁴⁰ The only purpose here is to show that the model converges to the predetermined steady state. Once at the baseline, the economy can then be shocked, scenarios can be simulated, and a meaningful interpretation to the responses can be given.

4.2 Impulse response functions to selected shocks

In this section we report selected dynamic responses of the system to standard shocks in deviation from the balanced growth path. In particular we focus on four shocks (with somewhat more emphasis on the first one): (i) a standard monetary policy shock; (ii) a term premium shock;

³⁹There is a third computation to check the steady state by calculating the numerical solution of a system of the target (long-run) equations of the model only. It can be shown that the solution of this system is nested in the equilibrium and balanced growth path of the first approach and is also the attractor in the simulation based solution.

⁴⁰As discussed in section 4.3.3 the residuals in the model are not always white noise. This relates to the dynamic equations and also to the long-run equations. For the steady-state all of these residuals are set to zero, implying a non-monotonic convergence path for some variables.

(iii) a foreign demand shock; and (iv) a cost-push shock. All impulse response functions are computed under exogenous fiscal and external blocks.

4.2.1 A standard monetary policy shock

A standard monetary policy shock is designed to increase the short-term policy rate by 100 basis points on impact and then let the Taylor rule react endogenously afterwards. The dynamic responses of the main endogenous variables are plausible from a quantitative and qualitative point of view. Figure D.3 shows that after such a shock the usual negative hump-shaped reactions of both nominal and real variables occur. These response functions are not only qualitatively in line with the macroeconomic theory but also in the quantitative ballpark of the responses to the same shock computed in benchmark macroeconometric models, such as the FRB/US, the ECB New Area Wide Model (NAWM), or the current ECB multi-country model (NMCM). Figure D.4 reports this comparison. The responses of the real variables are similar in shape and dynamics to the ones based on the FRB/US model. In terms of size, they are usually between the ones based on the core ECB structural model (NAWM) and the current multi country model (NMCM) both on the real and on the nominal side.

The transmission mechanism of monetary policy in this model is based on three different transmission channels: the financial amplification, the expectation channel, and the sensitivity of demand to interest rate.

The financial amplification. The endogenous modelling of a rich financial sector plays an important role in propagating the increase of the policy rate (figure D.1). As also described in section 3.3 both term premia and risk spreads are affected by expectations on future output gaps. Higher interest rates point to a negative output gap and consequently higher premia and spreads. The increase in the long-term interest rate is partly driven by the standard expectation hypothesis and partly by an increase in the term-premium. The individual rates such as the lending rate to households, the lending rate for non-financial corporations, the corporate bond yield, and the cost of equity, rise because of an increase in the short-term, the long-term rate and the increase in the risk spread (not shown). In fact, assuming constant all other rates, premia and spreads after the increase in the policy rate can dampen substantially the effect of monetary policy on real and nominal variables, as shown in figure D.5.

The expectation channel. As discussed in section 2 and detailed in A.2, expectations are based on projections from an estimated small-scale auxiliary VAR model where an increase in interest rates reduces inflation and output expectations. According to the logic of the PAC equations in the model, a downward revision of expected future targets leads to a downward adjustment of consumption and investment plans. Moreover, on the price side, an increase in interest rate affects the one-period-ahead expectation of inflation which in turn leads to lower price increases and lower inflation. These effects generate an expectation feedback loop that amplifies the responses to an interest rate shock. Figure D.6 illustrates the effects of this channel on top of the financial amplification and reports the impulse response functions when shutting down the expectation component in the price and wage Phillips curves beside assuming constant all rates (except the policy rate), premia and spreads. The effects of this additional channel are now significant on prices while still leaving additional real effects coming from a direct sensitivity of the demand side to interest rate movements.

Demand sensitivity to interest rate. Several equations of the model feature a direct sensitivity to changes in the interest rate. In the investment block, for instance, the user cost of capital (e.g. in equation (10)) depends on the policy rate. More specifically, the financing cost measure for business (residential) investment is constructed as a composite of the lending rate for non-financial corporations (mortgage), the corporate bond yield, and the cost of equity. As described in section 3.3, these rates are derived as a weighted average of short-term and long-term rates, including term premia and risk spreads. An increase in the rates as well as in the premia and spreads leads to higher user cost of capital and a reduced demand for investment. In the short-run consumption equation the spread on the consumer lending rate is introduced to capture the interest sensitive durable consumption part which is contained in the total household consumption but not modelled separately. The increased spread leads to a direct reduction in consumption, beside the one contained in the target.

4.2.2 Term premium shock

Discussing the effects of a term premium shock is a way to illustrate how unconventional monetary policies can impact the macroeconomy through e.g. a portfolio rebalancing channel. The ECB-BASE is equipped to compute these effects only to the extent that auxiliary models can provide a calibration of the effect of asset purchases on given asset prices or premia. Fed with

these calibrated paths, the model is then able to compute the macroeconomic impact.

Figure D.7 shows the response to a 100 basis points shock to the term premium. The term premium, in combination with the average expected short term rate over ten years, determines the long-term interest rate. The long-term interest rate therefore increases almost one-by-one to an increase in term premium. The increase of long-term interest rate is transmitted to the economy via four channels: the UIP condition, the lending rates, wealth, and income property.

The real long-term interest rate determines the exchange rate via the UIP condition. Following an increase of the term premium, the exchange rate appreciates, depressing exports and increasing imports. The deterioration of the trade balance implies an immediate drop of output and employment. Lower employment weighs negatively on income, leading to a drop in real consumption, while investment decreases due to the accelerator effect of output.

The second transmission channel works via the lending rates. The increase of lending rates has a direct impact on the user costs of capital which increase and depress investment further. Similarly, the lending rate for consumption loans increases, leading to an immediate drop of consumption over and above the drop of target consumption, whose fall is due to the drop in income and wealth.

The drop of wealth is a consequence of the third transmission channel. As long-term rates increase, asset prices drop, causing a decrease in wealth through the revaluation effect.

Finally, the fourth channel operates via property income. The increase of long-term rates affects mortgage rates and deposits rates that determine the property income of households. Given the maturity composition of mortgage rates and deposit rates, the mortgage rates increase more, leading to a drop of property income and further drag on consumption.

The drop in demand leads to a reduction of prices to which monetary authority reacts by decreasing short-term rates. The decrease of short term rates and the decaying effect from the term premium shock is bringing the economy back on the path to equilibrium.

4.2.3 World demand shock

Figure D.8 reports the impulse responses to a permanent 1% increase in world demand. The transmission of the world demand shock to exports is immediate and almost complete. The increased export activity affects the aggregate demand and output gap. In consequence, expectations related to labour and transfer incomes increase and this in turn positively affects private consumption. Similarly, aggregate investments increase due to the output accelerator effect. The

opening of the output gap in turn puts a pressure on the nominal side of the economy causing domestic products to become more expensive than imports. In combination with increased domestic and export demand, the relative price effect causes imports to increase. Finally, the stabilization of the economy is attained through reduced real income balances which is a result of elevated prices and interest rates.

4.2.4 Cost-push shock

Figure D.9 presents impulse responses to a cost-push shock reflected in a 1 p.p. increase in domestic inflation. An increase in consumer prices and corresponding counteractive increase in interest rates produce contraction of consumption and investment. The corresponding decrease in aggregate output implies a lower demand for imports, while export initial increases due to depreciation of exchange rate. Nominal wages increase with a one quarter delay, however the increase in domestic prices is relatively stronger, exerting a downward pressure on real wages. Lower real wages imply lower production costs and higher demand for labour, where the latter leads to a gradual stabilization of consumption response and eventual convergence of output to its baseline level.

4.3 Forecasting with ECB-BASE

The design of ECB-BASE is aligned to its role as workhorse model in the context of the forecasting and policy simulation exercises at the ECB. This section illustrates how the model can be used for forecasting, based on simple rules to mechanically introduce add-factors. The forecasting performance of the model is compared to standard benchmark models (naïve and BVAR).

4.3.1 Projections with different residual rules

A purely model-based prediction with zero residuals within this class of models can be challenging. Namely the model is misspecified in various dimensions, which could lead to biased forecasts. Instead, utilising the information from past model errors can improve the forecast.

In this section we evaluate the forecasting performance of the model and the corresponding residual settings in an *unconditional* and a *conditional* sense. In the first, the model is simulated under the endogeneity of all variables. In the second, we mimic the setup of a regular ECB projection exercise and project the main macroeconomic variables conditioning on the same

set of financial, fiscal and external variables whose forecast during the regular ECB projection exercises is exogenous to the model. To eliminate a potentially important source of mistake, we set the paths of all exogenous variables on the forecast horizon equal to their true realizations. Notice that the model can also be used for counterfactual simulations based on alternative paths of the exogenous variables.

Add-factors

In principle, a model-based forecast (unconditional or conditional) is obtained by setting the estimated errors of all endogenous variables to zero and simulating the model over a given horizon. Although the model has just been released and its estimation is up-to-date, misspecifications of the model, along dimensions which are difficult to ascertain and are also driven by the equation-by-equation estimation approach, might imply that the equation errors (residuals) are not white noise in-sample. These misspecifications imply that even in the case of a good in-sample fit, it is not guaranteed that the model can preserve its properties out-of-sample. Even if the estimated residuals have reasonable in-sample properties, the forecast of the variables of interest can be improved if information from past errors is properly retained. This can be done by deviating from a zero-residual rule for all, or a number of variables of interest over the forecast horizon. Add-factors are adjustments to the constant terms of the model's stochastic equations, that are typically used to improve the forecast accuracy in this class of models. They are a device to either account for deviations of the residuals from white noise properties in first and second moments (i.e. non-zero in sample mean or residual auto- and cross-correlations) or introduce reasoned expert judgment.

We evaluate the forecast performance of the ECB-BASE model with two simple *mechanical* add-factor rules. Specifically we either impose zero residuals for all variables or exploit some residual in-sample correlation to introduce an add-factor which is depreciating only gradually over the forecast horizon. For the second approach we are going to assume an unobserved-component representation of the residuals, given by the sum of a persistent component and an idiosyncratic one. The Appendix C describes the estimation of this auxiliary state space model for the residuals in detail and how it is used in the forecast. Just for the sake of illustration, Figure D.10 shows the residuals projected with the two rules for four (randomly chosen) residuals of the model. In the upper panel the residuals are set to zero over the forecast horizon. In the lower panel a UCM is fitted on the residuals over the entire sample and the persistent part of

the residual is projected out of sample on the forecast horizon. A 68-percent Bayesian region is also reported in this case.

The experiments we perform to compare the two add-factor rules at each point in time of the evaluation sample 2004Q1-2017Q4 once the model has been estimated on the full sample (precisely on 1999Q1-2016Q3).⁴¹ To illustrate the relative performance, Figure D.11 reports (modified) Theil-Us, namely the RMSE of the ECB-BASE model projected over a horizon of 12 quarters as a ratio to the RMSE obtained with a naïve benchmark (Random Walk for inflation and AR(2) for GDP growth). The ECB-BASE forecasts are obtained under (i) a zero-residuals rule, where all residuals of the endogenous variables are set to zero over the forecast horizon, and (ii) a mechanical unobserved-component-model rule (UCM rule) described above.⁴² The bands represent the 95% probability distribution of the Theil-Us computed from the forecasts (over the same horizon and evaluated over the same sample) of a BVAR model. We report results for unconditional and conditional forecasts. For the conditional forecast the BVAR has been estimated with four lags of four endogenous (real GDP growth, GDP deflator inflation, consumption growth and wage inflation) and five contemporaneous exogenous variables (short-term interest rate, government consumption growth, oil price inflation, exchange rate, and world demand growth). For the unconditional forecast all variables in the BVAR are endogenous and with two lags. A standard Litterman prior with general tightness equal to 0.15, a weight on other variables equal to 0.5, an harmonic decay with decay factor equal to 0, and a mean of the first own lag equal to 0.8 is used.

Looking at the charts, four remarks are in order. First, all forecast are on average better than the naïve forecast. Second, the difference between the zero-residuals and the UCM rule is not significant. In the conditional forecast for inflation zero residuals seem to provide better forecasts over the long-run and the UCM rule can be better at shorter horizons. Our practical suggestion would be to use a hybrid mechanical judgment that combines both approaches in real time. Third, the mechanical ECB-BASE forecast is as competitive as a BVAR-based forecast, being on the lower side of the BVAR distribution in the unconditional case. Fourth, the conditional forecast of GDP growth is better than the unconditional one. Instead, somewhat surprisingly,

⁴¹Some authors would refer to this exercise as a pseudo-recursive forecasting exercise. We acknowledge that we are relying on full sample information in several ways. In the choice of the specifications of the model and in the estimates which are based on the full sample.

⁴²For the case of the 'zero-residual rule' we are actually keeping some residuals at a constant over the projection horizon. More specifically the constant residual approach is restricted to explain the difference between the effective exchange rate and the exchange rate on the import and on the export side, and for the inventories.

for inflation the unconditional forecast has a lower RMSE than the conditional one.

The latter remark might have a sample-dependent explanation. Figure D.13 shows the relative performance of conditional and unconditional forecasts over the samples 2004-2009 and 2010-2017. Although the precision over a smaller sample is clearly lower, the exercise is meant to check the performance of these rules over two very different periods. It is clear that the forecast over the sample 2004-2009 is systematically better than the one over the sample 2010-2017, and this is true both for the BVAR and for the ECB-BASE forecasts. However, while the conditioning always improves the forecast of GDP growth as one would expect, it significantly worsens the one of inflation especially on the more recent period. In other words, after 2010 knowing the true realizations of the exogenous variables would not help making a better forecast, particularly of nominal variables. We examine this issue in the next subsection from an expectation perspective.

4.3.2 Medium-term inflation expectations and the forecast of nominal variables

The ECB-BASE model is equipped to dwell on interesting scenario analyses. As a way of example, this section elaborates on the importance of long-term inflation expectations in the forecast of nominal variables, namely GDP deflator inflation and nominal wage inflation. Medium-term inflation expectations in the model are governed by the following mechanism or rule:

$$\pi_t^e = (1 - \rho)\pi_{t-1}^e + \rho[(1 - \omega)\pi_{t-1}^* + \omega\pi_{t-1}] + \epsilon_t \quad (71)$$

where π_t^e is the long-term inflation expectation – whose observable for estimation is the 6-to-10 year consensus forecast – π_{t-1}^* is the inflation target – which can be time varying but we assume it is trending towards 1.9% over the estimation sample – and π_t is actual inflation. For a given degree of smoothing (ρ), expectations are a weighted average of the target and current inflation. A high weight before current inflation (ω) can be interpreted as a tendency of expectations deviating from target. On the other hand, for a given value of ω , small values of ρ imply that inflation expectations become very persistent and they might take long before they converge to the target. From a purely empirical perspective, the two cases are observationally equivalent and very difficult to discriminate in practice, as discussed in Ciccarelli and Osbat (2017). In a structural model, the two cases can imply different dynamics. The first case can be related to *low credibility* of the Central Bank inflation target. In this case, repeatedly low realizations of inflation drive inflation expectations down and lead to a self-enforcing loop between low inflation

and low inflation expectations. The second case simply implies a *high persistence* of inflation expectations which in practice can translate into a low credibility of the central bank instruments to achieve the target.

The benchmark values of ρ and ω have been calibrated to average values over the sample 2000-2017 and are equal to $\rho = 0.25$ and $\omega = 0.4$. However, the development of GDP deflator inflation from 2000 is clearly characterized by two regimes: one where average inflation was slightly above 2% (from 2000Q1 to 2008Q2) and one where average inflation has been 1% (2008Q3-2018Q4). Therefore, the assumption of an expectation mechanism with a fixed target and a constant degree of anchoring/persistence over the full sample can give rise to biased forecasts. To see this, Figure D.15 compares the RMSE of the conditional forecasts obtained with the ECB-BASE (with a "zero-residual" add-factor) over the two sub-samples with two degrees of anchoring, the benchmark one (with $1 - \rho = 0.75$ and $\rho\omega = 0.1$) and one where the dependence of inflation expectations on realized inflation is much higher (with $1 - \rho = 0.1$ and $\rho\omega = 0.75$). The charts report the ratios between the RMSE of the benchmark over the RMSE of the alternative scenario of much less anchored expectations. A value of the ratio below 1 means that the benchmark specification has a better forecast performance than the one with lower degree of anchoring. For the nominal variables this is clearly the case over the sample 2004-2009, i.e. when average inflation was at 2%. On the more recent sub-sample, instead, the forecast performance of the benchmark specification significantly worsened with respect to the previous sub-sample and, in relative terms, it is either equivalent (price inflation) or significantly inferior (nominal wage) to the alternative, suggesting that if we allow for a lower degree of anchoring over the more recent sample the forecast of nominal variables improves. For the real variables of the model (exemplified by real consumption and GDP growth) the forecast performance has been broadly similar over time and across expectation formations regardless of the inflation expectation rule. A lower degree of anchoring of inflation expectations can therefore help shed light on the finding of the previous subsection that over the most recent period a conditional forecast of inflation is worse than an unconditional one. The conditional forecast can indeed improve if we modify the assumption on the inflation expectation formation. This is also consistent with the findings on the causes and consequences of low inflation in the euro area discussed at length in Ciccarelli and Osbat (2017).

4.3.3 Stochastic simulations

As a final exercise, we briefly illustrate how the ECB-BASE model can be used to characterize uncertainty around baseline projections, either endogenously produced by the model or externally provided (such as the final ECB forecast of a given projection exercise). Figure D.16 presents intervals around an unconditional forecast over the sample 2016Q4-2019Q3. Notice that this is a purely out-of-sample forecast because the model has been estimated until 2016Q3. The model shows acceptable properties also out of sample and the true realizations of GDP growth and Inflation (blue solid lines) are compatible with the simulated distributions.

The nonlinear structure of the ECB-BASE does not permit closed-form solutions for the unconditional variance of the endogenous variables at different projection horizons. Therefore, to produce an interval like the one reported in the figure D.16, the model is simulated several times either drawing at random from the series of historical equation residuals (via bootstrap), or using the output of the Bayesian estimation of the residuals based on the UCM rule described above. The charts report both intervals.

5 Conclusion

ECB-BASE is the blueprint of a renewed generation of ECB semi-structural models for the projection process and the model-based policy advice at the ECB. The model will also provide a useful tool for a top down approach between euro area and country modelling.

The paper describes the basic features of the model and provides a detailed description of the specification and estimation choices. The model is also evaluated against a set of diagnostics, which include the derivation of a well-defined steady state to which the model converges, the dynamic responses of the system to selected shocks, and its forecasting capability and use. In all these dimensions the model is consistently in line with macroeconomic theory and standard empirical benchmarks.

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Technical Appendix

This appendix provides the technical details of general model features that are functional to the understanding of the whole model, such as the rigidities that apply to consumption, investment, and factor inputs in production specified as a generalized form of adjustment costs, polynomial adjustment costs (PAC), and the limited-information expectation formation via VAR models (Appendix A). Furthermore, the appendix contains the description and additional technical derivation of each model sector, following the same ordering of the description in the main text (Appendix B), as well as technical guidelines on the use of the model residuals for forecasting (Appendix C), and all tables and charts (Appendix D).

A General model features

A.1 PAC estimation

In this appendix we present the general setup for the PAC estimation and discuss how the approach is modified to include growth neutrality adjustment, non-optimising agents and additional independent variables. This section is based on Brayton, Davis, and Tulip (2000), developed on earlier work by Tinsley (1993) and Tinsley (2002). The reader can refer to those papers for the full algebraic derivations of what reported here.

The main idea of the PAC approach is that agents cannot costlessly adapt their behaviour to be in line with the optimality conditions instantaneously. Instead they choose an optimal path of their decision variable by minimizing an associated adjustment cost function. The cost function can be expressed as:

$$C_t = \sum_{i=0}^{\infty} \beta^i \left[(y_{t+i} - y_{t+i}^*)^2 + \sum_{k=1}^m b_k \left((1-L)^K y_{t+i} \right)^2 \right] \quad (\text{A.1})$$

where y_{t+i} is the decision variable, y_{t+i}^* is its desired level, m is the order of the polynomial, L is the lag operator, β is a discount factor on future penalties and b_k are cost parameters.

Minimizing the cost function yields the following first order condition:

$$(y_t - y_t^*) + \sum_{k=1}^m b_k [(1-L)(1-\beta F)]^K y_t = 0 \quad (\text{A.2})$$

where $F = L^{-1}$ is the lead operator. Note that $b_K [(1-L)(1-\beta F)]^K$ is a self-reciprocal polynomial, i.e. a polynomial with the property that the coefficients of the forward looking terms can be deduced from the backward-looking terms. This allows to introduce expectations and to estimate the final PAC equations with expectations.

Algebraic manipulations of this equation leads to the expression that is used in the empirical implementation:

$$\Delta y_t = a_0 (y_{t-1}^* - y_{t-1}) + \sum_{k=1}^{m-1} a_k \Delta y_{t-k} + \mathbb{E}_{t-1} \sum_{k=0}^{\infty} d_k \Delta y_{t+k}^* + \epsilon_t \quad (\text{A.3})$$

where y is the dependent variable, y^* is its own target, the coefficients a_k and d_k are functions of b_k , and $m - 1$ is the number of lags for the dependent variable. The number of lags depends on the assumed order of adjustment costs m which in each PAC equation is typically determined as part of the estimation process and is chosen to ensure that residuals are not serially correlated.

For the estimation of this equation, one needs forecasts of the target variable y^* . These forecasts can be obtained for instance with VAR models of the type

$$z_{t+1} = H z_t \quad (\text{A.4})$$

where the information vector z includes y^* and other variables that help forecast y^* .

It can be shown that we can rewrite the equation (A.3) as:

$$\Delta y_t = a_0 (y_{t-1}^* - y_{t-1}) + \sum_{k=1}^{m-1} a_k \Delta y_{t-k} + h z_{t-1} + \epsilon_t \quad (\text{A.5})$$

where h is a function of a_k , ($k = 0, 1, \dots, m - 1$), of the VAR coefficient matrix H , and of the discount factor.⁴³

Defining $Z_t = h z_{t-1}$, equation (A.5) becomes:

$$\Delta y_t = a_0 (y_{t-1}^* - y_{t-1}) + \sum_{k=1}^{m-1} a_k \Delta y_{t-k} + Z_t + \epsilon_t \quad (\text{A.6})$$

The estimation is performed through the following steps:

1. Start with a guess for a_k , ($k = 0, 1, \dots, m - 1$).⁴⁴
2. Construct initial estimates of h given values of the VAR coefficients H , the initial guess for a_k , ($k = 0, 1, \dots, m - 1$), and the discount factor;
3. Estimate the equation (A.5);
4. Repeat point 2 using the newly estimated a_k , ($k = 0, 1, \dots, m - 1$);

⁴³The exact derivation of h can be found in Brayton et al. (2000).

⁴⁴The guess is currently fixed at an ad-hoc number, as estimates converge after a few iterations only. Alternatively, one can set the initial guess by estimating an Error Correction Model (ECM).

5. Iterate until convergence.

Growth neutrality adjustment

The derivation of the PAC equation assumes zero growth rate of variables in the long run. To see this, we derive the steady state of equation (A.3):

$$a_0(y^* - y) = \left[1 - \sum_{k=1}^{m-1} a_k - \sum_{k=0}^{\infty} d_k \right] g \quad (\text{A.7})$$

where we assume a balanced growth equilibrium in which $\Delta y = \Delta y^* = g$. Given that in equilibrium $y = y^*$, in order to satisfy (A.7) there are usually two options. Either one would impose

$$1. \sum_{k=1}^{m-1} a_k + \sum_{k=0}^{\infty} d_k = 1$$

or assume

$$2. \Delta y = \Delta y^* = 0$$

Both options are problematic. The first one requires imposing additional constraints on a_i and d_k that would set to zero the first order adjustment costs in PAC (i.e. the adjustment of the level of variables is not costly) while leaving costly the higher order time derivatives. The second option ($\Delta y = \Delta y^* = 0$) would prevent modelling data with a positive growth rate along a balanced growth equilibrium.

For these reasons we prefer not impose either of these constraints and rather modify the PAC equation by adding the correction factor:

$$\left[1 - \sum_{k=1}^{m-1} a_k - \sum_{k=0}^{\infty} d_k \right] g_t$$

to the PAC equation, which then becomes:

$$\Delta y_t = a_0 (y_{t-1}^* - y_{t-1}) + \sum_{k=1}^{m-1} a_k \Delta y_{t-k} + h z_{t-1} + \left[1 - \sum_{k=1}^{m-1} a_k - \sum_{k=0}^{\infty} d_k \right] g_t + \epsilon_t \quad (\text{A.8})$$

where g_t is the observed measure of expected long-run growth of the variable along the balanced growth path.⁴⁵

⁴⁵In the case of consumption, we use the smoothed growth rate of potential output, which is also assumed to be the long-run growth rate of economy.

Defining $Z_t \equiv hz_{t-1} + \left[1 - \sum_{k=1}^{m-1} a_k - \sum_{k=0}^{\infty} d_k\right] g_t$, we get the general specification:

$$\Delta y_t = a_0 (y_{t-1}^* - y_{t-1}) + \sum_{k=1}^{m-1} a_k \Delta y_{t-k} + Z_t + \epsilon_t \quad (\text{A.9})$$

Operationally, the estimation is performed as described above since d_k , ($k = 0, 1, \dots, m-1$) are functions of a_k , ($k = 0, 1, \dots, m-1$) and the discount factor.

Non-optimising agents

It is also possible to enrich the approach and include additional features. For the case of consumption, for example, we include agents who follow a simple rule-of-thumb (ROT) or face liquidity constraints and cannot optimize consumption. The variable capturing the behaviour of these agents is x^{ROT} and $(1 - \theta)$ is the estimated (or calibrated) fraction of non-optimising agents.

In general we can write the PAC equation that includes non-optimising agents as:

$$\Delta y_t = \theta \left[a_0 (y_{t-1}^* - y_{t-1}) + \sum_{k=1}^{m-1} a_k \Delta y_{t-k} + \sum_{k=0}^{\infty} d_k \Delta y_{t+k}^* \right] + (1 - \theta) \Delta x_t^{ROT} + \epsilon_t \quad (\text{A.10})$$

Equation (A.10) is estimated with a non-linear regression using a Log-sigmoid transformation of the parameter in order to impose $\theta \in [0, 1]$.⁴⁶ Additionally, the growth neutrality adjustment is different from the one presented in Section A.1, with the adjustment term now being:

$$\frac{1}{\theta^{ROT}} \left[1 - \theta^{ROT} \left(\sum_{k=1}^{m-1} a_k + \sum_{k=0}^{\infty} d_k \right) \right] g_t$$

The empirical equation therefore becomes:

$$\Delta y_t = \theta \left[a_0 (y_{t-1}^* - y_{t-1}) + \sum_{k=1}^{m-1} a_k \Delta y_{t-k} + Z_t \right] + (1 - \theta) \Delta x_t^{ROT} + \epsilon_t. \quad (\text{A.11})$$

where $Z_t = hz_{t-1} + \frac{1}{\theta} \left[1 - \theta \left(\sum_{k=1}^{m-1} a_k + \sum_{k=0}^{\infty} d_k \right) \right] g_t$.

Note that this adjustment factor is derived assuming that the growth rate of the ROT variable along a balanced growth path is 0 ($\Delta x_{SS}^{ROT} = 0$). In other words, the variables are usually demeaned in the estimation.

⁴⁶We use the *nlinfit* function in Matlab that is based on iterative OLS.

Additional variables

The PAC equation could in principle also contain additional independent variables. For instance, in the consumption block we assume that the spread between the interest rate on consumer loans and the policy rate directly affects consumption's dynamics.

Additional (independent) variables can be included in two ways:

1. as a stationary part of the target
2. as independent/exogenous variables

When the variable is included as a stationary part of the target, it implies that its expectation will also matter for the dynamics of the dependent variable. On the other hand, when the variable is included as an exogenous variable, only the contemporaneous value of the variable will affect the dynamics of the dependent variable.

The equation now becomes:

$$\Delta y_t = \theta \left[a_0 (y_{t-1}^* - y_{t-1}) + \sum_{k=1}^{m-1} a_k \Delta y_{t-k} + Z_t + \psi x_t^{EXO} \right] + (1 - \theta) \Delta x_t^{ROT} + \epsilon_t \quad (\text{A.12})$$

When we assume that x_t^{EXO} is a stationary part of the target, it has to be included already in the VAR and we use a similar approach to the one that generates the expectations for the target variable:

$$x_t^{EXO} = h^0 z_{t-1}$$

where now the vector z_{t-1} also includes the additional independent variable. The only difference is that the h^0 vector is a slightly different function of a_k , ($k = 0, 1, \dots, m-1$), H , and the discount factor.⁴⁷

On the other hand, when the variable is assumed to be exogenous, x_t^{EXO} does not have to be included in the VAR. It is included in the equation as contemporaneous, lagged or instrumental, depending on the assumption about the variable.

We generally transform all additional, independent variables in a way that in the steady state their growth rates are zero. This implies that it is not necessary to change the adjustment factor neither in the general case nor in the ROT case.

A.2 Base VAR and Expectations

The expected part of the PAC equation can be obtained (and the model simulated) in two ways: With a Vector Autoregressive (VAR) model or in a model-consistent manner. The latter is based on the assumption that agents are rational and is a reasonable choice in empirical applications,

⁴⁷The exact derivation of h^0 can be found in Brayton et al. (2000).

such as permanent changes in policy rules, where one needs to avoid persistent expectational errors. The former is instead based on the assumption that agents have limited information and use a reduced form model with a limited set of variables (a VAR) to forecast the variables of interest.

The two types of expectations can be used separately or in combination when simulating the model. The simulation results of section 4 are derived under VAR-based expectations for the parts of the model that contain PAC equations. This appendix therefore explains the VAR expectation formation with some details.

The VAR used to forecast the target variables contains two blocks. The first block is simply the variable to forecast, typically the target or desired value of a given endogenous variable. The target value is usually constructed in the first step via co-integration or calibrated based on theory. For example, in the consumption equation we have to form expectations about future target consumption which is a function of permanent incomes and wealth. Target variables are specific to each equation. Given the large number of equations and the idea that VARs should be parsimonious, we use a different VAR for each specific equation.

The second part of the VARs is common to all equations and ensures consistent system properties. Following the FRB-US model we adopt the name Base VAR for this part of the VAR. The Base VAR contains three variables: inflation, interest rate and output gap. Following the structure of a basic New Keynesian model, the idea is that these are core states of the model that agents use to forecast all other target variables.

Base VAR

The exact setup of the Base VAR is:

$$\Delta z_t = \Lambda^0(z_{t-1} - z_{t-1}^*) + \sum_{k=1}^K \Lambda^k \Delta z_{t-k} \quad (\text{A.13})$$

where z_t is a $N \times 1$ vector of variables containing inflation, the level of the interest rate and the output gap ($N = 3$). Λ^0 is $N \times N$ matrix containing coefficients that control how the variables respond to the deviation of the lagged level from the long-term attractors. Λ^k are $N \times N$ matrices that collect auto-regressive coefficients for K lags.

Notice that the Base VAR does not contain any constant terms. Assuming that the VAR is stable, one can show that the VAR based forecasts will, in the long-term, converge to the long-term attractors $z_{t+h} \xrightarrow{h \rightarrow \infty} z_{t-1+h}^*$.

Long-term attractors

The main difference between the VARs used here and a standard setup is that the unconditional steady state is explicitly controlled by attractor variables and is not determined by the

estimated unconditional mean. From equation (A.13), the steady state of the variables included in the Base VAR is determined by the long-term attractors included in z_t^* . The attractors capture the long-term expectations of the variables included in the VAR. Specifically, we assume that inflation converges to long-term inflation expectations, the interest rate converges to the long-term interest rate expectations and that the output gap converges to zero. In practice, the long-term expectations for inflation are based on Consensus forecast and the interest rate expectations are based on interest rate swaps.

In order to close the model we have to determine the processes for the long-term expectations. We assume that the long-term targets follow a random walk process, $z_t^* = z_{t-1}^* + \epsilon_t$, while the long-term target for the output gap is fixed at zero.⁴⁸

Final VAR

The final VAR is obtained by augmenting the Base VAR with an additional equation for the target variable of interest. Following what described above for the Base VAR, we also constrain the long-term behaviour of the target variables. To this end, we assume that in the long-term the growth rate of the target variable will converge to the model consistent long-term attractor. For example, the long-term attractor for the target of consumption growth is the trend growth rate of potential GDP.

Given that the Base VAR is common to all equations, we block-exogenize it in all specific equations. In this way we achieve the same dynamics and forecasts of the variables from the Base VAR in all equations, strengthening the system properties of the expectation formation.

The final VAR can be represented as:

$$\begin{bmatrix} \textit{Target variable} \\ \textit{Inflation} \\ \textit{Output gap} \\ \textit{Interest rate} \\ \textit{Inflation target} \\ \textit{Interest rate target} \\ \textit{Long term growth rate} \end{bmatrix} \Longleftrightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (\text{A.14})$$

where the matrix on the right hand side shows which variables are included in each equation (1-variable is included, 0-variable is not included). The system is estimated with OLS.⁴⁹

⁴⁸Random walk for attractors is only assumed for VAR expectations. In the full model, attractors are assumed to slowly converge to the model-consistent steady state.

⁴⁹For estimation purposes, one can estimate the equation (A.13) directly in first difference and later solve for the level, $z_t = (I_n + \Lambda^0 + \Lambda^1)z_{t-1} + \sum_{k=2}^{K+1} \Lambda^k z_{t-k} - \Lambda^0 z_{t-1}^*$. Alternatively, the system can be estimated directly in levels, but then linear restrictions have to be imposed.

B Block-specific features

B.1 The Consumer's Problem

This appendix follows closely the exposition of Laubach and Reifschneider (2003). Consumers maximize the present value of current and future utility, subject to the lifetime budget constraint. We start by assuming that future income is known with certainty and that individuals are able to borrow freely. For all future periods $j = 0, \dots, D$ (where D is the number of periods left to live), we have:

$$\max V = \sum_{j=0}^D u(C_{t+j})\beta^j \quad (\text{B.15})$$

subject to

$$\sum_{j=0}^D \frac{C_{t+j}}{(1+R)^j} = HW_t + \sum_{j=0}^D \frac{Y_{t+j}}{(1+R)^j}, \quad (\text{B.16})$$

where C_{t+j} is consumption, $u(C_t)$ denotes the utility function, β is the rate of time preference, Y_{t+j} is labour income, r is the rate of return on savings, and HW_t is the value of capital assets at time t (start of period).

Under the assumption of constant relative risk aversion (CRRA), with γ as the risk aversion parameter, and $u(C_t) = C_t^\gamma/\gamma$, the first order condition of the problem gives:

$$\frac{C_{t+j}}{C_t} = [\beta^j(1+R)^j]^{-\frac{1}{\gamma-1}} \quad (\text{B.17})$$

We then substitute the FOC into the budget constraint to get:

$$c_t = \eta_c \left[HW_t + \sum_{j=0}^D \frac{Y_{t+j}}{(1+R)^j} \right] \quad (\text{B.18})$$

where

$$\eta_c = \left[\sum_{j=0}^D \frac{[\beta^j(1+R)^j]^{-\frac{1}{\gamma-1}}}{(1+R)^j} \right]^{-1} \quad (\text{B.19})$$

is the fraction of lifetime wealth consumed today.

Income Uncertainty

In general there is no closed-form solution when income flows are uncertain. We opt for an approximate solution similar to the certainty-equivalence case, except that the discount rate on future income is now augmented by a risk-adjustment factor. For expositional clarity we are

now transferring the consumption problem into a problem of choosing optimal savings, s_t .

The optimization problem can now be written as:

$$\max V = \sum_{j=0}^D \mathbb{E}_t u(C_{t+j}) \beta^j \quad (\text{B.20})$$

with respect to

$$S_{t+j} = Y_{t+j} - C_{t+j} \quad (\text{B.21})$$

subject to the lifetime budget constraint.

The only difference from the certainty equivalence case is that future labour income is now stochastic. The first order condition becomes:

$$(Y_t - S_t)^{\gamma-1} = \beta^j (1+R)^j E_t(Y_{t+j} - S_{t+j})^{\gamma-1} \quad (\text{B.22})$$

A second-order Taylor expansion is used to rewrite the nonlinear expectational first order condition:

$$\begin{aligned} \mathbb{E}_t(Y_{t+j} - S_{t+j})^{\gamma-1} &\approx (\mathbb{E}_t Y_{t+j} - \mathbb{E}_t S_{t+j})^{\gamma-1} (1 + \nu_{t+j}) \\ &\approx (\phi_{t+j} \mathbb{E}_t Y_{t+j} - \mathbb{E}_t S_{t+j})^{\gamma-1} \end{aligned} \quad (\text{B.23})$$

where

$$\phi_{t+j} = \psi_{t+j} + (1 - \psi_{t+j})(1 + \nu_{t+j})^{\frac{1}{\gamma-1}} \approx (1 + \nu_{t+j})^{\frac{1}{\gamma-1}} \quad (\text{B.24})$$

and

$$\nu_{t+j} = .5(\gamma - 1)(\gamma - 2)\sigma_{t+j}^2 \mu_{t+j}^2 \quad (\text{B.25})$$

and σ_{t+j} is the normalized standard deviation of income at time $t+j$.⁵⁰

We then substitute expression (B.23) back into the FOC and combine it with the budget constraint to arrive at the following approximate expression for consumption:

$$C_t \approx \eta_c \left[HW_t + \sum_{j=0}^D \frac{\phi_{t+j} \mathbb{E}_t Y_{t+j}}{(1+R)^j} \right] \quad (\text{B.26})$$

where η is the same as before (equation (B.19)). The difference with respect to the previous expression is the permanent income, which is now discounted by two factors: the real rate of interest and the risk-adjustment factor ϕ_{t+j} .⁵¹

⁵⁰Note that $\psi_{t+j} = \mathbb{E}_t s_{t+j} / \mathbb{E}_t y_{t+j}$ and $\mu_{t+j} = \mathbb{E}_t y_{t+j} / (\mathbb{E}_t y_{t+j} - \mathbb{E}_t s_{t+j}) \approx 1$.

⁵¹The risk-adjustment factor lies between 0 and 1 for risk adverse individuals. Moreover, it is inversely related

Introducing frictions via polynomial adjustment costs

In order to add frictions such as adjustment costs, habit persistence or other frictions preventing households from adjusting immediately to changes in permanent income, we proceed with the following steps.

We start by defining consumption according to equation (B.26), as target consumption c^* , ignoring adjustment costs. Then we assume that consumers try to balance the cost of adjusting their level of spending against the cost of deviating from the target path of consumption.

Adjustment costs are quadratic in logs:

$$\min \mathbb{E}_t \sum_{j=0}^{\infty} \left[(c_{t+j} - c_{t+j}^*)^2 + b \Delta c_{t+j}^2 \right] / (1 + R)^j \quad (\text{B.27})$$

where b is the adjustment cost parameter. The FOC for the second step is:

$$\mathbb{E}_t c_{t+j}^* = c_{t+j} + b \Delta c_{t+j} - \frac{b \Delta c_{t+j+1}}{1 + r} \quad (\text{B.28})$$

which can be rewritten as an error correction equation of the form:

$$\Delta c_t = (1 - \lambda) (c_{t-1}^* - c_{t-1}) + (1 - \lambda) \sum_{j=0}^{\infty} \mathbb{E}_t \log c_{t+j}^* \frac{\lambda^j}{(1 + R)^j} \quad (\text{B.29})$$

where λ is the backward root of the FOC.

Higher order adjustment costs give rise to a more general error correction specification.

Suppose that the optimization problem is the following:

$$\min \mathbb{E}_t \left[\sum_{j=0}^{\infty} (1 + R)^{-j} \left((c_{t+j} - c_{t+j}^*)^2 + \sum_{k=1}^m b_k ((1 - L^k) c_{t+j})^2 \right) \right] \quad (\text{B.30})$$

The solution to this expression is:

$$A(L)A(F/(1 + r))c_t = \mathbb{E}_t A(1)A(1/(1 + R))c_t^* \quad (\text{B.31})$$

where L is the lag operator, F is the lead operator and A an m^{th} order polynomial in either L or F .

This expression can be rewritten as:

$$\Delta \log c_t = A(1)(c_{t-1}^* - c_{t-1}) + A^*(L)\Delta c_{t-1} + \sum_{j=0}^{\infty} \delta_j \mathbb{E}_t \Delta c_{t+j}^* \quad (\text{B.32})$$

to the elasticity of substitution $(-\frac{1}{1-\gamma})$ and to the degree of income uncertainty σ_{t+j} .

where $A(1)$ and $A^*(L)$ result from the partition of the polynomial $A(L)$ into its level and difference components, i.e. $A(L) = A(1)L - A^*(L)(1 - L)$.

The weights on future growth in expected target consumption δ_j are non-linear functions of the discount rate, the error correction parameter, and the coefficients on lagged changes in consumption:

$$\delta_j = A(1)A(1/(1+r))\iota' [I - G]^{-1}G^j\iota \quad (\text{B.33})$$

where $\iota = [0, \dots, 0, 1]$, and G is an $m \times m$ companion matrix associated with the first order form of the lead polynomial $A(F/1 + r)$.⁵²

Age cohorts and aggregation

In a permanent income economy with a representative agent, a decomposition of income into different components is not affecting the dynamics, because households will base their decision on total income. Here we assume that the economy is populated by age cohorts, representing a certain age frame. The propensity to consume is homogeneous inside an age cohort but differs between age cohorts. Equation (B.19) shows that the share of lifetime income consumed in the current period depends on D , the number of remaining periods of life for the age group members, implying heterogeneous propensities to consume between age cohorts. Furthermore, we assume that the composition of income differs between age cohorts. Age cohorts representing prime-age workers will have a larger share of labour income than older cohorts receiving income from transfers and wealth primarily. Aggregating these heterogeneous age cohorts leads to different propensities to consume out of different income sources.⁵³

We can write the target spending of the j^{th} household at time t as a fraction of lifetime resources (split between property and labour):

$$C_{t,j}^* = \eta_{t,j}[W_{t,j}^P + W_{t,j}^L] \quad (\text{B.34})$$

We assume that the $\eta_{t,j}$ s are approximately equal for all members of the same age group and that an age group's propensity to spend is stable over time. If all members of age group i have identical wealth endowments, then the aggregate desired consumption for that group is given by:

$$C_{t,i}^* = \bar{\eta}_i [W_{t,i}^P + W_{t,i}^L] \quad (\text{B.35})$$

⁵²The property that the weights on future growth of the expected target can be written as a non-linear function of the parameters and the coefficients on lagged changes in consumption allows to model and estimate expectations in a backward looking equation.

⁵³It can be shown that the chosen approach of consumption of age cohorts is identical to the Blanchard-Yaari implementation in Smets and Wouters (2002) for the case of a constant interest rate. See also Yaari (1965) and Blanchard (1985).

where capitalized variables denote group (as opposed to individual) spending and wealth, and $\bar{\eta}_i$ represents the group's average propensity to spend. Summation across age groups yields:

$$C_t^* = W_t^P \sum_i \bar{\eta}_i \pi_i^P + W_t^L \sum_i \bar{\eta}_i \pi_i^L \quad (\text{B.36})$$

where π_i^P and π_i^L are the shares of aggregate capital and labour wealth held by the i^{th} age group. Generalizing to account for transfer income and using the identity $\eta_j = \sum_i \bar{\eta}_i \pi_i^j$, we can write the target consumption as:

$$C_t^* = \eta_L W_t^L + \eta_T W_t^T + \eta_P W_t^P \quad (\text{B.37})$$

where L , T and P denote its labour, transfer and property income.

The present discounted value of total income can be written as $W_t^i = \sum_{j=0}^{\infty} \frac{\phi_{t+j} \mathbb{E}_t Y_{t+j,i}}{(1+r)^j}$.

In contrast to labour and transfer income components, aggregate property wealth is only imperfectly observable in the data. We approximate property wealth as a weighted average of observed financial and housing wealth and the present value of property income.

$$W_t^P = \Psi \widehat{W}_t^P + (1 - \Psi) W_t^D \quad (\text{B.38})$$

Combining equations (B.37) and (B.38) yields the target equation to be estimated.

Liquidity Constraints

We now consider the case of relaxing the assumption that all consumers are free to borrow and lend against future income at rate r . We assume that there are two types of households: unconstrained (u) and constrained households (c). Spending by the unconstrained group evolves through the error correction equation:

$$\Delta c_{t,u} = a_0 (c_{t-1}^* - c_{t-1}) + \sum_{i=1}^1 a_i \Delta c_{t-i} + \beta_1 x_t + \mathbb{E}_{t-1} \sum_{j=0}^{\infty} d_j \Delta c_{t+j}^* + e_t^c \quad (\text{B.39})$$

Consumption growth for the constrained group, instead, equals the growth of income:

$$\Delta c_{t,c} = \Delta y_{t,c} \quad (\text{B.40})$$

Growth in aggregate consumption equals the sum of growth in constrained and unconstrained spending, weighted by their respective shares on the total. Denoting with θ the share of spending

and income attributable to constrained households, aggregate consumption growth is⁵⁴:

$$\begin{aligned} \Delta c_t = (1 - \theta) & \left(a_0 (c_{t-1}^* - c_{t-1}) + \sum_{i=1}^1 a_i \Delta c_{t-i} + \beta_1 x_t + \mathbb{E}_{t-1} \sum_{j=0}^{\infty} d_j \Delta c_{t+j}^* \right) \\ & + \theta \Delta (y h_t^L + y h_t^T) + \epsilon_t^C \end{aligned} \quad (\text{B.41})$$

where a spread on the lending rate on consumption, x , is also added to take into account the effects of financial factors on durable consumption, which is not modelled separately (see results in Table D.1).

Permanent income

The consumption target defined in equation (5) relies on measures of permanent income. Here we derive approximate expressions for permanent income, that can be evaluated with the extended version of the VAR used in the model.⁵⁵

We start by defining \widehat{W}_t as the the present value of total household income:

$$\widehat{W}_t \equiv \mathbb{E}_t \sum_{i=0}^{\infty} (1 + r + \phi_0)^{-i} Y_{t+i} \quad (\text{B.42})$$

We can relate this measure to permanent income by multiplying with $(1 - \check{\beta})$:

$$Y_t^P = (1 - \check{\beta}) \widehat{W}_t \quad (\text{B.43})$$

where $\check{\beta} = \frac{1}{1+r+\phi_0}$ is the discount factor and Y_t^P denotes permanent income.

We continue by defining the income to GDP ratio $\Omega = Y_t/X_t$. With this definition we can decompose total income into gap and trend terms of Ω and output:

$$\begin{aligned} Y_{t+i} &= \frac{Y_{t+i} X_{t+i}}{X_{t+i} \bar{X}_{t+i}} \bar{X}_{t+i} \\ &= \bar{\Omega}_{t+i} \bar{X}_{t+i} (1 + g)^i (1 + \tilde{\Omega}_{t+i}) (1 + \tilde{X}_{t+i}) \end{aligned} \quad (\text{B.44})$$

where $\bar{\Omega}$ is the average income-to-GDP ratio, $\tilde{\Omega}_t = \frac{\Omega}{\bar{\Omega}} - 1$ is its deviation from trend, and \bar{X}_t is potential output.

⁵⁴Two additional assumptions used here are: $\Delta \log C_{t,u}^* \approx \Delta \log C_t^*$ and $C_{t,u}^*/C_{t,u} \approx C_t^*/C_t$.

⁵⁵A detailed explanation of the structure of the Base VARs can be found in appendix A.2. Here it suffices to recall that the variables included in the Base VAR are: euro area interest rate and interest rate expectations; euro area inflation and inflation expectations; and euro area output gap. Permanent income variables are added to this base VAR to be forecasted.

Approximating equation (B.42) under the decomposition of Y_{t+i} in (B.44) we get

$$\log(\widehat{W}_t) \approx \log \left(\frac{\bar{\Omega}_t \bar{X}_t}{1 - \tilde{\beta}} \right) + (1 - \tilde{\beta}) \mathbb{E}_t \sum_{i=0}^{\infty} \tilde{\beta}^i \left[(\tilde{\Omega}_{t+i} + \tilde{X}_{t+i}) \right]$$

where $\tilde{\beta} = \frac{1+g}{1+r+\phi_0}$, r is the real interest rate, ϕ_0 is a risk-adjustment factor, and g is the growth rate of potential output \bar{X}_t .

By adding $\log(1 - \tilde{\beta})$ we get an expression for permanent income:⁵⁶

$$\log(Y_t^P) \approx \log(\bar{\Omega}_t \bar{X}_t) + (1 - \tilde{\beta}) \mathbb{E}_t \sum_{i=0}^{\infty} \tilde{\beta}^i \left[(\tilde{\Omega}_{t+i} + \tilde{X}_{t+i}) \right] \quad (\text{B.45})$$

To implement this setting in the VAR we define the extended VAR as:

$$\mathbb{E}_t Z_{t+i} = H^i Z_t$$

where Z_t is the stacked vector of the base VAR variables V_t and the gap of the income ratio $\tilde{\Omega}_t$. H^i denotes the estimated coefficient matrix of the VAR. Replacing the expected gaps in (B.45) we obtain:

$$\log(Y_t^P) \approx \log(\bar{\Omega}_t \bar{X}_t) + (1 - \tilde{\beta}) \left[(\iota_{\tilde{\Omega}_t} + \iota_{\tilde{X}_t})(I_n - \tilde{\beta}H)^{-1} Z_t \right] \quad (\text{B.46})$$

where $\iota_{\tilde{\Omega}_t}$ and $\iota_{\tilde{X}_t}$ define indicator vectors.

To evaluate this expression we calibrate the discount factor $\tilde{\beta}$ following the same approach and the same assumptions as in the FRB/US model. Following the definition of $\tilde{\beta} = \frac{1+g}{1+r+\phi_0}$ we need to calibrate the growth rate of potential output, the real rate of return on savings, and the risk adjustment parameter. The risk adjustment parameter itself is a function of the one-step-ahead conditional variance of income, the savings rate and the risk aversion parameter, as it is clear from equations (B.25) and (B.24). Evaluating these expressions we obtain a value of around 75 percent per year for $\tilde{\beta}$. Table D.2 gives the values underlying this calibration.

B.2 Investment Optimization Problem

The long-run demand for capital input into the production can be derived for a well know constrained profit maximization problem of an economic agent. Let us postulate a generic production function $F(N, K)$ satisfying standard properties with the two arguments denoting respectively labour and capital.⁵⁷ Firms maximize profits which are driven by relative (to price

⁵⁶For the subcomponents of permanent income the expression changes slightly. For example for the case of labour income we need to account for the labour income-to-total income ratio $\bar{\Omega}_t^L$, $Y_{L,t}^P \approx \bar{\Omega}_t \bar{\Omega}_t^L \bar{X}_t \exp \left((1 - \tilde{\beta}) \mathbb{E}_t \sum_{i=0}^{\infty} \tilde{\beta}^i \left[(\tilde{\Omega}_{t+i} + \bar{\Omega}_{t+i}^L + \tilde{X}_{t+i}) \right] \right)$.

⁵⁷Let us also ignore for the time being the issues of measuring capital, being raw capital or capital services.

of output) price of investment and real wage W .

What is important (from a purist perspective) is the timing assumption related to the capital law of motion. Namely, instead of assuming that investments are reflected in the capital stock within the same period, we adopt the time-to-build assumption according to which investments are projected onto capital in the next period. Considering the time-to-build assumption and its effect on capital accumulation, the profit maximization problem is given by:

$$\max_{\{K_t, I_t\}} \sum_{j=0}^{\infty} \left(\frac{1}{1 + R_{t+j}} \right)^j \{Y_{t+j} - W_{t+j}N_{t+j} - RP_{t+j}I_{t+j}\}$$

s.t.

$$K_{t+j} = (1 - \delta)K_{t+j-1} + I_{t+j-1} \quad (\text{B.47})$$

and

$$Y_t = F(N_t, K_t) \quad (\text{B.48})$$

Let λ_t denote the L-multiplier⁵⁸ on the evolution of capital so that we can write the maximization problem as:

$$\begin{aligned} \mathcal{L} = & \max_{\{K_{t+1}, I_t\}} \sum_{j=0}^{\infty} \left(\frac{1}{1 + R_{t+j}} \right)^j \{F(N_{t+j}, K_{t+j}) - W_{t+j}N_{t+j} - RP_{t+j}I_{t+j} \\ & + \lambda_{t+j} [I_{t+j} + (1 - \delta)K_{t+j} - K_{t+j+1}]\} \\ \frac{\partial \mathcal{L}}{\partial K_{t+1}} = & \frac{1}{1 + R_{t+1}} F'(N_{t+1}, K_{t+1}) - \left(\frac{1}{1 + R_t} \right)^0 g_t + \lambda_{t+1}(1 - \delta) \frac{1}{1 + R_{t+1}} \end{aligned} \quad (\text{B.49})$$

$$\frac{\partial \mathcal{L}}{\partial I_t} = R_t - \lambda_t \quad (\text{B.50})$$

Rearranging the FOC for capital

$$F'(N_{t+1}, K_{t+1}) - (1 + R_{t+1})\lambda_t + \lambda_{t+1}(1 - \delta) = 0$$

⁵⁸In this setting λ can be interpreted as the marginal effect of increased assets on profits and consequent market valuation of a firm, which offers a proxy for the Tobin's Q ratio.

$$\begin{aligned}
F'(N_{t+1}, K_{t+1}) &= RP_t(1 + R_{t+1}) - RP_{t+1}(1 - \delta) \\
&= RP_t \left[1 + R_{t+1} - (1 - \delta) \frac{RP_{t+1}}{RP_t} \right] \\
&= R_t \left[1 + R_{t+1} - (1 - \delta) - (1 - \delta) \left(\frac{RP_{t+1} - RP_t}{RP_t} \right) \right]
\end{aligned}$$

Under the assumption of constant returns to scales in the Cobb-Douglas production function we get:

$$(1 - \alpha) \frac{Y_{t+1}}{K_{t+1}} = RP_t \left\{ R_{t+1} + \delta - (1 - \delta) \left(\frac{RP_{t+1} - RP_t}{RP_t} \right) \right\} \equiv u_{t+1} \quad (\text{B.51})$$

where the right hand side represents the user cost of capital denoted by u .

B.3 Government

The structure of the sector

The government sector of the model can be organized around three parts as sketched in Table B.1). The first group of variables – non-financial accounts and gross debt – provides the decomposition of budget balance into underlying revenue and spending components, as well as the government debt with the linking debt-deficit-adjustment (DDA) variable. The suggested degree of disaggregation gives a comprehensive picture of the fiscal accounts, delivers variables of interest to the other parts of the model and takes data availability constraints into consideration.

The second group of variables – fiscal demand (consumption and investment) – includes all items necessary to calculate the demand components of the general government sector, namely, consumption and investment. Government consumption is disaggregated into categories of different nature, namely compensation, purchases (and a residual), which are subject to separate wage/employment and purchases shocks. Also, this group contains price variables (i.e. government consumption and investment deflators) that translate real demand components into nominal ones entering the budget balance.

The third group of variables – public labour sector – embeds elements that constitute input into the labour sector of the model.

The right-hand column of Table B.1 provides information on the type of equation associated with the particular variables in the block.

Table B.1: The overall structure of the fiscal block.

Non-financial accounts & gross debt		
Total revenue	<i>go_trn</i>	identity
Current taxes on income and wealth	<i>go_dtn</i>	identity

Table B.1 – continued from previous page

payable by corporations	<i>go_bu_dtn</i>	function of a macrobase
payable by households	<i>go_hh_dtn</i>	function of a macrobase
other	<i>go_rw_dtn</i>	share of potential GDP
Taxes on production and imports	<i>go_tin</i>	function of a macrobase
Net social security contributions	<i>go_scn</i>	function of a macrobase
Other revenue	<i>go_rrn</i>	nominal ECM equation
Total expend.	<i>go_toe</i>	identity
Interest, payable	<i>go_ipn</i>	specific equation
Social benefits in cash	<i>go_sbcn</i>	nominal ECM equation
Compensation of employees	<i>go_cen</i>	identity
Purchases ⁵⁹	<i>go_pun</i>	identity
Subsidies, payable	<i>go_sin</i>	nominal ECM equation
Gross fixed capital formation	<i>go_itn</i>	identity
Other expend.	<i>go_ren</i>	share of potential GDP
Net lending(+)/borrowing(-)	<i>go_b9n</i>	identity
DDA	<i>go_dda</i>	share of potential GDP
Government debt	<i>go_mal</i>	identity
Fiscal demand (consump. & invest.)		
Final consump. expend. (nominal)	<i>go_con</i>	identity
Final consump. expend. (nominal)/ resid.	<i>go_rcon</i>	identity
Final consump. expend. (real)	<i>go_cor</i>	identity
Compensation of employees (real)	<i>go_coer</i>	real ECM equation
Purchases (real)	<i>go_pur</i>	real ECM equation
Final consumption expend. (real)/ resid.	<i>go_rcor</i>	share of potential GDP
Gross fixed capital formation (real)	<i>go_itr</i>	real ECM equation
Government consump. deflator	<i>go_cod</i>	identity
Government invest. deflator	<i>go_itd</i>	specific equation
Government labour sector		
Employment (number of persons)	<i>go_lnn</i>	identity

⁵⁹Purchases consist of government intermediate consumption and social transfers in kind.

Table B.1 – continued from previous page

Compensation per head	<i>go_c_cen</i>	specific equation
Productivity in the government sector	<i>tfplg</i>	specific equation

The behavioural equations that govern the fiscal block can be classified into four main types. First, all variables on the revenue side with the exception of other revenue and other direct taxes are linked to macrobases and modelled through implicit tax rates, while most of the spending components and the other revenue category are modelled using ECM-type equations. The modelling of these parts have been detailed in section 3.1.3. Second, a majority of residual items (i.e. other direct taxes, other expenditure, DDA and the residual of final consumption expenditure), which exhibit large volatility in the data and therefore are difficult to forecast, stay constant as a share of potential GDP. Third, there are variables of idiosyncratic nature, such as interest payments, which require a specific equation to describe their dynamics. Finally, in addition to the behavioural equations, the fiscal block also contains multiple identities.

Revenue items and macro bases

Tax revenue variables in the fiscal block evolve as a function of macrobases. This is consistent with the nature of these variables, which in reality are strictly linked to relevant tax bases. Given no data availability on actual tax bases the usual approach in economic forecasting is to approximate them with macrobases. The modelling of this part has been formalised in 3.1.3. The following Table B.2 provides a list of all tax revenue variables in the model and the corresponding macrobases.

Table B.2: Revenue items and their corresponding macro bases.

Revenue item	Relevant macro base
Current taxes on income and wealth	
payable by corporations	Gross operating surplus & mixed income
payable by households	Total economy compensation of employees
Taxes on production and imports	Final private consumption & gov. purchases
Net social security contributions	Total economy compensation of employees

Fiscal variables as a share of potential GDP

Fiscal variables exhibiting erratic behaviour are modelled with processes that rapidly revert to a certain GDP share. This applies to other direct taxes, other spending, government consumption residual and Deficit-Debt Adjustment (DDA). The equations in these cases postulate that the variables immediately reach a ratio with respect to potential GDP observed in the past (or determined exogenously in the case of DDA) and remain there over the simulation horizon.⁶⁰ This implies that any large falls and surges observed in the data for these variables have no persistence and hence are treated as one-offs.

Specific equations

For the three following fiscal variables, we use specific equations. First, interest payments (INT_t) follow the equation

$$INT_t = (1 - 0.06)B_{t-1}IIR_{t-1} + 0.035\frac{1}{2}\left(IIR_{t-1} + \frac{AMIR_t}{4}\right) + \frac{1}{2}(-BB_t + INT_t)\frac{AMIR_t}{4} \quad (B.52)$$

$$IIR_t = \frac{INT_t}{B_{t-1}} \quad (B.53)$$

where B_t represents the government debt in nominal terms, $\frac{AMIR_t}{4}$ is a quarterly average market interest rate calculated as an average of market interest rates of various maturities, IIR_t denotes quarterly implicit interest rate on the existing stock of debt and BB_t stands for the budget balance of the general government sector. The coefficient of 0.06 is the average share of outstanding debt maturing per quarter in the euro area.⁶¹

Second, government labour productivity ($tfpl_t^G$) follows the pace of the labour productivity growth in the total economy ($tfpl_t$) based on the equation below.

$$\Delta(tfpl_t^G) = \Delta(tfpl_t) \quad (B.54)$$

Finally, average public compensation (cen_t^G) evolves in line with average total economy compensation (cen_t).

$$\Delta(cen_t^G) = \Delta(cen_t). \quad (B.55)$$

⁶⁰The assumption of zero share in the case of DDA used in the model implies zero values for this variable over the simulation period.

⁶¹The value of the coefficient is based on the average share of outstanding government debt securities maturing withing next 90 days observed during the 2014-17 period.

Identities

Table B.3 provides the full lists of identities of the fiscal block.

Table B.3: List of identities in the fiscal block.

Total revenue
$go_{trn_t} = go_{dtn_t} + go_{tin_t} + go_{scn_t} + go_{rrn_t}$
Current taxes on income and wealth
$go_{dtn_t} = go_{bu_dtn_t} + go_{hh_dtn_t} + go_{rw_dtn_t}$
Total expend.
$go_{toe_t} = go_{ipn_t} + go_{sbcn_t} + go_{cen_t}$
$+ go_{pun_t} + go_{sin_t} + go_{itn_t} + go_{ren_t}$
Compensation of employees
$go_{cen_t} = go_{lnn_t} \times go_{c_cen_t}$
Purchases
$go_{pun_t} = go_{pur_t} \times hh_{cod_t}$
Gross fixed capital formation
$go_{itn_t} = go_{itr_t} \times go_{itd_t}$
Net lending(+)/borrowing(-)
$go_{b9n_t} = go_{trn_t} - go_{toe_t}$
Government debt
$go_{mal_t} = go_{mal_{t-1}} - go_{b9n_t} + go_{dda_t}$
Final consump. expend. (nominal)/ resid.
$go_{rcon_t} = go_{rcor_t} \times hh_{cod_t}$
Final consump. expend. (nominal)
$go_{con_t} = go_{cen_t} + go_{pun_t} + go_{rcon_t}$
Final consump. expend. (real)
$go_{cor_t} = go_{cer_t} + go_{pur_t} + go_{rcor_t}$
Government consump. deflator
$go_{cod_t} = go_{con_t} \div go_{cor_t}$
Employment (number of persons) ⁶²

⁶²The equation for employment originates from the Cobb Douglas production function for the government sector, which embeds the assumption that output is equal to real compensation and takes the following form:

Table B.3 – continued from previous page

$\log(go_lnn_t) = \frac{3}{2} \log(go_cor_t) - \frac{1}{2} \log(tfplg_t)$

B.4 An auxiliary model for wages and inflation

Introduction

While the main part of the ECB-BASE follows the logic of the FRB-US model, a system estimation approach is chosen for the joint determination of output gap, inflation, wages and unemployment. This part formulates a semi-structural general equilibrium model, where the structure of the equations follows economic reasoning, but several economic restrictions are relaxed to improve the empirical fit and allow for an intuitive use of the model. This approach is very close to the approach chosen by the IMF in their 'QPM' series, Carabenciov et al. (2008). The model is centered around a **Wage-Price** and **Output gap** (WAPRO) specification.

The strategy of estimating the equations in the main model in subblocks, combined with a general equilibrium approach for the price, wage, output gap nexus allows us to merge the advantages from two different modelling classes. The modelling strategy of the main part of the ECB-BASE follows the structure of the FRB-US and combines a high degree of flexibility, a good data-fit and an intuitive and easy use of the model. The equations of the model are estimated in small subsystems such as the investment block, the consumption block or the trade block. While this approach features the advantages discussed above, the system properties and general equilibrium properties of the model are introduced in a later step only, when the model parts are combined.

To improve the system properties, especially in the nexus of prices, wages, output gap and interest rates, a small general equilibrium model is estimated. Key features of the approach such as Bayesian system estimation, some microfoundations and the explicit modelling of expectations are borrowed from the dynamic stochastic general equilibrium literature. However, several of the economic restrictions are relaxed to improve the data fit and to allow for an easy an intuitive use of the model. While the structure of the equations is guided by optimizing behaviour of the economic agents, the cross-equation restrictions of the structural parameters are ignored and instead reduced form estimates of the loading coefficients are estimated.

The chosen framework allows for a Kalman filter based, general equilibrium estimation strategy in a Bayesian setting. The advantages of this approach relate to an explicit focus on system properties and the possibility to estimate unobserved concepts such as the output gap or mea-

$$go_cor_t = tfplg_t^{\frac{1}{3}} go_lnn_t^{\frac{2}{3}}.$$

asures of natural unemployment measures such as the NAIRU. The output gap in the main model will then be consistent with the estimated Phillips curves for price and wage inflation.

Furthermore the chosen approach explicitly focuses on the open economy dimension of the euro area. Also, the WAPRO model features a clear structure with steady state and balance growth path, which is identical to the steady state implemented in the ECB-BASE. The model consistent estimates of unobserved variables such as the output gap and the equations for price and wage inflation are exported to the ECB-BASE.

The WAPRO model relates to the ECB-BASE in two respects. First, unobserved variables such as output gap and unemployment gap are estimated as state variables in the WAPRO model. This allows to identify these measures in a model consistent way and ensure a joint determination of output, prices, wages and various measures of slack. More concretely, the WAPRO implied measure of potential output can serve as alternative measures for output gap and natural unemployment to the ones used in the projection process. Second, the WAPRO model produces the equations and estimated parameters of prices and wages that will be exported to the ECB-BASE.

Equations

The model is a two country model. The first country is defined as the euro area, the second country is defined by the world except the euro area. The cross country relations between these two entities are kept simple and comprise the determination the exchange rate based on an uncovered interest rate parity (UIP) condition and trade as a function of foreign demand. To capture the effect of exchange rates and oil prices on headline inflation, an import price Phillips curve is introduced. In the following equations, notation is such that euro area variables are denoted without a superscript, while all variables with a star refer to the ROW.

Inflation

In the price block we model three different price indices: the GDP deflator as the core-price deflator, the import price deflator and the consumer price inflation as a function of GDP deflator and import deflator.

The GDP deflator is modelled via a reduced form NK Phillips Curve, as in Galí et al. (2012) combined with a variant of Cogley and Sbordone (2008). We follow this approach because it features involuntary unemployment and is therefore consistent with the assumptions inside the ECB-BASE. The equation shows that actual inflation depends both on a measure of expected inflation and on past inflation. Marginal costs are approximated via a combination of the output

gap and the wage gap.

$$\begin{aligned} \pi_t = \frac{1}{1 + \beta_\pi \delta_\pi} & \left\{ (1 - \delta_\pi + \delta_\pi \beta_\pi - \beta_\pi) \bar{\pi}_t \right. \\ & + \beta_\pi (\mathbb{E}_t \pi_{t+1}) + \delta_\pi \pi_{t-1} \\ & \left. + \beta_y^\pi \left(\hat{w}_t + \left(\frac{\alpha}{1-\alpha} \right) \hat{y}_t \right) \right\} + e_t^\pi \end{aligned} \quad (\text{B.56})$$

where α is the capital share in the model.

Note that inflation is modelled with reference to an inflation attractor $\bar{\pi}$. To accommodate medium term deviation of inflation from the inflation target, we model the time-varying inflation attractors as follows:

$$\bar{\pi}_t = (1 - \beta_{\bar{\pi}}) \pi_{ss}^{EA} + \beta_{\bar{\pi}} \bar{\pi}_{t-1} + \delta_{\bar{\pi}} (\pi_{t-1} - \bar{\pi}_{t-1}) + e_{\bar{\pi},t} \quad (\text{B.57})$$

In addition to the GDP deflator, which is largely capturing price developments of domestic goods, we introduce an additional Philips curve for the import price deflator. Here we follow the specification of the New Area Wide Model (NAWM, Christoffel, Coenen, and Warne 2008), where marginal cost are approximated as a combination of oil prices and foreign prices. Unlike the NAWM, we use inflation measures rather than the price level gaps of oil and foreign goods.

$$\begin{aligned} \pi_t^m = & (1 - \delta_\pi^m + \delta_\pi^m \beta_\pi^m - \beta_\pi^m) \bar{\pi}_t^m \\ & + \beta_\pi^m (\mathbb{E}_t \pi_{t+1}^m - \delta_\pi^m \pi_t^m) + \delta_\pi^m \pi_{t-1}^m \\ & + \beta_y^{\pi^m} [\gamma_{oil}^m (\hat{p}_t^{oil}) + (1 - \gamma_{oil}^m) (\pi_t^* - \pi_{ss}^* + \hat{s}_{4,t})] + e_{\pi,t}^m \end{aligned} \quad (\text{B.58})$$

The trend of import price inflation is

$$\bar{\pi}_t^m = (1 - \beta_{\bar{\pi}}^m) \pi_{ss}^m + \beta_{\bar{\pi}}^m \bar{\pi}_{t-1}^m + e_{\bar{\pi},t}^m \quad (\text{B.59})$$

Note that we are modelling extra euro area import prices and augment them with a measurement error.

Exchange rate and consumption deflator

The exchange rate is modelled via an uncovered interest parity assumption:

$$(i_t - i_t^*) = \tilde{\mathbb{E}} s_{t+1} - s_t + E_t (\pi_{t+1} - \pi_{t+1}^*) + e_t^s$$

where $\tilde{\mathbb{E}}_{s_{t+1}}$ is a weighted average of the rational expectation of the exchange rate and its trend level:

$$\tilde{\mathbb{E}}_{s_{t+1}} \equiv \alpha_s \mathbb{E}_{s_{t+1}} + (1 - \alpha_s) \bar{s}_t \quad (\text{B.60})$$

Oil prices are treated in a non-structural way by defining an oil inflation gap in deviation from an oil inflation trend, where both the deviation from the gap and the trend are modelled as stationary autoregressive components.⁶³

Finally, private consumption inflation is modelled as a weighted average of import price inflation and GDP deflator inflation:

$$\pi_t^C = \beta_{\pi^C}^{\pi} \pi_t + (1 - \beta_{\pi^C}^{\pi}) \pi_t^m + e_{\pi,t}^C \quad (\text{B.61})$$

Wage Inflation

The wage inflation gap is modelled via a reduced form NK Phillips Curve. The equation relates actual wage inflation gap depends to future wage inflation, past, present and future domestic price inflation, present and future euro area price inflation, the deviation of the labour share of income from its trend, the deviation of trend output growth from its steady state and the deviation of the unemployment rate from its trend. In formula:

$$\begin{aligned} \pi_t^w + \pi_t^C = \frac{1}{1 + \beta_w \delta_w} & \left\{ (1 - \beta_{\pi}^w) (1 - \delta_{\pi}^w) (\bar{\pi}_t + \Delta \bar{y}_t) \right. \\ & + \beta_{\pi}^w \mathbb{E}_t (\pi_{t+1}^w + \pi_{t+1}^C) + \gamma_{\pi}^w (\pi_{t-1}^w + \pi_{t-1}^C) - \beta_u^{\pi^w} \hat{u}_t \Big\} \\ & + \xi_w \hat{w}_t + e_t^{\pi^w} \end{aligned} \quad (\text{B.62})$$

Here we follow the approach by Galí et al. (2012) and relate the unemployment gap to the gap in the marginal rate of substitution and a variant of Cogley and Sbordone (2008).

Output

An IS curve is added to approximate aggregate demand in the euro area.

$$\hat{y}_t = (1 - \rho_{\hat{y}}) \mathbb{E}_t \hat{y}_{t+1} + \rho_{\hat{y}} \hat{y}_{t-1} - \rho_{\hat{r}}^{\hat{y}} \hat{r}_t + \rho_{\hat{y}^*}^{\hat{y}} \hat{y}_t^* + e_t^{\hat{y}} \quad (\text{B.63})$$

In addition to the usual determinants in the form of the expected and lagged output gap and the real interest rate \hat{r}_t we have added the foreign output gap (\hat{y}_t^*) to proxy for export demand.

⁶³Note also that oil inflation is stationarized and assumed growing at the rate of domestic inflation in the measurement part of the model.

Policy Rule

The monetary authority sets the nominal interest rate according to a policy rule which contains its value in the previous period, the nominal interest rate trend, a weighted average of home and foreign inflation and a weighted average of home and foreign output gaps.

$$i_t = (1 - \rho)(\bar{\pi} + \bar{r} + \phi_\pi(\hat{\pi}_{4,t} - \bar{\pi}_t) + \phi_y \hat{y}_t) + \rho i_{t-1} + \epsilon_t^i \quad (\text{B.64})$$

The real interest rate is given by the difference between the nominal interest rate for the euro area and the expected domestic price inflation rate: $r_t = i_t - \mathbb{E}_t \pi_{t+1}$ where \hat{r}_t is the log deviation of the real interest rate from its trend; $\hat{r}_t = r_t - \bar{r}_t$. The trend real interest rates are modelled via an AR process:

$$\bar{r}_t = (1 - \rho_{\bar{r}}) \bar{r}_{ss} + \rho_{\bar{r}} \bar{r}_{t-1} + e_t^{\bar{r}}$$

Unemployment

The unemployment gap follows a flexible interpretation of Ogun's law and is positively related to its level in $t - 1$ and it is negatively related to the output gap:

$$\hat{u}_t = \rho_{\hat{u}} \hat{u}_{t-1} + \beta_{\hat{y}}^{\hat{u}} \hat{y}_t + e_t^{\hat{u}} \quad (\text{B.65})$$

where the unemployment gap (\hat{u}) is defined as the difference between actual unemployment rate and the natural level of the unemployment rate (\bar{u}): $\hat{u}_t = u_t - \bar{u}_t$; and the natural level of the unemployment rate follows an autoregressive structure in its own level and the growth rate of natural unemployment (GU_t) which is in turn modelled as an AR(1) process:

$$\bar{u}_t = (1 - \rho_{\bar{u}}) \bar{u}_{t-1} + \rho_{\bar{u}} \bar{u}_{ss} + GU_t + e_t^{\bar{u}} \quad (\text{B.66})$$

$$GU_t = (1 - \rho_{GU}) GU_{t-1} + e_t^{GU} \quad (\text{B.67})$$

Rest of the WORLD

The rest of the world (ROW) is modelled via three key equations. Specifically, an IS curve:

$$\hat{y}_t^* = (1 - \rho_{\hat{y}^*}) \mathbb{E}_t \hat{y}_{t+1}^* + \rho_{\hat{y}^*} \hat{y}_{t-1}^* - \rho_{\hat{r}^*}^{\hat{y}^*} \hat{r}_t^* + e_t^{\hat{y}^*};$$

A price Phillips curve

$$\begin{aligned}\pi_t^* &= (1 - \delta_{\pi^*} + \delta_{\pi^*}\beta_{\pi^*} - \beta_{\pi^*})\bar{\pi}_t^* + \beta_{\pi^*}(\mathbb{E}_t\pi_{4,t+4}^* - \delta_{\pi^*}\pi_{4,t}^*) \\ &\quad + \delta_{\pi^*}\pi_{4,t-1}^* + \beta_{\hat{y}^*}^{\pi^*}(\hat{y}_t^*) + e_t^{\pi^*};\end{aligned}\tag{B.68}$$

and a Taylor rule:

$$i_t^* = (1 - \beta_1^{i^*})[\bar{r}_t^* + \bar{\pi}_t^* + \beta_2^{i^*}(\pi_t^* - \bar{\pi}_t^*) + \beta_3^{i^*}\hat{y}_t^*] + \beta_1^{i^*}i_{t-1}^* + \epsilon_t^{i^*}.$$

For the relation between euro area and ROW we assume that, with the exception of the exchange rate, ROW variables affect the euro area, but euro area variables do not affect the ROW. Furthermore we take the US-federal funds rate as the ROW interest rate.

Relation between WAPRO and the main model

As discussed above the WAPRO model can be used as a self-sustained model of the euro area embedded into a global model. This use of the model might lend itself to analyze questions around world demand, oil prices and exchange rates. In addition to this, the wage Phillips curve and the GDP deflator Phillips curve are used as the central price and wage equations inside the ECB-BASE. Obviously, WAPRO and ECB-BASE differ in some respects, but share the most important aspects, and this ensure a consistent connection between the two models.

Modelling of expectations

In WAPRO expectations are modelled in a model-consistent rational expectations framework, based on the solution of the model. In the standard version of the ECB-BASE expectations are modelled via a VAR. More specifically use a small VAR to estimate a reduced form representation of these expectations. To evaluate the impact of expectation-formation on the performance of the wage and price equations, we have evaluated WAPRO under the same expectations formation process as ECB-BASE. More specifically we have introduced the estimated VAR equations that define the one-step ahead forecast for inflation and wage inflation into WAPRO, estimated under rational expectations, and conducted pseudo-recursive forecast exercises. The differences in the forecast performance between the model under the different expectations formation processes are not very pronounced and qualitative analysis as impulse responses also produces very similar outcomes.

An additional difference between WAPRO and the main model is in the timing of expectations. In the ECB-BASE the equation are estimated individually and contemporaneous variables can lead to a simultaneity bias. To avoid expectations contributing to the simultaneity bias, expectations are dated in $t - 1$. WAPRO, instead, is estimated as a system and offers greater flexibility in terms of the information set for inflation expectations.

In a future version of the ECB-BASE, the wage price block will be evaluated under model consistent expectations, bridging the currently existing differences.

Unobserved components

WAPRO is estimated in a state space system, allowing for the identification of unobserved concepts such as the output gap and the natural rate of unemployment. In contrast to this, in the ECB-BASE these concepts are treated as exogenous. To increase the comparability between the two models and ensure consistency of the output gap between ECB-BASE and WAPRO, potential output growth is included as an additional observable variable in WAPRO.⁶⁴ Furthermore, the wage equation contains a measure of the wage gap derived from the ECB-BASE. This is a key series to ensure convergence in the ECB-BASE and is therefore added as an additional observable in WAPRO.

Steady state and long-run

Steady state and long-run are identical between ECB-BASE and WAPRO.

Disaggregated modelling and aggregate modelling of GDP

The ECB-BASE is obviously a large scale model with disaggregated demand components and modelling many channels that are not present in WAPRO, where GDP is modelled in the aggregate via an IS curve.

Data and estimation

WAPRO is estimated on the following observable variables for the euro area: GDP deflator, GDP growth, growth of potential output, extra euro area import price deflator, private consumption deflator, unemployment rate, wage per employee, wage gap (from ECB-BASE), euro area interest rate and the euro area nominal effective exchange rate. Furthermore we use several international variables: GDP deflator (world without euro area), GDP growth (world without euro area), oil price, federal funds rate (US).

The model is estimated with Bayesian techniques. The details of the estimation and its results can be found in Table D.13.

B.5 Details on equity premium

The cost of equity represents the return a firm's pays to its equity investors (shareholders) to compensate the risk they undertake by investing their capital with the company. It is unobservable and must thus be inferred from other observable variables and a theoretical model.

⁶⁴For the natural rate of unemployment no clear consensus exists for the euro area and the measures obtained in WAPRO and in the projection process are very similar. We therefore abstained from using natural unemployment as an additional observable variable in WAPRO.

The Dividend Discount Model: The value of any asset is the present value of expected future cash flows discounted at a rate appropriate to the riskiness of the cash flows:

$$P_t^E = \sum_{i=1}^{\infty} \frac{D_{t+i}}{(1 + r_t^{coe})^i} \quad (\text{B.69})$$

where P^E is a stock price, D are corresponding dividend payments, and r^{COE} is a discount factor associated with future dividends payments.

An investor that buys a stock, expects to get two types of cashflows: dividends during the holding period of the stock and an expected price at the end (itself determined by future dividends). Thus, the value of the stock is the present value of dividends discounted through infinity at a rate appropriate to the riskiness of cash flows. The cost of equity is estimated as the discount rate r_t^{coe} implied by the realized price.

Since projections of dividends in practice cannot be made through infinity, several versions of the dividend discount model have been developed based upon different assumptions about future growth. We use the Hsia-Fuller Divident Discount Model that is based on two stages of growth, an extraordinary growth phase (g_{st}) that lasts n years ($n = 2H$) and a stable growth phase that lasts forever afterwards:⁶⁵

$$r^{coe} = \frac{D_t}{P_t^E} [(1 + g_{LT}) + H(g_{ST} - g_{LT})] + g_{LT} \quad (\text{B.70})$$

B.6 Accounting and closing conditions

The individual blocks presented in the model are linked into a coherent macroeconomic framework through the main accounting aggregates related to the System of National Accounts, namely the GDP, the gross disposable income, the financial account and the change in net worth position. With these accounts we attempt to represent the modelled economy in terms of production of goods and services, generation and allocation of income in that process, and changes in the net worth position. The structure of the model allows a full encompassing closure of accounts for the households sector, while a bridge to the total economy is provided in a non-systematic fashion as explained below.

To provide a full accounting framework of the model we start by defining the **nominal GDP** from the expenditure side as:

$$Y_t^d = P_t^c C_t + P_t^{ib} IB_t + IHD_t IH_t + P_t^G G_t + XTD_t \cdot XTR_t + MTD_t \cdot MTR_t \quad (\text{B.71})$$

where P_t^{ib} is the deflator related to aggregate business investment IB , IHD is residential in-

⁶⁵Fuller and Hsia (1984).

vestment deflator related to the residential investment IH , P^G is the deflator related to the government spending G , XTR and MTR are total exports and imports, while XTD and MTD are their respective deflators.

The nominal supply side of the economy is given by:

$$Y_t^S = A_t (K_t)^\alpha (\zeta^t N_t)^{(1-\alpha)} \quad (\text{B.72})$$

where K_t denotes aggregate capital stock and N_t denotes aggregate employment (headcount). A_t is total factor productivity and ζ_t is a labour augmenting technology.

Note that in the short run:

$$Y_t^d \neq Y_t^s \quad (\text{B.73})$$

The demand-supply equality is only imposed in the long-run with the dynamics of several variables linked to the one of the output gap which is assumed to close in steady-state.

The nominal income generated in the domestic production process corresponds to the nominal GDP from the income side:

$$Y_t = CEN_t + GOSMIN_t + TIN_t \quad (\text{B.74})$$

where CEN captures aggregate compensation allocated to the labor, $GOSMIN$ is the aggregate gross operating surplus and mixed income, and TIN represents the tax revenues related to production and imports. The aggregate compensation for employees is obtained within the Supply block in Section 3.2, while the tax revenues are determined in the Government block in Section 3.1.3. The only variable that is therefore not endogenously specified within the model and remains to be defined is the gross operating surplus and mixed income, which is simply given as a residual term, $GOSMIN = Y - CEN - TIN$.

A shift from domestic to national economy perspective is provided by the **gross national income**:

$$GNI_t = Y_t + NPI_t \quad (\text{B.75})$$

where NPI represents the net property income. The NPI , however, is not endogenously specified within the current structure of the model nor is it provided exogenously. The existing specification of the model does, however, allow a complete representation of accounts for the household sector.

A GNI equivalent for the household sector is given by the Gross Balance of Personal Income, $GBPI$, defined as:

$$GBPI_t = GOS_t^{HH} + MIN_t^{HH} + CEN_t + NPI_t^{HH} \quad (\text{B.76})$$

where

$$GOS^{HH} = GOSMIN^{HH} - MIN^{HH} = share \cdot GOSMIN - MIN^{HH} \quad (B.77)$$

and

$$MIN_t^{HH} = MIN_{t-1}(1 + gr_t^{CEN}) \quad (B.78)$$

where gr_t^{CEN} is a growth rate of compensation for employees and $GOSMIN^{HH}$ is expressed as a fixed share of the total economy's $GROSMIN$. Note that in the $GBPI$ for the household sectors CEN is the same as the one for the total economy, while tax revenues for the household sector are assumed to be zero.

By correcting $GBPI$ for direct taxes, social contribution and other transfers, we can construct the **gross disposable income** for the household sector:

$$\begin{aligned} GDI_t^{HH} = & \underbrace{(CEN_t + GOS_t^{HH} + MIN_t^{HH} + NPI_t^{HH})}_{GBPI} - \underbrace{(PITL_t^{HH} + PITt_t^{HH} + WTP_t^{HH})}_{Direct\ taxes} \\ & - \underbrace{(SCL_t^{HH} + SCT_t^{HH} + SCP_t^{HH})}_{Social\ contributions} + SB_t^{HH} + OCT_t^{HH} \end{aligned} \quad (B.79)$$

where $PITL^{HH}$ and $PITt^{HH}$ are personal taxes on labor and transfer income, WTP^{HH} denotes taxes on property wealth, SCL^{HH} , SCT^{HH} and SCP^{HH} are social contributions on respective labor, transfer and property incomes, while SB^{HH} and OCT^{HH} denote social benefits and other current transfers. Individual direct tax and social contribution components are derived within the Government block in Section 3.1.3.

By rearranging (B.79) we can express GDI^{HH} in terms of endogenously specified income components in the ECB-BASE model. Specifically, components of GDI^{HH} can be grouped into disposable labor income (DINL), disposable transfer income (DINT) and disposable property income (DINP):

$$\begin{aligned} GDI_t^{HH} = & \underbrace{CEN_t + MIN_t^{HH} - PITL_t^{HH} - SCL_t^{HH}}_{DINL} + \underbrace{SB_t^{HH} - PITt_t^{HH} - SCT_t^{HH}}_{DINT} \\ & + \underbrace{GOS_t^{HH} + NPI_t^{HH} + OCT_t^{HH} - WTP_t^{HH} - SCP_t^{HH}}_{DINP} \end{aligned} \quad (B.80)$$

What remains to be defined to fully represent the household sector in terms of national accounts are the accounting identities for the **financial account** and the **net worth position**. The financial account is given by the net borrowing/lending position ($NBLP^{HH}$) of the

household sector:

$$NBLP_t^{HH} = GDI_t^{HH} - C_t - ITN_t^{HH} + OINC_t^{HH} \quad (\text{B.81})$$

where C and ITN^{HH} denote the private consumption and investment of the household sector respectively, while $OINC^{HH}$ captures residual terms not fully accounted by the model (i.e. net capital transfers, changes in net worth of pension funds and net acquisitions of non-produced financial assets).

The changes in the net worth position of the household sector can be represented by

$$\Delta HHW_t = NBLP_t^{HH} + HHW_{t-1}REV_t \quad (\text{B.82})$$

where REV captures accounts related to other changes in volume of assets and net acquisitions, and is specified within the Financial block in Section 3.3. Finally, the household net worth position, HHW , also specified in the Financial block, is:

$$HHW_t = GHW_t + NFW_t \quad (\text{B.83})$$

where GHW is the gross housing wealth and NFW is the net financial wealth, defined respectively by equations (60) and (61).

The above identities therefore allow for a full representation of accounts related to the household sector. The transition to the total economy is provided by two steps: first, by specifying a residual related to the net lending/borrowing position whose dynamics can be used for forecasting applications; second, by assuming that the revaluation of the total economy's wealth follows dynamics of other changes in wealth related to the household sector.

C Use of residuals for forecasting in the ECB-BASE model

This appendix sketches a possible way of using (a subset of) the residuals of the ECB-MC model to generate a “mechanical-judgement” set of forecasts. It starts from the assumption that the residuals have some remaining structure due to two factors: (1) the model is estimated equation by equation; and (2) the model is misspecified. These assumptions imply that some remaining auto- and cross-correlations might still be present in the residuals.

C.1 A simple state space model

We assume the following unobserved-component representation of the residuals, given by the sum of a more “persistent” component and an idiosyncratic one:

$$e_t = c_t + \varepsilon_t \quad \varepsilon_t \sim N(0, \Sigma) \quad (\text{C.84})$$

where the vector $c_t = (c_{1t}, \dots, c_{kt})'$ evolves according to the following transition equation

$$c_t = Ac_{t-1} + u_t \quad (\text{C.85})$$

for $t = 2, 3, \dots, T$, with $u_t \sim N(0_k, \Omega)$, and the transition equation is initialised with

$$c_1 \sim N(c_0, \Omega_0).$$

We make the following assumptions:

1. The matrix Σ is symmetric and positive definite;
2. The matrix A is diagonal, with $\rho = \text{diag}(\rho_1, \dots, \rho_k)$ denoting the vector of its diagonal elements;
3. The matrix Ω is diagonal, with $\omega^2 = \text{diag}(\omega_1^2, \dots, \omega_k^2)$ denoting the vector of its diagonal elements;
4. The initialization hyperparameter c_0 and Ω_0 are known.

Given the above assumptions, the possible remaining cross-correlation of the residuals is captured by Σ .

C.2 Bayesian estimation

To complete the model and estimate our main object of interest, c_t , we need to complement the likelihood implied by the assumption (1) above with a prior for the unknown parameters.

Re-write Eq. (C.84) in matrix notation

$$\mathbf{e} = \mathbf{c} + \varepsilon \quad \varepsilon \sim N(\mathbf{0}, I_T \otimes \Sigma)$$

where $\mathbf{e} = (e_1, \dots, e_T)'$, $\mathbf{c} = (c'_1, \dots, c'_T)'$, $\varepsilon = (\varepsilon_1, \dots, \varepsilon_T)'$.

The (log) likelihood is therefore given by:

$$\ln p(\mathbf{e} \mid \mathbf{c}, \Sigma) = -\frac{T}{2} \ln |\Sigma| - \frac{1}{2} (\mathbf{e} - \mathbf{c})' (I_T \otimes \Sigma)^{-1} (\mathbf{e} - \mathbf{c}) + \text{const} \quad (\text{C.86})$$

or, equivalently, by

$$\ln p(e_t | c_t, \Sigma) = -\frac{T}{2} \ln |\Sigma| - \frac{1}{2} \sum_t (e_t - c_t)' \Sigma^{-1} (e_t - c_t) + \text{const} \quad (\text{C.87})$$

For convenience we also stack the transition equation and assume $c_0 = 0_k$. Eq. (C.85) can then be written as:

$$\mathbf{H}\mathbf{c} = \mathbf{u} \quad \mathbf{u} \sim N(\mathbf{0}, S) \quad (\text{C.88})$$

where

$$\mathbf{H} = \begin{pmatrix} I & 0 & 0 & \cdots & 0 \\ -A & I & 0 & \cdots & 0 \\ 0 & -A & I & \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & \cdots & -A & I \end{pmatrix} \quad S = \begin{pmatrix} \Omega_0 & 0 & 0 & \cdots & 0 \\ 0 & \Omega & 0 & \cdots & 0 \\ 0 & & \Omega & \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \Omega \end{pmatrix}$$

Notice that \mathbf{H} is invertible. Therefore the prior distribution of \mathbf{c} is

$$\mathbf{c} = \mathbf{H}^{-1}\mathbf{u} \sim N\left[\mathbf{0}, \left(\mathbf{H}'S^{-1}\mathbf{H}\right)^{-1}\right] \quad (\text{C.89})$$

where $|\mathbf{H}| = 1$, and $|S| = |\Omega_0| |\Omega|^{T-1}$.

The joint (log) density for \mathbf{c} can be written as

$$\ln p(\mathbf{c} | \Omega) = -\frac{T-1}{2} \ln |\Omega| - \frac{1}{2} \mathbf{c}' \mathbf{H}' S^{-1} \mathbf{H} \mathbf{c} + \text{const} \quad (\text{C.90})$$

To complete the prior specification we assume

$$p(\rho, \Sigma, \omega^2) = p(\rho) p(\Sigma) p(\omega^2)$$

with:

$$\rho_j \sim N(\bar{\rho}, \sigma_\rho^2) \text{ and } E(\rho_i, \rho_j) = 0 \quad (\text{C.91})$$

$$\Sigma \sim IW(S^{-1}, \nu) \propto |\Sigma|^{-(\nu+k+1)/2} \exp\left[-\frac{1}{2} \text{tr}(S\Sigma^{-1})\right] \quad (\text{C.92})$$

$$\omega_j^2 \sim IG(\alpha_{\omega^2}, \lambda_{\omega^2}) \propto (\omega_j^2)^{-(\alpha_{\omega^2}+1)} \exp(-\lambda_{\omega^2}/\omega_j^2) \quad (\text{C.93})$$

where IW denotes an Inverse-Wishart and IG denotes an Inverse-Gamma distribution whose hyperparameters are all known. An exchangeability assumptions holds for ρ and ω_j^2 . This means

that the elements of $\rho(\omega^2)$ are assumed to come from the same distribution and that their prior variance determines the degree of the cross-residual heterogeneity.

The joint posterior distribution is

$$p(\mathbf{c}, \rho, \Sigma, \omega^2 | \mathbf{y}) \propto p(\mathbf{y} | \mathbf{c}, \Sigma) p(\mathbf{c} | \rho, \omega^2) p(\rho) p(\Sigma) p(\omega^2)$$

This distribution does not have a known form and marginal distributions of c , ρ , Σ , and ω^2 are analytically not available in standard form. It can be shown, however, that standard conditional posterior distributions $p(\mathbf{c} | \mathbf{y}, \rho, \Sigma, \omega^2)$, $p(\rho | \mathbf{y}, \mathbf{c}, \Sigma, \omega^2)$, $p(\Sigma | \mathbf{y}, \mathbf{c}, \rho, \omega^2)$, and $p(\omega^2 | \mathbf{y}, \mathbf{c}, \rho, \Sigma)$ can be easily derived and a Gibbs Sampling used.

C.3 Gibbs sampling

The distribution $p(\mathbf{c} | \mathbf{y}, \rho, \Sigma, \omega^2)$ can be derived from the combination of (C.86) and (C.90)

$$\begin{aligned} \ln p(\mathbf{c} | \mathbf{y}, \rho, \Sigma, \omega^2) &\propto -\frac{1}{2} \left[(\mathbf{e} - \mathbf{c})' (I_T \otimes \Sigma)^{-1} (\mathbf{e} - \mathbf{c}) + \mathbf{c}' (\mathbf{H}' S^{-1} \mathbf{H}) \mathbf{c} \right] \\ &\propto -\frac{1}{2} \left[(\mathbf{c} - \hat{\mathbf{c}})' K_c (\mathbf{c} - \hat{\mathbf{c}}) \right] \end{aligned}$$

which implies that

$$p(\mathbf{c} | \mathbf{y}, \rho, \Sigma, \omega^2) = N(\hat{\mathbf{c}}, K_c^{-1}) \quad (\text{C.94})$$

where it is easy to show that

$$K_c = \mathbf{H}' S^{-1} \mathbf{H} + (I_T \otimes \Sigma)^{-1} \quad \hat{\mathbf{c}} = P^{-1} (I_T \otimes \Sigma)^{-1} \mathbf{e}$$

The derivation of $p(\rho | \mathbf{y}, \mathbf{c}, \Sigma, \omega^2)$ comes from the product of (C.91) and (C.90). Given the diagonality assumption of A , a conditional posterior distribution for each ρ_j can be derived:

$$\begin{aligned} \ln(\rho_j | \mathbf{y}, \mathbf{c}, \Sigma, \omega^2) &\propto -\frac{1}{2} \left[\omega_j^{-2} \sum_{t=2}^T (c_{jt} - \rho_j c_{jt-1})^2 + \sigma_\rho^{-2} (\rho_j - \bar{\rho})^2 \right] \\ &\propto -\frac{1}{2} \left[(\rho_j - \hat{\rho}_j)^2 / \hat{\sigma}_\rho^2 \right] = N(\hat{\rho}_j, \hat{\sigma}_\rho^2) \end{aligned} \quad (\text{C.95})$$

where

$$\hat{\sigma}_\rho^2 = \left(\omega_j^{-2} \sum_{t=2}^T c_{jt-1}^2 + \sigma_\rho^{-2} \right)^{-1} \quad \hat{\rho}_j = \hat{\sigma}_\rho^2 \left(\omega_j^{-2} \sum_{t=2}^T c_{jt} c_{jt-1} + \sigma_\rho^{-2} \bar{\rho} \right)$$

To avoid an explosive residual setting on the forecast horizon, there might be a need to truncate this distribution to the stationary region $(-1, 1)$.

The derivation of $p(\Sigma | \mathbf{y}, \rho, \mathbf{c})$ proceeds from the product of (C.87) and (C.92):

$$\begin{aligned} \ln p(\Sigma | \mathbf{y}, \mathbf{c}, \rho) &\propto -\frac{(\nu + T + k + 1)}{2} \ln |\Sigma| - \frac{1}{2} \left[\text{tr} \left(\sum_t (e_t - c_t)(e_t - c_t)' \Sigma^{-1} \right) + \text{tr} (S \Sigma^{-1}) \right] \\ p(\Sigma | \mathbf{y}, \mathbf{c}, \rho) &= IW \left[\left(S + \sum_t (e_t - c_t)(e_t - c_t)' \right)^{-1}, \nu + T \right] \end{aligned} \quad (\text{C.96})$$

Finally, $p(\omega_j^2 | \mathbf{y}, \rho, \mathbf{c})$ can be obtained from the product of (C.93) and (C.90). Given the diagonality assumption of Ω , we can derive a conditional posterior distribution for each ω_j^2 :

$$\begin{aligned} \ln p(\omega_j^2 | \mathbf{y}, \mathbf{c}, \rho) &\propto -\frac{T-1}{2} \ln \omega_j^2 - \frac{1}{2\omega_j^2} \sum_{t=2}^T (c_{jt} - \rho_j c_{jt-1})^2 + (\alpha_{\omega^2} + 1) \ln \left(\frac{1}{\omega_j^2} \right) - \frac{\lambda_{\omega^2}}{\omega_j^2} \\ p(\omega_j^2 | \mathbf{y}, \mathbf{c}, \rho) &= IG \left[\alpha_{\omega^2} + \frac{T-1}{2}, \lambda_{\omega^2} + \frac{1}{2} \sum_{t=2}^T (c_{jt} - \rho_j c_{jt-1})^2 \right] \end{aligned} \quad (\text{C.97})$$

A Gibbs sampling algorithm cycles through (C.94) to (C.97).

Efficient sampling

Note that inverting the matrix K_c could be problematic if the dimension of \mathbf{e} is sizeable. An efficient sampling from $p(\mathbf{c} | \mathbf{y}, \rho, \Sigma, \omega^2) = N(\hat{\mathbf{c}}, K_c^{-1})$ can be obtained from the following steps:

1. Obtain a Cholesky decomposition $K_c = CC'$
2. Let $\mathbf{x} = (C')^{-1} \mathbf{z}$ with $\mathbf{z} \sim \mathbf{N}(\mathbf{0}, \mathbf{I})$. Hence $\mathbf{x} \sim \mathbf{N}(\mathbf{0}, K_c^{-1})$
3. Obtain $\hat{\mathbf{c}}$ efficiently by solving $K_c \hat{\mathbf{c}} = (I_T \otimes \Sigma)^{-1} \mathbf{e}$
4. Compute $\mathbf{c} = \hat{\mathbf{c}} + (C')^{-1} \mathbf{z}$

C.4 Model-based forecasting

Additional steps can be added to the Gibbs sampling to derive an out-of sample forecast of both \mathbf{c} and \mathbf{e} .

In particular, conditional on data until T and on the parameters' draws, a sequence of additional distributions in the Gibbs sampling for c_{jT+h} and e_{jT+h} for all j and over the forecast horizon $T+1, \dots, T+h$, can be obtained.

In particular, given linearity and under the assumption of no structural breaks, from (C.84) and (C.85) it is easy to show that:

$$p(c_{jT+h} | \mathbf{y}_{(T)}, \mathbf{c}_{(T)}, \rho, \omega^2) = N(\rho_j c_{jT+h-1}, \omega^2) \quad (\text{C.98})$$

and

$$p\left(e_{T+h} \mid \mathbf{y}_{(T)}, \mathbf{c}_{(T+h)}, \rho, \omega^2\right) = N\left(c_{T+h}, \Sigma\right) \quad (\text{C.99})$$

Given these draws, we have a forecast for all relevant variables with the corresponding model uncertainty which can be used to do stochastic simulation. These projections will be conditional on a given path of the exogenous variables. For different paths of the exogenous variables we can have desired scenario analyses.

D Tables and Charts

Table D.1: Consumption

Target			
Parameter	Description	Estimate	st.error
η_0	Constant of target consumption	0.504	0.3069
η_T	Weight of permanent transfer income	0.271	0.0750
η_P	Weight of permanent property income	0.107	0.0830
η_D	Weight of financial and housing wealth	0.078	0.0670
η_L	Weight of permanent labour income	0.545	0.0820
γ^T	Coeff. of time trend	-0.0004	0.0005
Dynamic PAC Equation			
Parameter	Description	Estimate	st.error
a_0	Coeff. of past deviation from target consumption	0.343	0.0596
a_1	Coeff. of lagged consumption growth	0.011	0.1122
β_x	Weight of spread on the lending rate on consumption	-0.003	0.0007
θ	Share of liquidity constrained households	0.361	0.0699
R^2		0.592	

Note: Note that the estimation of the dynamic part is done using iterative least squares on the non-linear model in a non-standard way, where the true distribution of standard errors of this estimation is not available. Instead we report the standard errors according to the linear OLS at the last iteration taking the expectation term as given.

Table D.2: Calibration of discount factor

Target		
Parameter	Description	calibration
g	Growth rate potential GDP	2%
s	Savings rate	7%
γ	Risk aversion	-3
$\sigma_{t,t+1}$	Conditional variance of income	9%
r	Real rate of return on savings	8%

Note: The calibration of $\sigma_{t,t+1}$ is based on Christelis, Georgarakos, Jappelli, and van Rooij (2016) while γ follows evidence provided by Guiso, Sapienza, and Zingales (2013) and Guiso and Paiella (2008).

Table D.3: Business Investment

Target			
Calibration			
Dynamic PAC Equation			
Parameter	Description	Estimate	st.error
a_0^{ib}	Coeff. of past deviation from target investment	0.169	0.0530
a_1^{ib}	Coeff. of lagged investment growth	0.665	0.1638
R^2		0.662	

Table D.4: Residential Investment

Target			
Parameter	Description	Estimate	st.error
β_1^{ih}	Coeff. of user cost of capital	-0.16	0.0600
β_2^{ih}	Coeff. of relative prices growth	1.06	0.1000
γ^{ih}	Coeff. of time trend	-0.01	0.0003
α^{ih}	Weight of housing capital on residential investment production	-2.22	0.1560
Dynamic PAC Equation			
Parameter	Description	Estimate	st.error
a_0^{ih}	Coeff. of past deviation from target residential investment	0.096	0.0464
a_1^{ih}	Coeff. of one-period lagged investment growth	0.049	0.0925
a_2^{ih}	Coeff. of two-period lagged investment growth	0.227	0.0923
θ_1^{ih}	Coeff. of de-trended real GDP growth	1.05	0.2349
R^2		0.546	

Table D.5: Estimates for total import volumes

Target			
Parameter	Description	Estimate	st.error
α_0^m	Constant of target import	0.3406	0.0082
α_1^m	Coeff. of relative import prices (without oil)	-1.1663	0.1170
Dynamic Error Correction Equation			
Parameter	Description	Estimate	st.error
β_c^m	Constant of import growth	0.0012	0.0021
ρ^m	EC Coeff. related to target deviations	-0.0842	0.0626
β_1^m	Coeff. related to import demand changes	1.6938	0.3206
β_2^m	Coeff. related to exchange rate changes	-0.1187	0.1573
R^2		0.456	
SE		0.013	
SSR		0.010	
D-W		1.792	

Table D.6: Estimates for extra-EA import volumes

Target			
Parameter	Description	Estimate	st.error
α_c^{mx}	Constant of target extra-EA import	-0.2501	0.0131
α_1^{mx}	Coeff. of relative extra-EA import prices (without oil)	-1.2668	0.1503
Dynamic Error Correction Equation			
Parameter	Description	Estimate	st.error
β_c^{mx}	Constant of extra-EA import growth	0.0034	0.0024
β_{ecm}^{mx}	EC Coeff. of target extra-EA import	-0.0701	0.0396
β_I^{mx}	Coeff. of import demand growth	1.5520	0.3674
β_X^{mx}	Coeff. of exchange rate growth	-0.1039	0.1687
R^2		0.395	
SE		0.015	
SSR		0.014	
D-W		1.76	

Table D.7: Estimates for extra-EA export volumes

Target			
Parameter	Description	Estimate	st.error
α_0^{xx}	Constant of target extra-EA export	1.8909	0.0399
α_1^{xx}	Coeff. of relative extra-EA prices	-0.5445	0.1184
α_2^{xx}	Coeff. of time trend	-0.0021	0.0003
Dynamic Error Correction Equation			
Parameter	Description	Estimate	st.error
β_c^{xx}	Constant of extra-EA export growth	0.0005	0.0025
ρ^{xx}	EC coefficient	-0.0720	0.0680
β_1^{xx}	Coeff. of world demand for extra-EA export growth	0.8921	0.1259
β_2^{xx}	Coeff. of variation in exchange rates	0.1972	0.4158
R^2		0.684	
SE		0.014	
SSR		0.012	
D-W		1.99	

Table D.8: Estimates for extra-EA export deflators

Target			
Parameter	Description	Estimate	st.error
α_0^{xd}	Constant of target extra-EA export prices	0.0843	0.0092
β_1^{xd}	Coeff. of relative extra-EA cost competitiveness	0.7317	0.0411
Dynamic Error Correction Equation			
Parameter	Description	Estimate	st.error
β_c^{xd}	Constant of extra-EA export prices growth	-0.0025	0.0014
ρ_1^{xd}	EC Coeff. of target extra-EA export prices	-0.0853	0.0330
β_1^{xd}	Coeff. of foreign prices growth	0.1211	0.0298
β_2^{xd}	Coeff. of extra-EA import prices growth	0.2224	0.0418
β_3^{xd}	Coeff. of domestic prices growth	0.9482	0.3151
R^2		0.685	
SE		0.014	
SSR		0.0122	
D-W		2.086	

Table D.9: Estimates for extra-EA import deflators

Target			
Parameter	Description	Estimate	st.error
α_0^{mxd}	Constant of target extra-EA import prices	0.2144	0.0044
β_1^{mxd}	Coefficient of energy prices	0.1165	
β_2^{mxd}	Coefficient of relative prices	0.3000	
Dynamic Error Correction Equation			
Parameter	Description	Estimate	st.error
β_c^{mxd}	Constant of extra-EA import prices growth	-0.0020	0.0012
ρ^{mxd}	EC Coeff. of target extra-EA import prices	-0.0526	0.0324
β_1^{mxd}	Coeff. of energy prices	0.1165	
β_2^{mxd}	Coeff. of lagged energy prices growth	0.0235	0.0102
β_3^{mxd}	Coeff. of competitors price growth	0.1715	0.0378
β_4^{mxd}	Coeff. of lagged extra-EA export prices growth	0.5763	0.1310
R^2		0.893	
SE		0.006	
SSR		0.001	
D-W		2.24	

Table D.10: Estimates for intra-EA import deflators

Target			
Parameter	Description	Estimate	st.error
α_0^{mnd}	Constant of target intra-EA import prices	0.0195	0.0049
α_1^{mnd}	Coeff. of relative domestic prices	0.4461	0.0877
Dynamic Error Correction Equation			
Parameter	Description	Estimate	st.error
β_c^{mnd}	Constant of intra-EA import prices growth	0.0007	0.0012
ρ^{mnd}	EC Coeff. of target intra-EA import prices	-0.0970	0.0313
β_1^{mnd}	Coeff. of lagged intra-EA import prices growth	-0.0008	0.1074
β_2^{mnd}	Coeff. of foreign prices growth	0.0379	0.0482
β_3^{mnd}	Coeff. of energy prices growth	0.0495	0.0115
R^2		0.308	
SE		0.011	
SSR		0.009	
D-W		2.07	

Table D.11: Exchange rate equation

Parameter	Description	Estimation	st. error
β_1^s	Elasticity To Domestic Prices	1	0.000
β_2^s	Elasticity To Foreign Prices	-1	0.000
β_3^s	Elasticity To ex-ante EA 10-year rate	-0.042	0.008
β_4^s	Elasticity to ex-ante US 10-year rate	0.042	0.008
β_5^s	Constant	0.175	0.006

Table D.12: Employees

Target			
Calibration			
Dynamic PAC Equation			
Parameter	Description	Estimate	st.error
α	Weight of capital on aggregate production (calibrated)	0.372	
a_0	Coeff. of past deviation from target labour	0.038	0.0125
a_1	Coeff. of lagged labour growth	0.874	0.0555
R^2		0.824	

Table D.13: Domestic Inflation

Parameter	Description	Estimate	st.error
β_y^π	Coeff. of marginal cost	0.1214	0.0275
δ_π	Indexation on past inflation	0.3873	0.0792
β_π	Coeff. of expectations	0.6295	0.0749

Note: These estimates are part of a system estimation. For further details see D.16.

Table D.14: Wage Inflation

Parameter	Description	Estimate	st.error
β_π^w	Coeff. of expectations	0.8112	0.0877
γ_π^w	AR coeff.	0.3913	0.0979
δ_π^w	Indexation on con. defl.	0.1409	0.0397
$\beta_u^{\pi^w}$	Coeff. of unempl. gap	0.0921	0.0453
ξ_w	Coeff. of wage gap	0.5000	0.2773

Note: These estimates are part of a system estimation. For further details see D.16.

Table D.15: Interest Rate Rule

Parameter	Description	Value
Φ_i	Smoothing coefficient	0.8
$\bar{\pi} + \bar{r}$	Target Interest Rate	3.2
Φ_{Π}	Response to inflation gap	1.9
$\Phi_{\Delta\Pi}$	Response to change in inflation	0.0250
Φ_y	Response to change in output gap	0.5000

Note: Parameters and the real natural rate are calibrated as in the New Area-Wide Model according to the version updated with data until 2011(Christoffel, Coenen, and Vetlov (2012)).

Table D.16: Wapro Parameters

Variable		prior dist.	mean	s.d.	posterior mode	s.d.
<i>Domestic Inflation:</i>						
β_y^{π}	Coeff. of marginal cost	beta	0.200	0.050	0.1214	0.0275
δ_{π}	Indexation on past inflation	beta	0.650	0.100	0.3873	0.0792
β_{π}	Coeff. of expectations	beta	0.750	0.100	0.6295	0.0749
$\delta_{\bar{\pi}}$	Coeff. of past dev. of target	beta	0.200	0.080	0.0973	0.0470
$\beta_{\bar{\pi}}$	AR coeff. of target	beta	0.800	0.100	0.7955	0.0965
<i>Foreign Inflation:</i>						
$\beta_{y^*}^{\pi}$	Coeff. of marginal cost	beta	0.300	0.080	0.0996	0.0317
δ_{π}^*	Indexation on past inflation	beta	0.250	0.050	0.2045	0.0418
β_{π}^*	Coeff. of expectations	beta	0.970	0.010	0.9736	0.0094
$\beta_{\bar{\pi}}^*$	AR coeff. of target	beta	0.800	0.100	0.8755	0.1204
<i>Wage Inflation:</i>						
β_{π}^w	Coeff. of expectations	beta	0.750	0.100	0.8112	0.0877
γ_{π}^w	AR coeff.	beta	0.650	0.100	0.3913	0.0979
δ_{π}^w	Indexation on con. defl.	beta	0.150	0.040	0.1409	0.0397
$\beta_u^{\pi^w}$	Coeff. of unempl. gap	beta	0.250	0.100	0.0921	0.0453
ξ_w	Coeff. of wage gap	beta	0.100	0.020	0.1030	0.0206
<i>Consumption and Import Prices:</i>						
$\beta_{\pi^c}^{\pi}$	Weight on domestic infl. (con.)	beta	0.800	0.100	0.8127	0.0115
$\beta_{\bar{\pi}}^m$	AR coeff. of target (imp.)	beta	0.500	0.100	0.4999	0.1066
β_y^m	Coeff. of marginal cost (imp.)	beta	0.500	0.200	0.4381	0.1782
δ_{π}^m	Indexation on imp. defl. (imp.)	beta	0.500	0.200	0.3492	0.0725
β_{π}^m	Coeff. of expectations (imp.)	beta	0.800	0.150	0.6523	0.2508
γ_{oil}^m	Weight on oil prices (imp.)	beta	0.500	0.200	0.1658	0.0640

<i>Oil Price:</i>						
ρ_{π}^{oil}	AR coeff. of target	beta	0.500	0.150	0.5000	0.1756
ρ_{π}^{oil}	AR coeff.	beta	0.500	0.150	0.2686	0.0853
<i>Output:</i>						
$\rho_{\hat{y}}$	AR Coeff. of output (dom.)	beta	0.200	0.100	0.5548	0.0187
$\rho_{\hat{r}}^{\hat{y}}$	Coeff. of int. rate gap (dom.)	beta	0.050	0.020	0.0350	0.0123
$\rho_{y^*}^{\hat{y}}$	Coeff. of foreign output (dom.)	beta	0.100	0.030	0.0460	0.0103
$\rho_{\Delta \bar{y}}$	AR coeff. of change in target (dom.)	beta	0.950	0.015	0.9519	0.0146
$\rho_{\hat{y}^*}$	AR Coeff. of output (for.)	beta	0.500	0.100	0.6484	0.0356
$\rho_{\hat{r}^*}^{\hat{y}^*}$	Coeff. of int. rate gap (for.)	beta	0.100	0.030	0.0817	0.0195
$\rho_{\Delta \bar{y}^*}$	AR coeff. of change in target (for.)	beta	0.950	0.015	0.9470	0.0159
<i>Unemployment:</i>						
$\rho_{\hat{u}}$	AR coeff.	beta	0.800	0.100	0.9276	0.0493
$\beta_{\hat{y}}^{\hat{u}}$	Coeff. of output	norm	1.000	0.500	0.0184	0.0439
<i>Exchange Rate:</i>						
α_s	Coeff. on expectations	beta	0.100	0.050	0.0912	0.0584

Table D.17: Standard deviation of shocks

Variable		prior dist.	mean	s.d.	posterior mode	s.d.
e_t^π	Domestic Inflation	inv. gam.	0.500	0.050	0.5359	0.0360
$e_t^{\pi^*}$	Foreign Inflation	inv. gam.	0.100	0.100	1.1192	0.1058
$e_t^{\pi^w}$	Wage Inflation	inv. gam.	0.100	0.050	0.7580	0.0752
$e_t^{\pi^{\hat{w}}}$	Wage Gap	inv. gam.	0.100	0.050	0.2731	0.0244
$e_{\pi,t}^m$	Import Prices	inv. gam.	0.500	0.050	0.4234	0.0297
$e_{\pi,t}^{\bar{m}}$	Target Import Prices	inv. gam.	0.500	0.300	0.3391	0.1121
$e_{\pi,t}^{ME,m}$	Import Prices Mesurment Errors	inv. gam.	0.500	0.100	2.1149	0.1673
$e_t^{\hat{y}}$	Output Gap	inv. gam.	0.100	0.030	0.1528	0.0185
$e_t^{\Delta \hat{y}}$	Eq. Output Growth Rate	inv. gam.	0.100	0.030	0.0389	0.0032
$e_t^{\Delta \hat{y}^*}$	Eq. Foreign Output Growth Rate	inv. gam.	0.100	0.030	0.1016	0.0256
$e_t^{\hat{u}}$	Unemployment Gap	inv. gam.	1.000	0.100	0.9582	0.0899
$e_t^{\bar{u}}$	Eq. Unemployment	inv. gam.	0.100	0.100	0.0573	0.0214
e_t^{GU}	Natural Unemployment	inv. gam.	0.100	0.100	16.6943	1.3990
$e_t^{\bar{r}}$	Domestic Trend Real Int. Rate	inv. gam.	0.100	0.100	0.0572	0.0215
e_t^i	Domestic Interest Rate	inv. gam.	0.100	0.100	0.4079	0.0350
$e_t^{i^*}$	Foreign Interest Rate	inv. gam.	0.100	0.100	0.3448	0.0343
$e_t^{\bar{r}^*}$	Foreign Trend Real Int. Rate	inv. gam.	0.100	0.100	1.2977	0.2036
e_t^s	Exchange Rate	inv. gam.	0.100	0.100	2.1888	0.2755
$e_{p,t}^{oil}$	Trend Oil Price	inv. gam.	0.100	0.100	0.0573	0.0214
$e_{p,t}^{oil}$	Oil Price	inv. gam.	0.100	0.100	1.1317	0.0950

Table D.18: Term and Risk Premium Estimates

Parameter	Description	Estimate	st.error
<i>Term premium $R(40)$:</i>			
ρ^{TP}	Autoregressive term	0.59	0.090
β_1^{TP}	Expected output gap	-0.11	0.050
β_2^{TP}	US spill-over effects	0.20	0.046
α^{TP}	Constant	0.62	0.144
Risk spreads			
<i>Consumption rate:</i>			
ρ^{PR}	Autoregressive term	0.65	0.073
β^{PR}	Expected output gap	-0.52	0.109
α^{PR}	Constant	1.51	0.310
<i>Mortgage rate:</i>			
ρ^{PR}	Autoregressive term	0.65	0.082
β^{PR}	Expected output gap	-0.30	0.074
α^{PR}	Constant	0.49	0.117
<i>NFC lending rate:</i>			
ρ^{PR}	Autoregressive term	0.84	0.053
β^{PR}	Expected output gap	-0.13	0.042
α^{PR}	Constant	0.20	0.066
<i>Corporate bond rate:</i>			
ρ^{PR}	Autoregressive term	0.85	0.088
β^{PR}	Expected output gap	-0.01	0.066
α^{PR}	Constant	0.14	0.096
<i>Equity cost:</i>			
ρ^{PR}	Autoregressive term	0.90	0.037
β^{PR}	Expected output gap	-0.30	0.098
α^{PR}	Constant	0.55	0.181
<i>Deposit rate:</i>			
ρ^{PR}	Autoregressive term	0.94	0.057
β^{PR}	Expected output gap	-0.05	0.076
α^{PR}	Constant	0.03	0.028

Table D.19: Gross Operating Surplus

Parameter	Description	Estimate	st.error
α^{GS}	Constatnt	0.000	0.000
β_1^{GS}	Nominal housing stock	0.007	
β_2^{GS}	Relative house prices	0.015	0.001

Table D.20: Net Interest Income

Parameter	Description	Estimate	st.error
α^{IRN}	Constatnt	0.001	0.0006
ρ_1^{IRN}	Autoregressive term	0.949	0.0300
β_2^{IRN}	Net foreign assets	0.065	0.0584
β_3^{IRN}	Short term rate	0.000	0.0004
β_4^{IRN}	Mortgage-deposit spread	-0.000	0.000

Table D.21: Dividends estimation results using Iterative OLS method

Target			
Calibration			
Dynamic Error Correction Equation			
Parameter	Description	Estimate	st.error
a_0	EC Coeff. of target dividends	0.25	0.0871
a_1	Coeff. of one-period lagged dividend growth	-0.08	0.1270
a_2	Coeff. of two-period lagged dividend growth	0.15	0.1181
a_3	Coeff. of three-period lagged dividend growth	0.33	0.1138
R^2		0.22	

Table D.22: Revaluation of Financial Wealth - % shares

Parameter	Description	Calibration
s_0	Not subject to revaluation	
s^{LT}	Gov. debt securities	
s^{CB}	Corporate debt securities	
s^{EQP}	Equity	

Table D.23: Revaluation of Financial Wealth

Parameter	Description	Estimation	st. error
α^{nfa}	Constant	0.0109	0.01
β_1^{nfa}	Spread between rates on foreign liabilities and assets	-0.2165	-3.14
β_2^{nfa}	Exchange rate	0.2897	1.21
β_3^{nfa}	Domestic prices	-1.1018	-1.18
β_4^{nfa}	Foreign prices	1.1559	2.52

Table D.24: PAC - House Prices

Target			
Parameter	Description	Estimate	st.error
α_0^{H*}	Constant of target consumption	4.23	0.219
β_1^{H*}	Elasticity to income to housing capital ratio	0.98	0.035
β_3^{H*}	Elasticity to user costs of housing ownership	-0.60	0.071
Dynamic PAC Equation			
Parameter	Description	Estimate	st.error
α^{rph}	Coeff. of past deviation from target	0.034	0.007
ρ_1^{rph}	Coeff. of lagged prices	0.674	0.060
R^2	0.742		

Table D.25: Net Foreign Assets

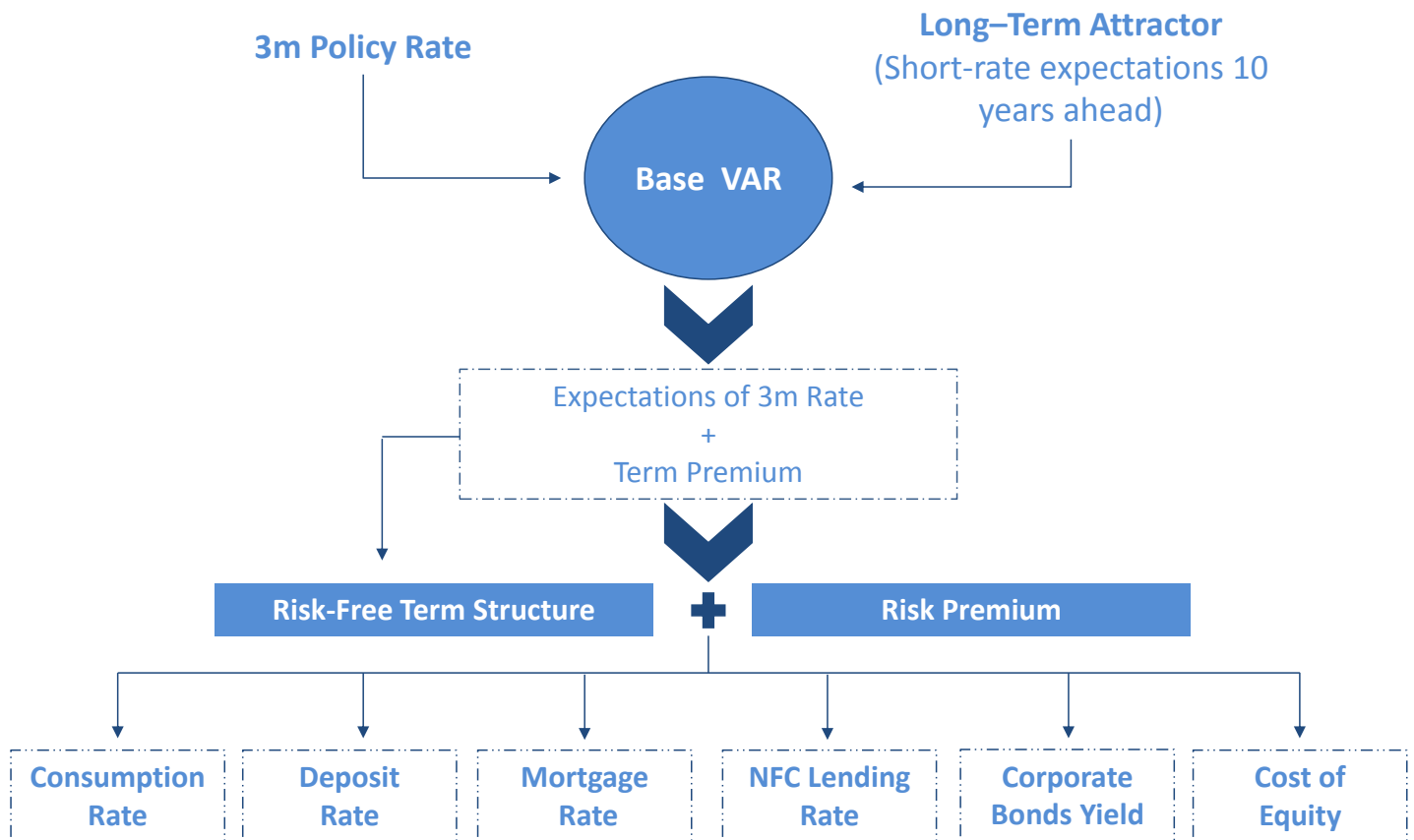
Parameter	Description	Estimation	st. error
β^{ifa}	Elasticity To US 10-year rate	0.4151	0.055
β^{ifl}	Elasticity to EU 10 year rate	0.2447	0.024

Table D.26: Long-run and balanced growth path

Balanced Growth Path Assumptions				
Variable	Mean (long samp.)	Mean (est. samp.)	Model	
Productivity	0.29	0.16	0.3	<i>quart. growth rate in %</i>
Population growth	0.1	0.06	0.2	<i>quart. growth rate in %</i>
Inflation	4.68	1.56	1.9	<i>annual growth rate in %</i>
Unemployment rate	7.8	9.5	8.4	<i>Rate in %</i>
Steady State Ratios w.r.t. GDP				
Variable	Mean (long samp.)	Mean (est. samp.)	Model	
Consumption	55.9	55.1	59.8	<i>share of GDP in %</i>
Government consumption	19.5	19.5	20.6	<i>share of GDP in %</i>
Trade balance	2.4	3.6	-0.6	<i>share of GDP in %</i>
Total investment	22.3	21.9	20.3	<i>share of GDP in %</i>
Employee compensation	48.4	47.0	56.3	<i>share of GDP in %</i>

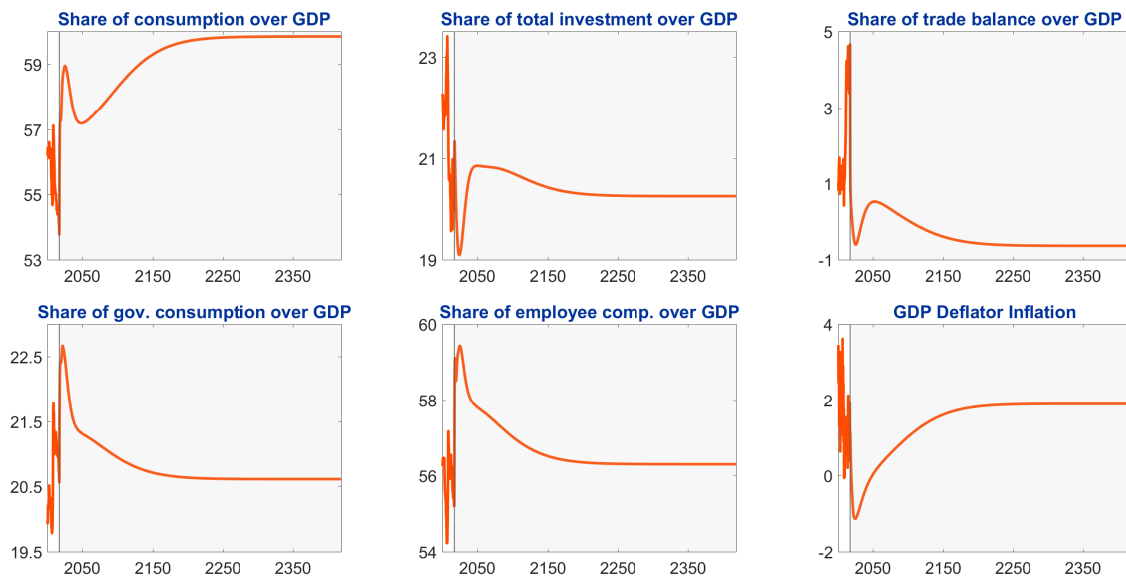
Note: This table shows selected properties of the model along the balanced growth path. The moments are compared to the moments in the data over two samples. The long sample goes from 1970Q1 to 2017Q4, while the estimation sample is truncated to 2000Q1 to 2017Q4.

Figure D.1: Interest rates in the ECB-BASE



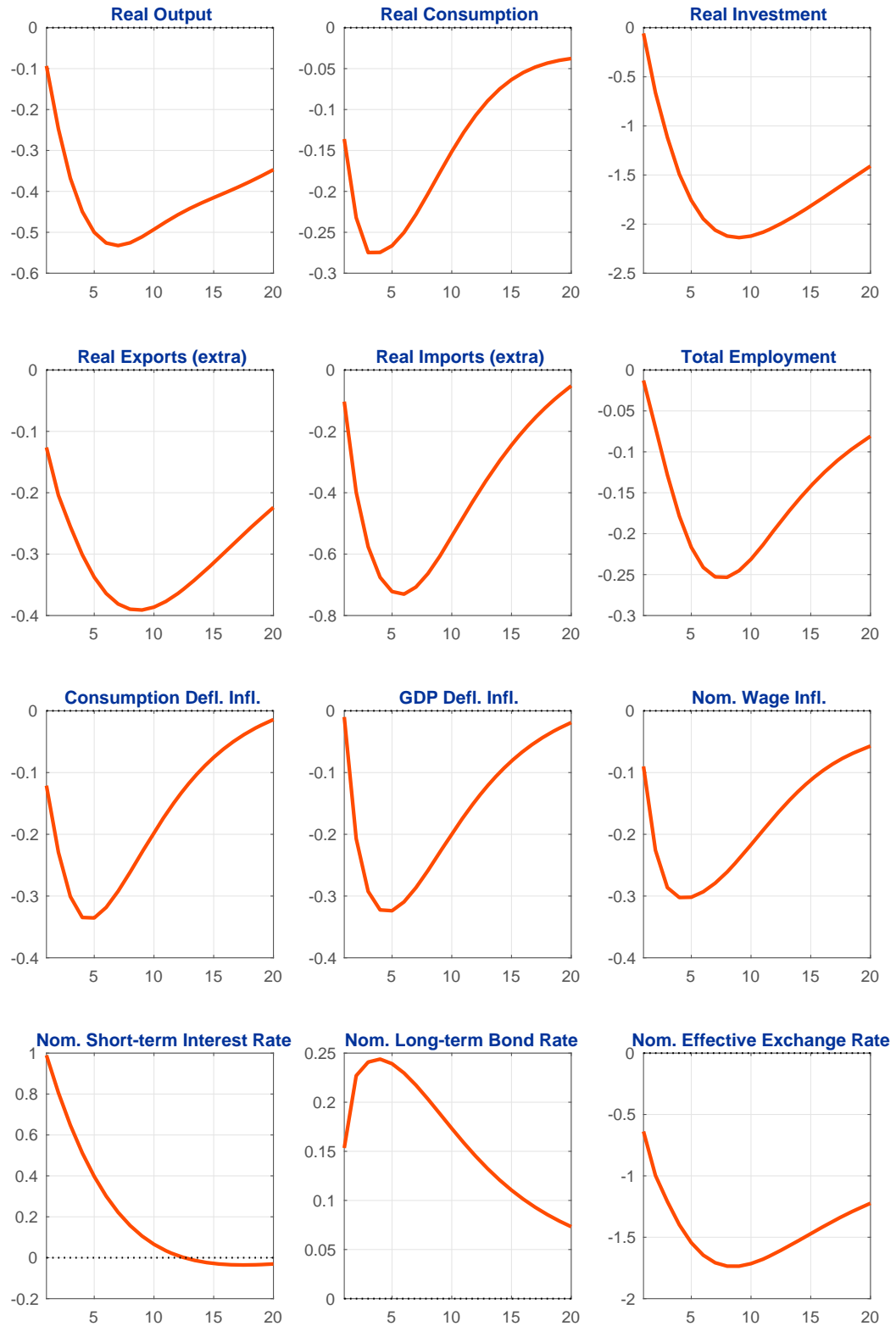
Note: This figure shows the set-up of the interest rates inside the financial block.

Figure D.2: Long-run behaviour



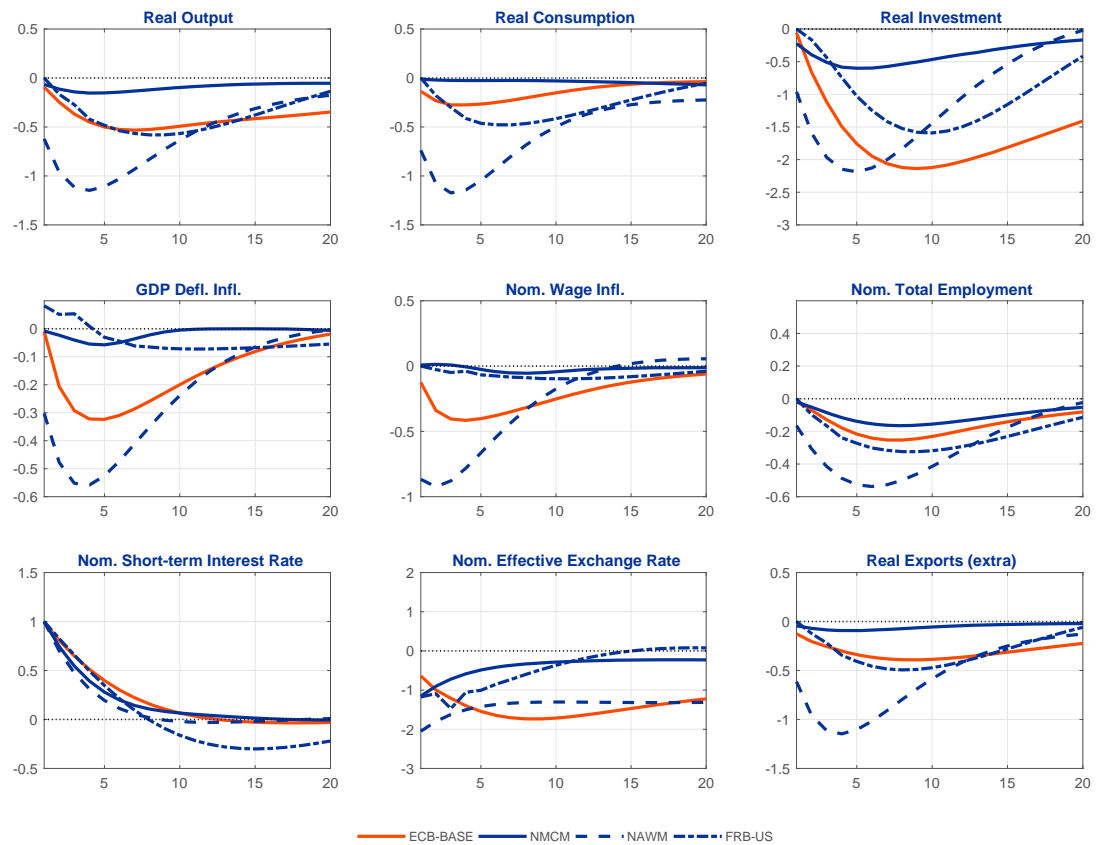
Note: This figure shows the convergence paths of selected variables and ratios in percent in the model. The grey shaded denotes the simulation horizon.

Figure D.3: Response to a monetary policy shock



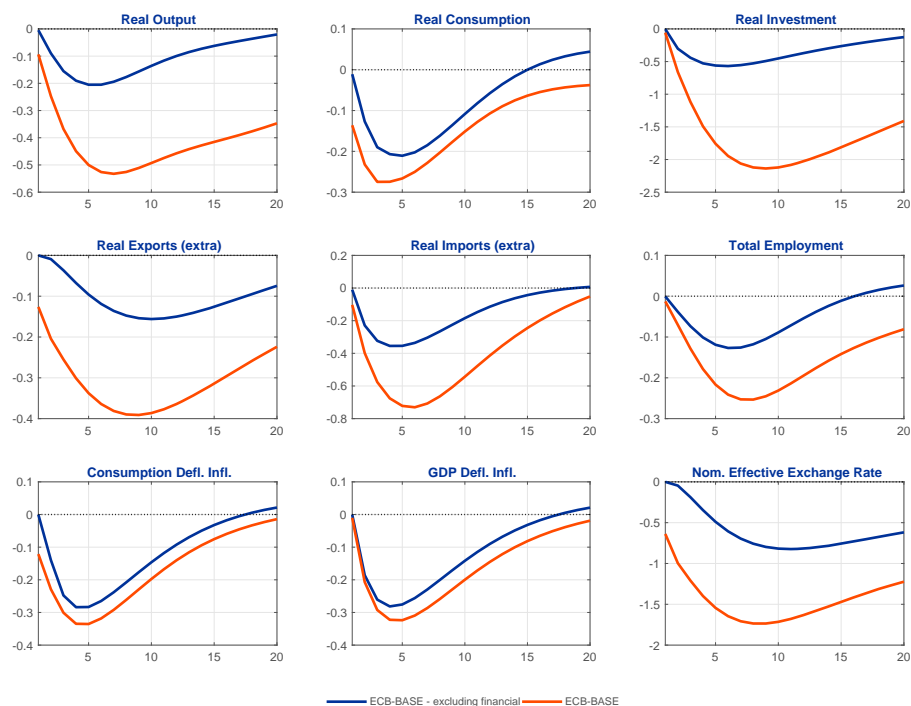
Note: The figure shows impulse responses to an increase in short-term nominal rate equal to 100 bp. All impulse responses are reported as percentage deviations from the steady state, except for the impulse responses of inflation and interest rates which are reported as annualised percentage-point deviations.

Figure D.4: Response to a monetary policy shock: Comparison with benchmarks



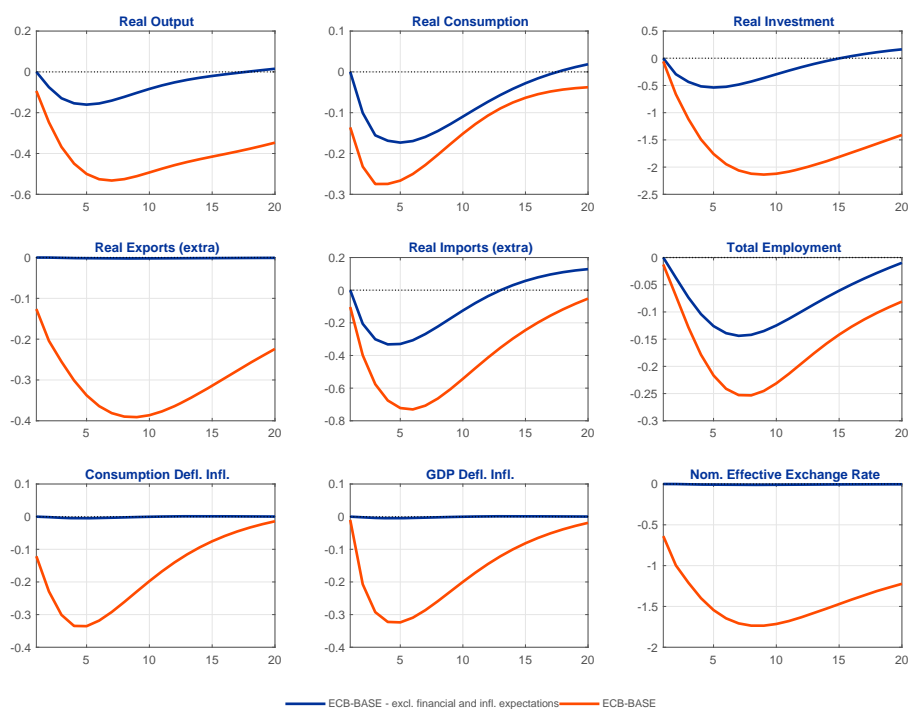
Note: This figure depicts the impulse to a short-term interest rate shock (100 bp) for the ECB-BASE and selected benchmark models. Responses corresponding to the ECB-BASE are depicted in solid red, responses related to the ECB New Multi-Country Model (NMCM) are depicted in solid blue, responses related to the ECB New Area Wide Model (NAWM) are depicted in dashed blue, and responses related to the FRB-US model are depicted by dotted blue lines.

Figure D.5: Response to a monetary policy shock shutting down financial propagation



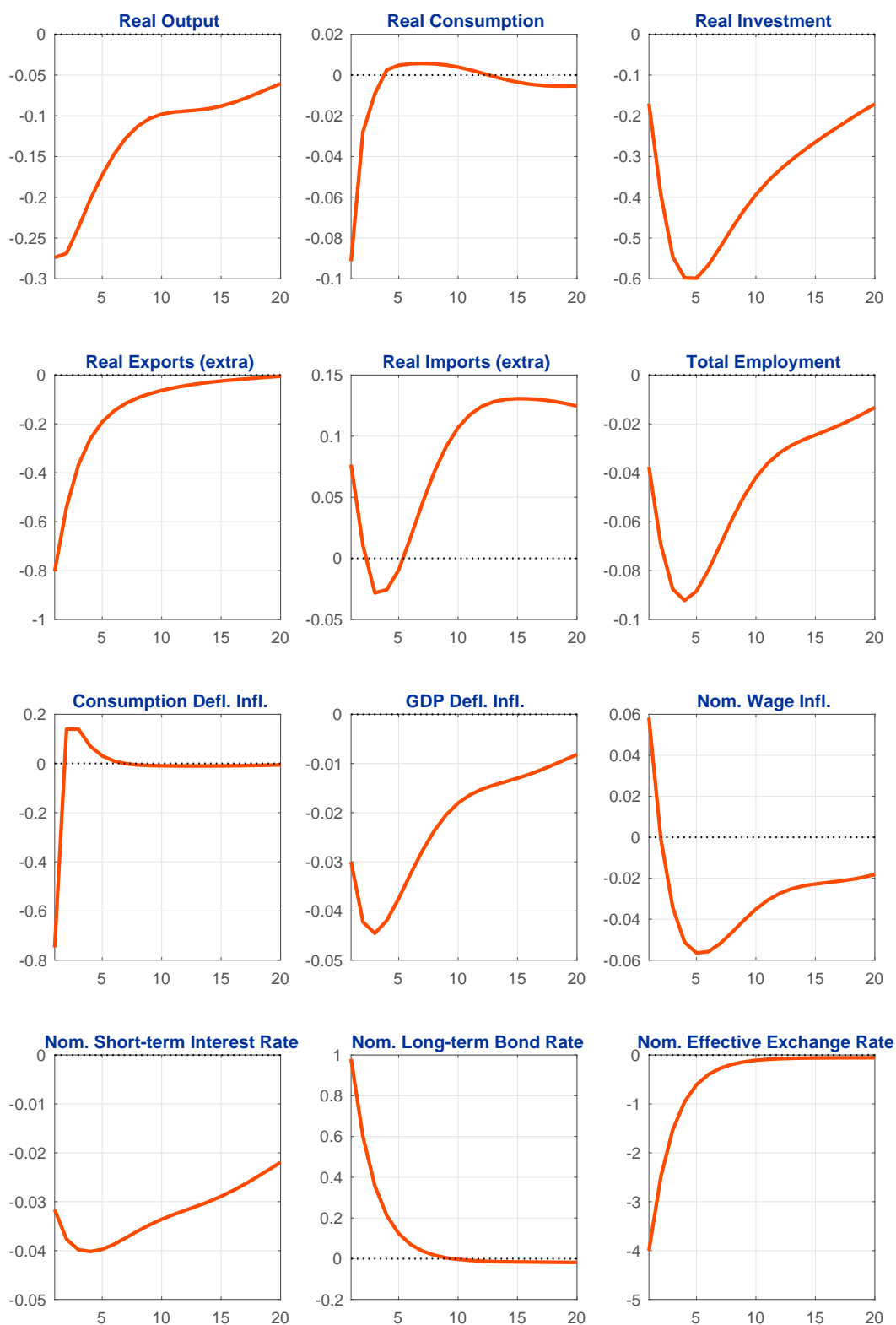
Note: This figure depicts the impulse responses of selected domestic variables to a short-term interest rate shock increasing interest rates by 100 basis points (red line). The blue solid line shows the response if the endogenous spreads in the financial part of the model are kept constant.

Figure D.6: Response to a monetary policy shock shutting down financial propagation and expectation channel in the Phillips curve



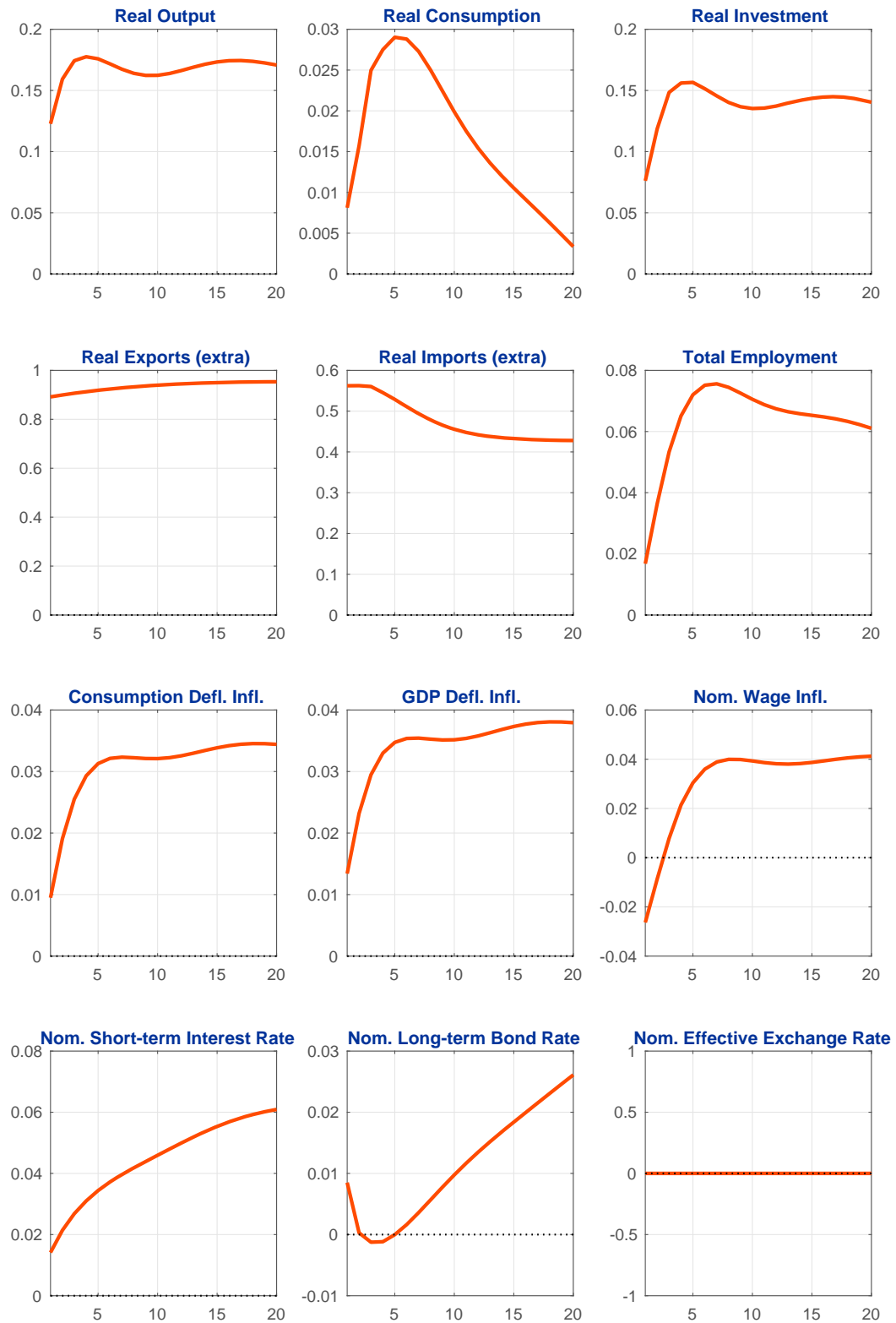
Note: This figure depicts the impulse responses of selected domestic variables to a short-term interest rate shock increasing interest rates by 100 basis points (red line). The blue solid line shows the response if the endogenous spreads in the financial part of the model are kept constant and the expectation channel in the Phillips curve is shut down.

Figure D.7: Response to a term premium shock



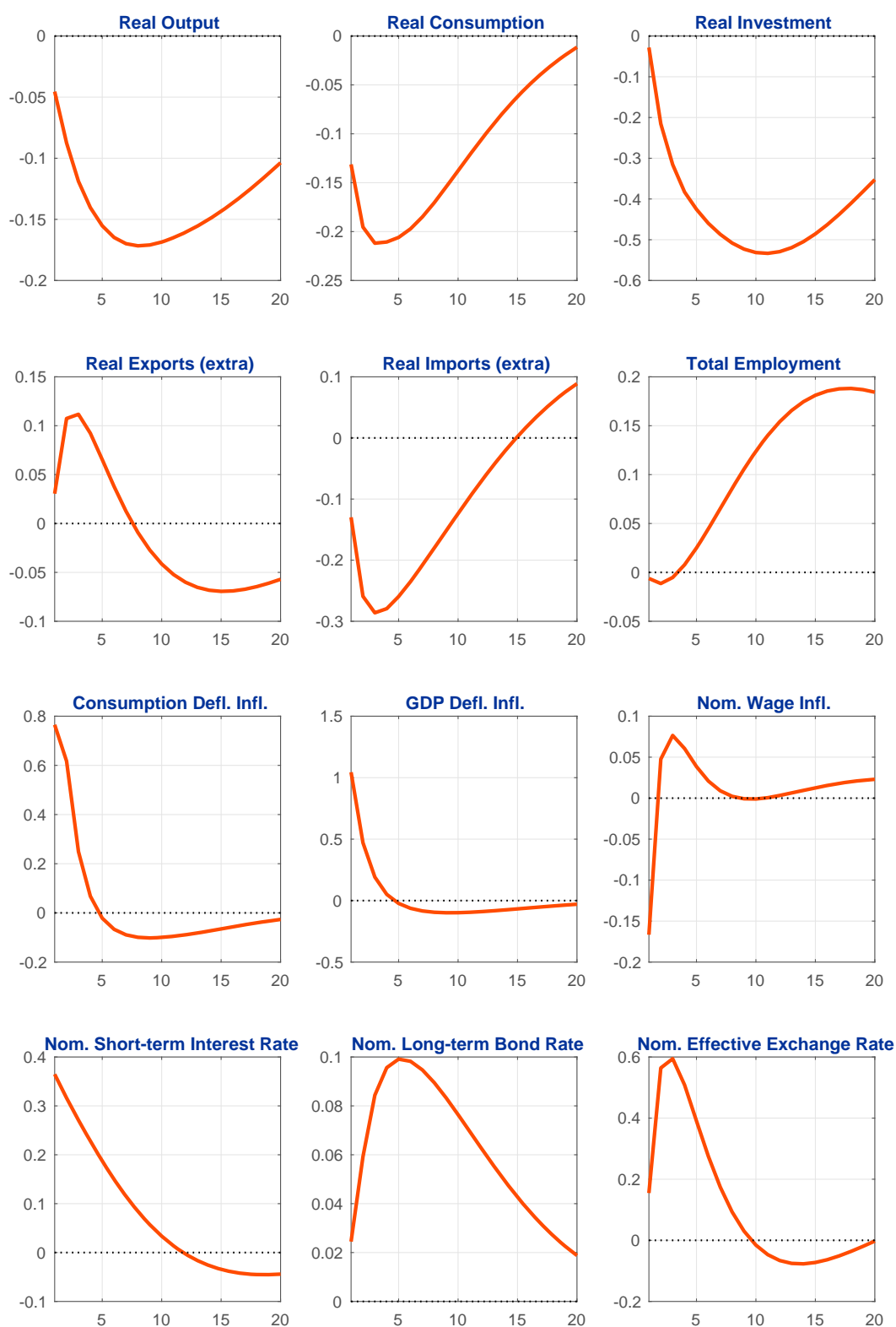
Note: The figure shows impulse responses to an increase in 10-year term-premium equal to 100 bp. All impulse responses are reported as percentage deviations from the steady state, except for the impulse responses of inflation and interest rates which are reported as annualised percentage-point deviations.

Figure D.8: Response to a world demand shock



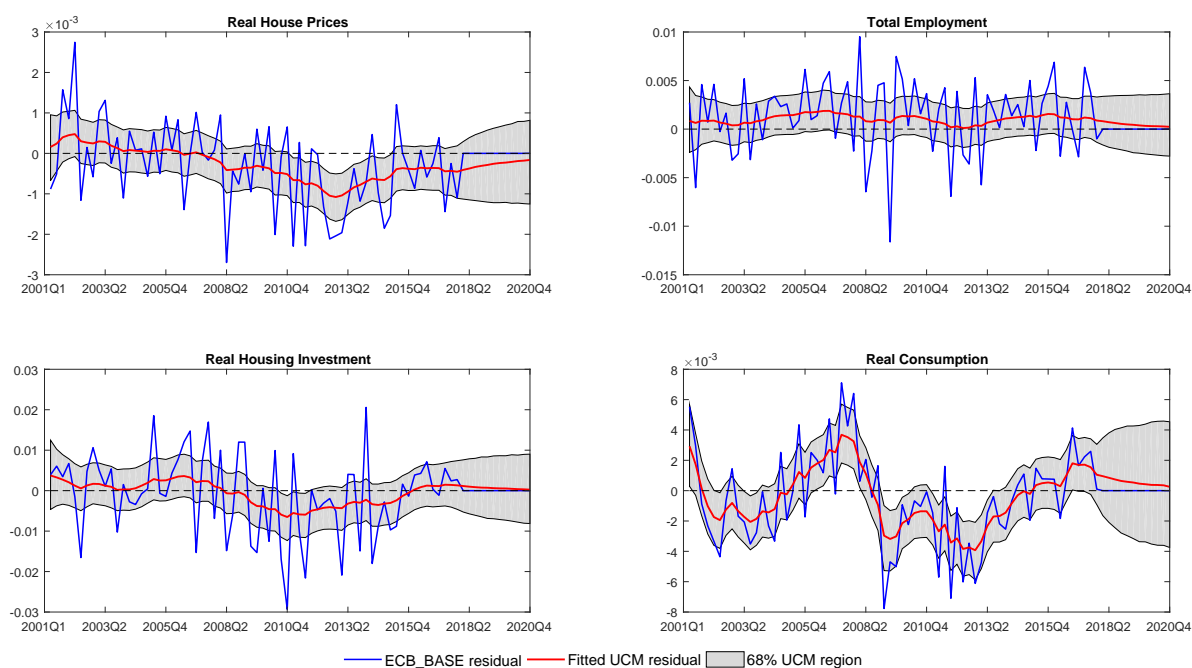
Note: The figure shows impulse responses to permanent increase in world demand equal to 1%. All impulse responses are reported as percentage deviations from the steady state, except for the impulse responses of inflation and interest rates which are reported as annualised percentage-point deviations.

Figure D.9: Response to cost-push shock



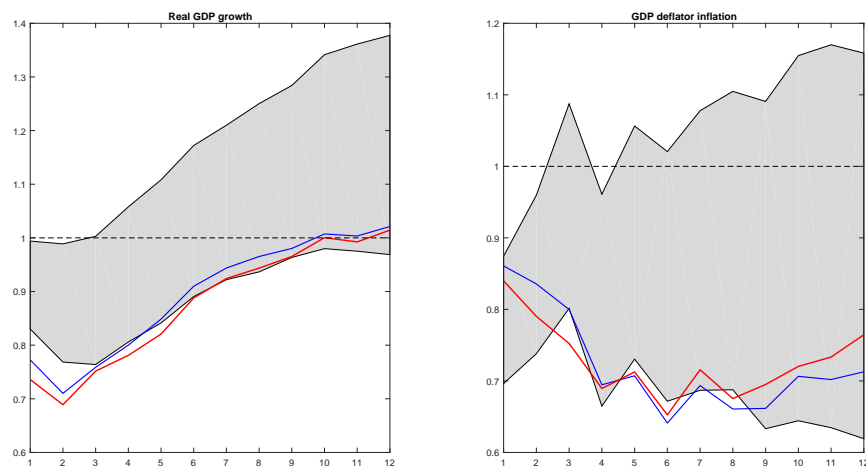
Note: The figure shows impulse responses to a 1 p.p. increase in the annual GDP deflator inflation rate. All impulse responses are reported as percentage deviations from the steady state, except for the impulse responses of inflation and interest rates which are reported as annualised percentage-point deviations.

Figure D.10: Residual add-factors

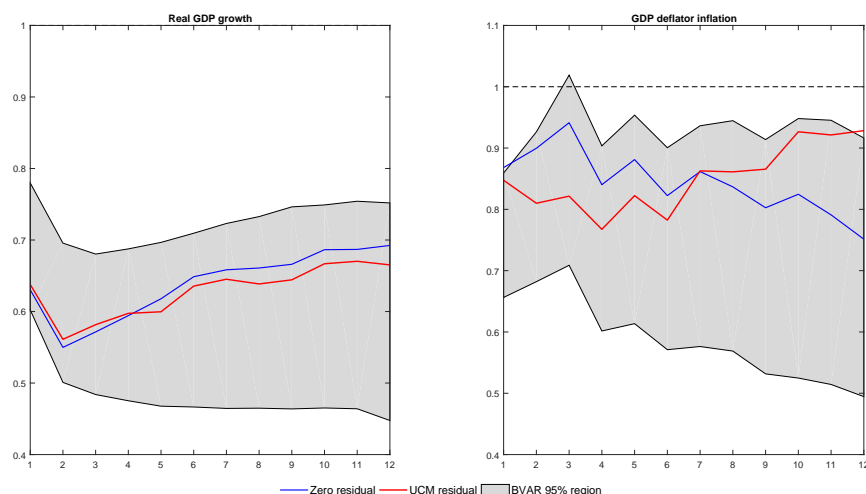


Note: The figure exemplifies the two add-factors rules with four randomly chosen residuals. The blue lines depict residual series set to zero over the forecast horizon. The red series depict the UCM fitted residual projected out of sample in the forecast horizon. The shaded area corresponds to 68% Bayesian region.

Figure D.11: Theil-Us of ECB-BASE forecasts with two add-factor rules and a reference BVAR model. Evaluation Sample: 2004-2017



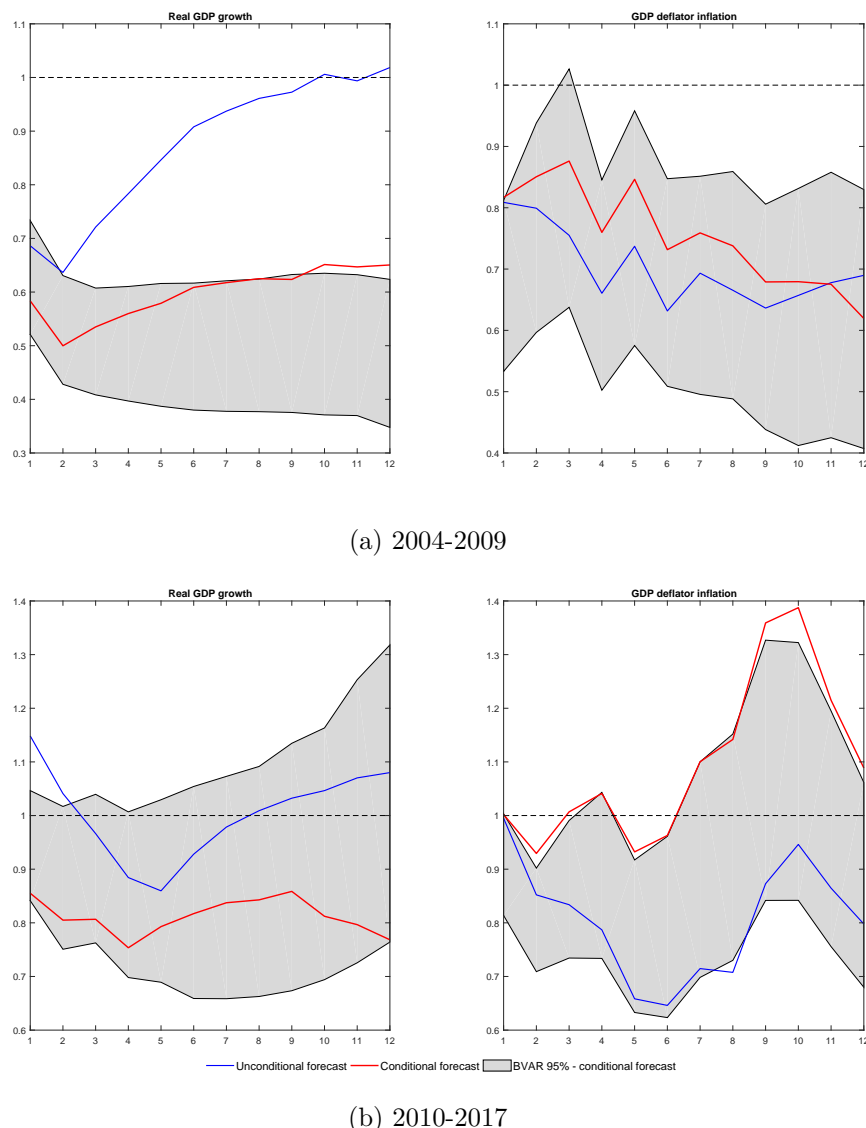
(a) Unconditional Forecast



(b) Conditional Forecast

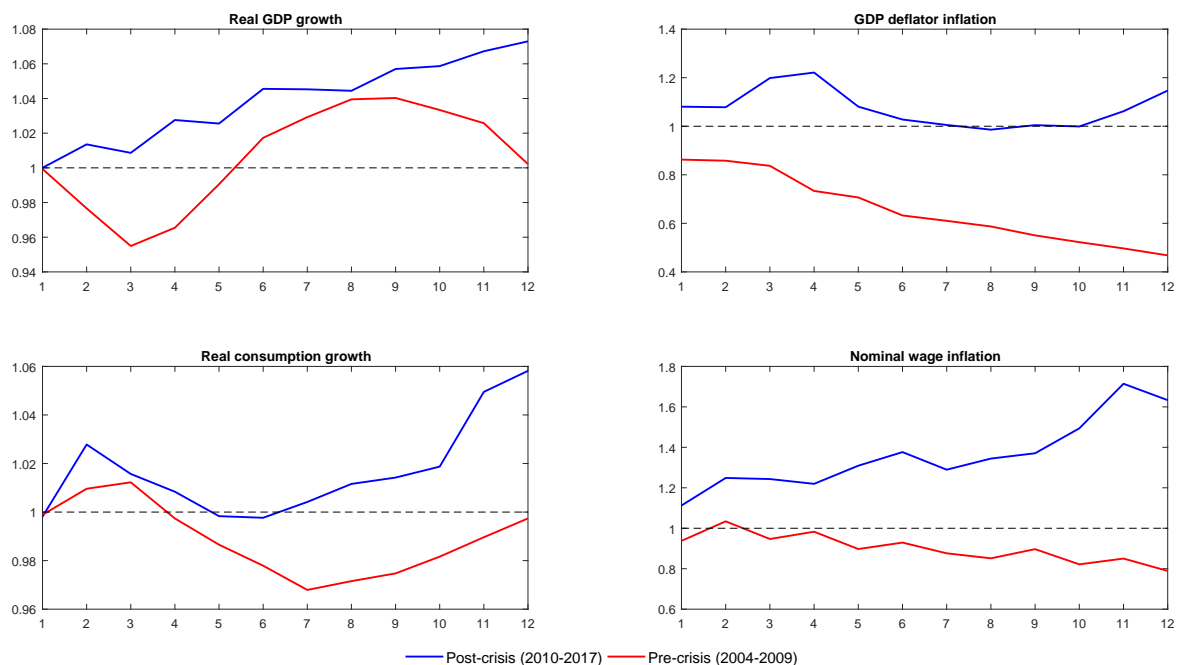
Note: The charts report the RMSE of the ECB-BASE model projected for 12 horizons and evaluated over the sample 2004-2017 as a ratio of the RMSE obtained with a naive benchmark (Random Walk forecast for inflation and AR(2) for GDP growth). The bands are derived from the forecast over the same horizon and sample of a BVAR model. In the conditional setup the BVAR has four endogenous variables (GDP growth, GDP deflator inflation, consumption growth and wage inflation) and five exogenous variables (short term interest rate, government consumption growth, oil price inflation, exchange rate, and world demand growth) whose values on the forecast horizon are the realizations. In the unconditional setup all variables becomes endogenous. The BVAR is estimated with four lags (two lags in the full endogenous setup) and a standard Litterman prior with general tightness equal to 0.15, a weight on other variables equal to 0.5, an harmonic decay with decay factor equal to 0, and a mean of the first own lag equal to 0.8.

Figure D.13: Relative performance over two samples



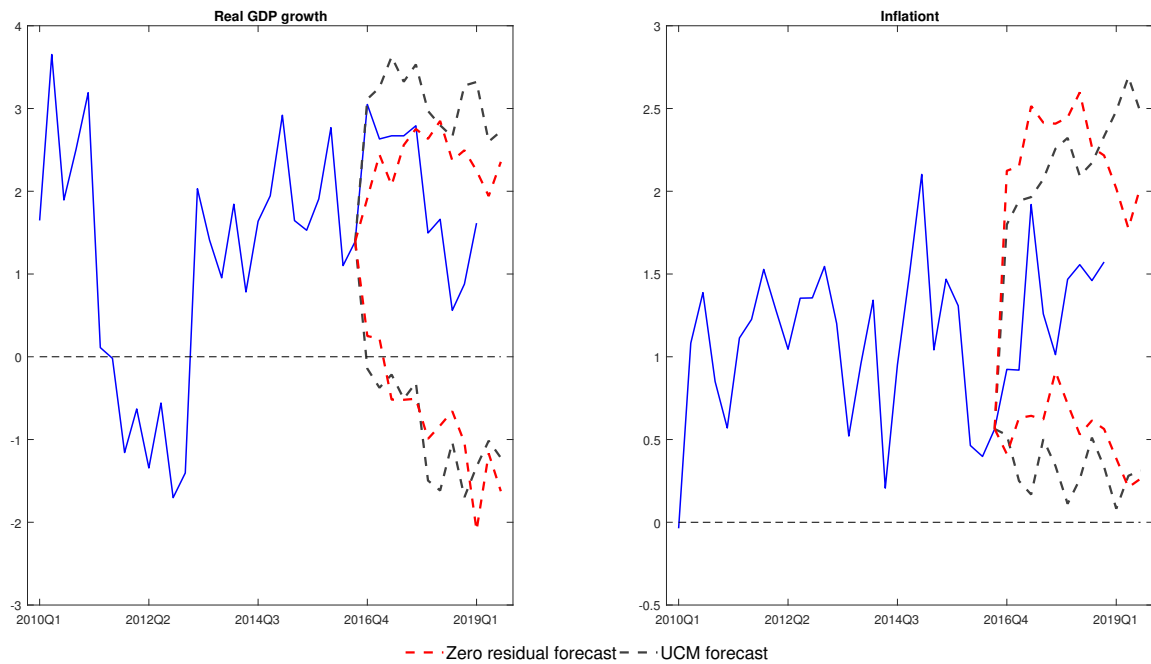
Note: The charts report the RMSE of the ECB-BASE model projected for 12 horizons and evaluated over two samples (2004-2009 and 2010-2017) as a ratio of the RMSE obtained with a naïve benchmark (Random Walk forecast for inflation and AR(2) for GDP growth). The bands are derived from the forecast over the same horizon and sample of a BVAR model. In the conditional setup the BVAR has four endogenous variables (GDP growth, GDP deflator inflation, consumption growth and wage inflation) and five exogenous variables (short term interest rate, government consumption growth, oil price inflation, exchange rate, and world demand growth). In the unconditional setup all variables becomes endogenous. The BVAR is estimated with four lags and a standard Litterman prior with general tightness equal to 0.15, a weight on other variables equal to 0.5, an harmonic decay with decay factor equal to 0, and a mean of the first own lag equal to 0.8.

Figure D.15: Comparing the forecast performance over time and across different degrees of anchoring



Note: The chart plots the ratio between the RMSE of the baseline forecast and the RMSE of the forecast allowing for a lower degree of anchoring of medium-term inflation expectations. A ratio lower than 1 favors the baseline forecast.

Figure D.16: Stochastic simulation



Note: The figure shows stochastic simulations around unconditional forecast over the sample 2016Q4-2019Q3. The model is simulated 1000 times, both drawing at random from the series of historical equation residual (bootstrap - red dashed line), and using the output of the of the Bayesian estimation of residuals based on the UCM rule (grey dashed line.)

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