# Endogenous Bank Fragility in a Macroeconomic Model<sup>a</sup>

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#### Abstract

A consistent feature of financial crises is stress in the money market. To what extent does this account for the persistent economic contractions that follow these episodes? This paper builds a calibrated model of endogenous bank fragility and embeds it in a canonical macroeconomic framework. Fragility is a measure of a bank's vulnerability to a run. It depends on fundamentals and determines the bank's funding costs in the money market, the TED spread between a LIBOR-like interest rate and the yield on government bonds. Because fragility and funding costs are endogenous, they amplify the effects of fundamental shocks. A policy of supplying more liquid assets reduces fragility and the associated distortion in the money market. However, too much liquidity can impair the build up of sufficient bank-capital buffers.

Keywords: bank runs, liquidity, bank funding spreads, balance-sheet policies.

JEL Codes: E4, E5, G2.

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### 1 Introduction

Maturity transformation, a key function of financial intermediation, results in a maturity mismatch on the balance sheets of intermediaries. This is a vulnerability that makes them prone to "runs" by panicked creditors, as first formalized in Diamond and Dybvig (1983). In the recent past, runs played a central role in the global financial crisis in 2007 and the crisis of US money-market funds in 2020 (Shin, 2009; Bernanke, 2010; Li et al., 2020).

Vulnerability to runs has macroeconomic consequences for two reasons. First, when they take place, runs bankrupt banks. Several banks, including Bear Sterns and Lehman Brothers, went bankrupt during the global financial crisis, reducing the economy's capacity to efficiently channel savings to productive uses. This first effect is studied in Gertler et al. (2020). Second, run risk distorts the money market. The coordination failure that generates run risk leads to higher funding costs for banks. Empirically, banking crises, episodes with heightened run risk, are characterized by large spikes in money-market rates as displayed in figure 1. In the average banking crisis, the spread between the rate at which banks borrow short-term in the money market and the short-term treasury rate goes up by around one percentage point. This distortion in the money market chokes the credit supply of all banks, even if they do not fail, and therefore reduces economic activity. The macroeconomic literature has so far overlooked this transmission mechanism for shocks.

This paper studies the role of a distortion in the money market, caused by the risk of runs, in amplifying and propagating real shocks to the economy. We can study the effects quantitatively and use a canonical real business cycle model, which is nested in our framework, as benchmark. We find that a variety of standard shocks have an impact on run risk and thereby on the distortion in the money market. This impact amplifies and propagates their effect on macroeconomic outcomes. For instance, a capital-destruction shock, commonly analyzed in the literature, increases run risk by reducing the net worth of banks. The distortion in the money market implies higher bank-funding costs and hence less credit supply. As a consequence, output falls by approximately one quarter more than in the frictionless benchmark. The persistence of the drop in output is also greatly increased since higher funding costs make it more difficult for banks to accumulate net worth.

Run risk causes the distortion in the money market. Elevated risk of runs implies banks must pay a high interest on their debt to convince creditors to hold it instead of a risk-free bond. This is a distortion because it creates a wedge in the funding costs of fundamentally sound banks, which could borrow at a risk-free rate in the absence of runs. The modelling challenge is to determine the equilibrium run risk. Indeed, incentives to run are undetermined in a coordination game with full information and no impediment to coordination, as is well known in the literature.

This paper models the creditors' coordination game, which determines their incentives to run, as a global game.<sup>1</sup> A global game features a special form of imperfect information. Creditors have an infinitesimally small uncertainty about the economy's fundamentals as well as about other creditors' views about the fundamentals. Because of strategic complementarities in running, any small uncertainty about other creditors' information plays a key role in an individual's optimal strategy. First, the lack of common knowledge makes it impossible to coordinate on an arbitrary sunspot, ruling out indeterminacy. Second, even if a creditor is perfectly confident that a bank is solvent in the absence of runs, uncertainty about other creditors' views makes it risky to hold the bank's debt when just a few other creditors running are enough to bankrupt the bank. This is run risk as perceived from an individual creditor.

A creditor's perceived run risk in equilibrium depends on bank fundamentals. In particular, it is determined by a bank's fragility, defined as the minimum share of creditors that must choose not to run on a bank's debt for the bank to survive. If the bank can survive even if all creditors run, then the bank has zero fragility and creditors see no run risk. This bank can borrow at the risk-free rate and thus there is no distortion. However, positive bank fragility means that, if enough creditors run on the bank, then the bank goes bankrupt. The bank has no solvency problem. If its debt were not runnable, then the bank could borrow at the risk-free rate. Nonetheless, the bank must compensate creditors for run risk. Interestingly, as long as the bank pays appropriate compensation for run risk, no run takes place in equilibrium because no creditor has an incentive to start the run that they fear. In other words, fragile banks faces a distortion in the money market, where they funds themselves, in the form of a spread over the risk-free rate.<sup>2</sup>

Bank fragility, the key notion that determines run risk and thus the distortion in the money market, is endogenous in the model. It depends on banks' balance sheets. In particular, more levered banks and banks with fewer liquid assets as a share of total

<sup>&</sup>lt;sup>1</sup>See Morris and Shin (2003) for a survey of the literature on global games and applications.

<sup>&</sup>lt;sup>2</sup>The fact that run risk is endogenously driven by fundamentals is an attractive feature of this specification of the coordination game. According to Baron et al. (2021), there is no evidence of non-fundamental crises since financial panics, as identified in narrative accounts, are found to generally follow in time large reductions in bank-equity values. See also Gorton (1988) for a first empirical study casting doubt on the empirical relevance of non-fundamental bank panics.

assets are more fragile. Indeed, the distortion in the money market can be summarized in a constraint on banks, which maps higher leverage and more illiquid-asset holdings into higher funding costs. This is the model's key financial friction. In equilibrium, banks make an optimal choice by trading off the higher returns that they earn thanks to leverage and illiquid assets against the higher funding costs caused by greater fragility.

Because of the financial friction, banks' net worth is a state variable of the economy. Consider a shock that reduces banks' net worth and nothing else. The efficient level of investment in the economy is unchanged. Banks could keep investing as much as before the shock, but this implies higher leverage now, and, because of the financial friction, higher leverage is costly. It implies higher funding costs. Optimally, the bank levers up and becomes more fragile, but not as much as needed to invest the same amount as before the shock. The negative correlation between bank net worth and funding costs, which the model generates in equilibrium, is in line with the empirical regularities of



Figure 1: Panics and bank funding spreads in banking crises.

The figure plots the average evolution of bank-equity returns (in log-points) and bank-funding spreads (in percentage points) around banking-crisis dates. A banking-crisis year is identified either as having bank-equity returns cumulatively below -30% and widespread bank failures, or narrative accounts of a financial panic. The variables are normalized to 0 at event time 0, which is January of the banking-crisis year. Bank-equity returns correspond to the left axis, while interest-rate spreads to the right axis. Both the data and the list of banking-crisis episodes are from Baron et al. (2021). We use the 66 episodes with monthly data on bank-funding spreads available. The list is in table 3 in appendix B. Figure 7, showing the dynamics for the US around 2007, is also available in appendix B.

banking crises as portrayed in figure 1. It is also in line with the data from 2007 in the US as reported in figure 7.

Alongside net worth, liquid assets play an important role in mitigating the financial friction. Banks with high leverage can keep down their funding costs by holding liquid assets. Therefore, even if they have a lower return than illiquid assets, banks demand liquid assets. And they demand more when net worth is scarce, as after an adverse shock. In other words, the framework delivers a well-defined demand for liquid assets coming from banks on account of their fragility. This motive for banks to hold liquid assets is novel in a macroeconomic setting. It is separate from regulatory requirements and payments-system needs, already explored in the literature.

Banks can use liquid assets to pay off running creditors without incurring liquidation costs. We consider monetary and fiscal liabilities of the government as the natural source of such liquidity. Banks create liquid assets for other sectors of the economy but they cannot produce liquid assets for their own use in case of a systemic run. Therefore, the supply of liquid assets is a policy variable. In the model, the supply of liquid assets is a powerful tool to lessen the distortion in the money market. More plentiful liquidity ends up on banks' balance sheets and reduces their fragility all else equal. In equilibrium, this has a multiplier effect on banks' balance sheets since banks react to the lower funding costs by levering up and investing more.

The optimal supply of liquidity is done by targeting the liquidity premium. Such policy is the relevant "Friedman rule" of the model.<sup>3</sup> It successfully dampens the amplification of fundamental shocks via the model's financial friction. Consider a shock that reduces banks' net worth. As discussed above, this drives up bank fragility and funding costs for a constant quantity of liquidity in the economy. It also drives up the liquidity premium, since banks demand more liquid assets, which mitigate the distortion, when the distortion is large. By satisfying the extra demand with more supply, the government stabilizes the liquidity premium. Most importantly from a welfare perspective, it also stabilizes investment by allowing banks to have higher leverage without the extra fragility. In fact, the larger liquidity buffers on banks' balance sheets fully compensate for the higher leverage. An interesting result is that, even if it reacts to a one-off shock, the expansion of liquidity necessary to keep the liquidity premium on target is permanent. If interest-rate spreads do not increase after an adverse shock to banks' net worth, then net worth does not grow back to its steady-state level. Given that net worth remains scarce indefinitely, bank liquidity buffers need to remain

<sup>&</sup>lt;sup>3</sup>Woodford (1990) explains meaning and implications of the "Friedman rule" in the context of modern macroeconomic models.

large indefinitely to avoid giving rise to run risk in the future. If at any future point the government unwinds its large balance sheet, money-market stress arises.

Literature review. An important novelty in our analysis is the central role of liquid assets held by banks. The existing literature has focused on two motives for banks' demand of reserves and other liquid assets. First, reserve holdings are necessary for the payments system (Poole, 1968; Arce et al., 2020; Bianchi and Bigio, 2022). Second, banks may be subject to liquidity requirements to correct inefficiencies (Vives, 2014; Ahnert, 2016). In contrast, we study banks' demanding liquid assets to mitigate the risk of runs. The empirical literature has uncovered a demand for liquid and safe assets manifested in a liquidity premium that suggests broader uses for liquid assets beyond payments-system demand (Krishnamurthy and Vissing-Jorgensen, 2012). Theories often take such demand by putting government bond holdings in the utility function (analogous to money in the utility function from Sidrauski (1967)) or in a broader cash-in-advance constraint (Bansal and Coleman, 1996). The cross-country aspects of safe-asset demand are studied by Caballero et al. (2017).

There is an extensive literature that introduces financial frictions in banking to macroeconomic models.<sup>4</sup> Typically, the friction is one of moral hazard for bankers, which implies banks are subject to a leverage constraint to ensure bankers do not misappropriate assets (Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011; Brunnermeier and Sannikov, 2014). This friction implies bank capital is scarce, but does not naturally give rise to a role for liquid assets, in contrast to what we do. The friction we study implies both capital and liquidity are relevant in a single constraint, pointing to interactions between the two. Moreover, our framework makes predictions about the dynamics of bank-funding spreads (TED spread) in addition to the credit spreads explored in the literature.

Bank runs have been prominent in microeconomic analyses of banking, most famously by Diamond and Dybvig (1983).<sup>5</sup> That paper explains the possibility of banking panics, but as there are multiple equilibria, it does not speak to the determinants of runs themselves. Gertler et al. (2020) and Amador and Bianchi (2021) have adopted the multiple-equilibrium approach to banking panics in a macroeconomic model. Using global-game techniques developed in Carlsson and Van Damme (1993), Goldstein and Pauzner (2005) pin down a unique equilibrium to the coordination game among bank

<sup>&</sup>lt;sup>4</sup>This builds on and complements an earlier literature on fire sales and the financial accelerator (Kiyotaki and Moore, 1997; Bernanke et al., 1999).

<sup>&</sup>lt;sup>5</sup>Calomiris and Kahn (1991) present an alternative approach in which short-term debt and bank runs provide market discipline and help overcome a moral hazard friction between bankers and creditors.

creditors. This allows them to study the determinants of bank runs. In their model, banks trade off maturity transformation with the risk of runs. We share their focus on the management of run risk to which banks are vulnerable on account of maturity transformation.

### 2 Coordination game

This section sets up and solves the coordination game that justifies a key constraint on the economy's intermediaries.

#### 2.1 Household problem

A unit mass of households indexed by h is born at time zero with an infinite time horizon. They own the firms and intermediaries in the economy. The households' utility is

$$E_{0}\sum_{t=0}^{+\infty}\beta^{t}\left[\frac{C_{h,t}^{1-\frac{1}{\sigma}}-1}{1-\frac{1}{\sigma}}-\chi\frac{L_{h,t}^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}}\right],$$
(1)

where  $\beta \in (0, 1)$  is the discount factor,  $\sigma > 0$  is the elasticity of intertemporal substitution and  $\psi > 0$  is the Frisch elasticity of labour supply. In every period, households choose consumption  $C_{h,t}$ , labour  $L_{h,t}$  and risk-free bond holdings  $B_{h,t}$ . Moreover, they choose whether to hold an intermediary's debt  $D_{bt}$ . This choice is captured by indicator function  $H_{h,b,t}$ , which is equal to 1 if the household chooses to hold the debt and equal to 0 if it refuses to do so. Holding an intermediary's debt exposes the household to credit risk. It pays a better return than the risk-free bond if the intermediary does not fail. If the intermediary does fail, the household suffers a loss  $\theta$  on the debt's principal. When deciding whether to hold an intermediary's debt, a household speculates about the likelihood of the intermediary failing. As it turns out, this is a coordination game. In this economy, an intermediary fails when too many households decide not to hold its debt. We can write an indicator function for an intermediary's failure as

$$\phi_{b,t} = \phi(F_{b,t}, H_{b,t}) = \begin{cases} 0 & \text{if } H_{b,t} \ge F_{b,t}, \\ 1 & \text{otherwise.} \end{cases}$$
(2)

Variable  $H_{b,t} = \int_0^1 H_{h,b,t} dh$  is the share of households who decide to hold the debt and  $F_{b,t}$  is the minimum share of households who must hold the intermediary's debt for

it not to fail. We can interpret  $F_{b,t}$  as a measure of the intermediary's fragility, which depends on the intermediary's portfolio and leverage decisions as we will see below.  $F_{b,t} = 1$  means that intermediary *b* needs each and every household to trust it and hold its debt in order to survive. On the other hand, an intermediary with  $F_{b,t} = 0$  is not fragile at all. It will not fail even if all households refuse to hold its debt. The households' flow budget constraints are given by

$$C_{h,t} + B_{h,t} + \int_0^1 H_{h,b,t} D_{b,t} \, \mathrm{d}b = w_t L_{h,t} + \Pi_t + T_t + (1 + \rho_{t-1}) B_{h,t-1} + \int_0^1 \phi_{b,t} (1 - \theta) H_{h,b,t} D_{b,t} + (1 - \phi_{b,t-1}) (1 + j_{b,t-1}) H_{h,b,t-1} D_{b,t-1} \, \mathrm{d}b \,, \quad (3)$$

where { $w_t$ ,  $\Pi_t$ ,  $T_t$ ,  $\rho_t$ ,  $j_{b,t}$ } are respectively the wage rate, profits from intermediaries and firms, lumpsum taxes, the risk-free rate and the interest on an intermediary's debt. All these quantities are taken as given by the households. Boundary condition

$$\lim_{t \to +\infty} \frac{B_{h,t+1}}{\prod_{s=0}^{t} (1+\rho_s)} = 0$$
(4)

also applies. In terms of information, households cannot observe  $F_{b,t}$  or  $H_{b,t}$  for any of the intermediaries. They observe an independent and arbitrarily precise signal  $\hat{F}_{b,t}$ centered around the true fragility  $F_{b,t}$ . This information structure makes this coordination game a global game. Households are virtually certain about the economic fundamental. But they are uncertain about the information held by other households and this makes it hard to coordinate. As is well-established in the literature, this rules out sunspot equilibria, in which households can coordinate both on holding the debt or not regardless of the fundamentals, and leaves the game with a unique equilibrium that depends on the economic fundamentals.

#### 2.2 Global game.

Before we use results from the literature on global games to solve our coordination game, we must derive an intermediate step. In particular, it is key to work out a households' net payoff from holding an intermediary's debt given the intermediary's fragility and the share of households holding the debt.

**Lemma 1.** At any point in time t, a household's net payoff from holding an intermediary's debt given intermediary fragility  $F_{b,t}$  and the share of households holding the debt  $H_{b,t}$  is

given by

$$\pi_t(H_{b,t}, F_{b,t}) = \frac{\mu_h^*}{\prod_{s=0}^{t-1} (1+\rho_s)} \left[ (1-\phi_{b,t}) \frac{j_{b,t} - \rho_t}{1+\rho_t} - \phi_{b,t} \theta \right] D_{b,t} \, \mathrm{d}b \tag{5}$$

with  $\phi_{b,t}$  given by equation (2). Variable  $\mu_h^*$ , the household's equilibrium marginal value of wealth, is strictly positive and independent of both  $F_{b,t}$  and  $H_{b,t}$ .

*Proof.* Please refer to the appendix.

With an expression for the net payoff of holding intermediary debt in hand, we can verify the game is indeed of the form required by global-games methods. In practice, there are three requirements on individual net payoffs. First, households must have strategic complementarity in holding debt. That is, the more households hold the intermediary's debt, the more every individual household has an incentive to do so too, given the intermediary's fundamentals. Second, the intermediary's fragility, which summarizes its fundamentals, reduces an individual households' incentive to hold the debt given other households' behaviour. Third, that there are levels of the intermediary's fragility such that a household's optimal debt-holding behaviour is independent of other households' behaviour. This is the case when the intermediary's fragility is greater than unity or smaller than zero. In the former case, it is dominant to not hold the debt because the intermediary fails even if all households decide to hold its debt. In the latter, it is dominant to hold the debt because the intermediary find the household's unique equilibrium debt-holding strategy.

**Lemma 2.** Given  $j_{b,t} \ge \rho_t$  and  $D_{b,t} > 0$ , the household's unique equilibrium strategy satisfies  $H_{h,b,t} = 0$  if  $\hat{F}_{h,b,t} > F^*_{h,b,t}$  and  $H_{h,b,t} = 1$  if  $\hat{F}_{h,b,t} < F^*_{h,b,t}$ . The threshold is given by

$$F_{h,b,t}^{*} = \left[1 + \theta \left(\frac{j_{b,t} - \rho_t}{1 + \rho_t}\right)^{-1}\right]^{-1}.$$
(6)

*Proof.* Please refer to the appendix.

A household looks at its signal for the intermediary's fragility and implements a rule for holding the debt. Suppose the debt pays no spread over the risk-free rate. In this case, the household refuses the debt unless the intermediary is not fragile at all with  $F_{b,t} = 0$ . Notice that the household is virtually certain about the intermediary's fragility level. It is just not sure exactly how many households observe a signal marginally above its own. As the intermediary pays a larger spread over the risk-free rate, the household accepts higher levels of fragility while choosing to hold the debt. Wider spreads alleviate

the coordination problem because each household understands other households are also incentivized to hold the intermediary's debt.

#### 2.3 Aggregation.

Now that we derived the equilibrium debt-holding behaviour of households, we can move on to determining conditions on intermediaries' portfolios and leverage which result in financial crises.

Since intermediaries are ex-ante identical, from here on we drop the *b*-subscript. At each date, intermediaries have a predetermined level of net worth  $N_t$ . They choose how much to invest in capital  $K_t$  and how many liquid assets  $M_t$  to hold. This implies a level of debt that they seek to issue determined by the flow budget constraint

$$K_t + M_t = D_t + N_t. ag{7}$$

After the portfolio allocation decision is made, households determine whether they want to hold the debt. At that point time, portfolio allocations are already made. If households choose not to hold a bank's debt, then the bank pays them off with liquid assets. If liquid assets are not enough, the bank must liquidate capital. If the bank cannot pay them off with liquid assets and capital liquidation, then it goes bankrupt. So, the condition for bank bankruptcy is

$$(1 - H_t)D_t > M_t + \lambda K_t, \tag{8}$$

where  $\lambda$  denotes the efficiency of capital liquidation. We use this expression to find the bank's fragility, defined above as the minimum share of households that must hold the debt for the bank not to default. This is given by

$$F_{t} = \frac{(1-\lambda)K_{t} - N_{t}}{K_{t} + M_{t} - N_{t}}.$$
(9)

Net worth and liquid assets make an intermediary less fragile. An intermediary that has no net worth and only holds illiquid capital has fragility  $1 - \lambda$ .

Notice that if there is no liquidation cost of capital, i.e.  $\lambda = 1$ , intermediaries can never be fragile. Thus, liquidation costs are key to the existence of a coordination problem. Also, it is key that the portfolio decision is made before debt markets decide whether to hold the intermediary's debt. Only once capital is installed, there is strategic complementarity in debt holding via the threat of liquidation. This timing assumption

could capture the fact that banks literally issue deposits when they make a loan and then someone in the economy must be willing to hold the bank's deposits so that it can hold the loan on its balance sheet rather than sell it. More generally, it could be interpreted as a mismatch between the timing of capital investment, which is typically long-term, and intermediaries' more short-term funding sources.

Having laid out the relationship of an intermediary's fragility with its balance sheet, we can use the individual household's debt-holding behaviour from lemma 2 to identify conditions on intermediaries' balance-sheet that lead to generalized refusal to hold an intermediary's debt and thus a crisis.

**Lemma 3.** For  $j_t \ge \rho_t$  and  $D_t > 0$ , the share of households that hold an intermediary's debt is given by

$$H_{t} \in \begin{cases} 1 & \text{if } K_{t} < \frac{1}{1 - \frac{\lambda}{\theta} \left(\frac{j_{t} - \rho_{t}}{1 + \rho_{t}} + \theta\right)} \left(N_{t} + \frac{1}{\theta} \frac{j_{t} - \rho_{t}}{1 + \rho_{t}} M_{t}\right), \\ [0, 1] & \text{if } K_{t} = \frac{1}{1 - \frac{\lambda}{\theta} \left(\frac{j_{t} - \rho_{t}}{1 + \rho_{t}} + \theta\right)} \left(N_{t} + \frac{1}{\theta} \frac{j_{t} - \rho_{t}}{1 + \rho_{t}} M_{t}\right), \\ 0 & \text{otherwise.} \end{cases}$$
(10)

*Proof.* Please refer to the appendix.

The lemma shows that intermediaries have a limit on the amount of capital they can hold without households refusing to buy their debt for a given level of equity. If the intermediary pays the risk-free rate on its debt, then it can invest a fixed multiple of its equity  $1/(1 - \lambda)$ . This limit becomes slacker as the intermediary pays a higher return on its debt. Also, the intermediary can make the limit slacker by holding more liquid assets.

The limit on illiquid asset holdings acts as a constraint on the intermediary because it is a sufficient condition for its survival.

**Proposition 1.** For  $j_t \ge \rho_t$  and  $D_t > 0$ , an intermediary does not fail at time t only if

$$K_t \le \frac{1}{1 - \frac{\lambda}{\theta} \left(\frac{j_t - \rho_t}{1 + \rho_t} + \theta\right)} \left( N_t + \frac{1}{\theta} \frac{j_t - \rho_t}{1 + \rho_t} M_t \right). \tag{11}$$

*Proof.* Please refer to the appendix.

In the rest of the paper, the above condition enters the intermediary's problem as a constraint. It is never optimal for an intermediary to violate it since in case of failure it makes zero profits forever. On the other hand, it is always possible to make positive

profits while satisfying the constraint as long as initial ex-dividend equity is strictly positive.

We conclude this section with some intuition into the constraint on intermediaries' balance sheets implied by the coordination game. For this, we formulate the constraint in terms of financial ratios that we are familiar with. In particular, we can define the intermediaries' capital ratio

$$\mathbf{n}_t = \frac{N_t}{K_t + M_t} \tag{12}$$

and the liquidity ratio

$$\mathbf{m}_t = \frac{M_t}{K_t + M_t}.$$
(13)

Then, we can write the constraint as

$$\frac{\mathbf{n}_t}{1-\lambda} + \mathbf{m}_t + \frac{\lambda + (1-\lambda)\mathbf{m}_t}{\theta(1-\lambda)} \frac{j_t - \rho_t}{1+\rho_t} \ge 1.$$
(14)

There are three ways for an intermediary to stay safe. First, it can keep a moderate level of leverage. In particular, it can pay the risk-free rate on its debt and hold an entirely illiquid asset portfolio as long as it backs at least a share  $1 - \lambda$  of its assets with equity. Second, it can keep a lot of liquidity. If it only holds liquid assets, then the intermediary needs no equity at all and can pay the risk-free rate on its debt. Third, it can pay a large enough return on its debt to convince households to hold it despite the intermediaries' fragility. Indeed, an intermediary with no equity and no liquid assets at all can survive by paying a sufficiently high return on its debt. As the capital ratio and liquidity ratio go up, the intermediary can get by with lower levels of interest on its debt. Interestingly, the effect of the interest on debt is reduced by loss given default  $\theta$ . An intermediary whose debt is guaranteed only has to pay an infinitesimal extra return on its debt in order to always satisfy the condition even with no equity and no liquid-asset holdings.

The coordination game creates a motive for intermediaries to hold liquid assets, keep a buffer of equity and pay extra-returns on debt. These three courses of action are substitutes and costly for intermediaries. The rest of the paper studies how intermediaries choose among these options on the basis of economic conditions and in turn how this choice affects macroeconomic outcomes.

### 3 Equilibrium

The core of the economy is a real business cycle model as in Kydland and Prescott (1982), to which we add financial intermediaries. The key friction is the coordination problem faced by holders of the intermediaries' debt. Since the coordination problem is made worse by leverage and asset illiquidity, it results in a constraint on the portfolio and leverage choice of intermediaries. If the constraint holds, then households coordinate on holding the debt. If it is violated, they coodinate on refusing to hold it and the intermediary fails.

**Firm.** Perfectly competitive static firms produce a homogeneous good with a Cobb-Douglas production function with total factor productivity  $A_t$ . At any given time t, factors of production are capital  $K_{f,t}$  and labour  $L_t$  with capital share  $\alpha$ . Firms rent capital at rate  $z_t$  and hire labour at wage  $w_t$  in competitive markets. They maximize their profits given by

$$A_t K_{f,t}^{\alpha} L_{f,t}^{1-\alpha} - z_t K_{f,t} - w_t L_{f,t}.$$
 (15)

Optimally, the firm sets the marginal product of capital equal to the cost of borrowing

$$\alpha A_t \left(\frac{L_{f,t}}{K_{f,t}}\right)^{1-\alpha} = z_t \tag{16}$$

and the marginal product of labour equal to the wage rate

$$(1-\alpha)A_t \left(\frac{K_{f,t}}{L_{f,t}}\right)^{\alpha} = w_t.$$
(17)

**Intermediaries.** In each period, financial intermediaries have a pre-determined level of net worth  $N_t$  and take as given the lending rate  $r_t$ , interest on liquid assets  $i_t$  and the risk-free rate  $\rho_t$ . They choose how many illiquid assets  $K_t$  and liquid assets  $M_t$  to hold, the interest rate on their debt  $j_t$ , and dividends  $\Pi_t$ . They maximize the present discounted value of profits

$$\sum_{t=0}^{+\infty} \frac{\prod_{t+1}}{\prod_{s=0}^{t} (1+\rho_s)}.$$
(18)

subject to flow budget constraint

$$K_t + M_t = D_t + N_t \tag{7}$$

and the law of motion for net worth

$$N_{t+1} = (1 + r_{t+1})K_t + (1 + i_t)M_t - (1 + j_t)D_t - \Pi_{t+1}.$$
(19)

The ex-post return on the capital stock is defined as

$$1 + r_t = (1 - \xi_t)(z_t + 1 - \delta), \tag{20}$$

where  $\xi_t$  is a capital-destruction shock. The interest-rate on debt cannot be lower than the return on the risk-free bond, which is the households' alternative:

$$j_t \ge \rho_t. \tag{21}$$

The economy's financial friction is captured by the no-run constraint

$$K_t \le \frac{1}{1 - \frac{\lambda}{\theta} \left( \frac{j_t - \rho_t}{1 + \rho_t} + \theta \right)} \left( N_t + \frac{1}{\theta} \frac{j_t - \rho_t}{1 + \rho_t} M_t \right), \tag{11}$$

which we derive in the previous section. Moreover, dividend payouts are constrained by

$$\Pi_t \ge \gamma N_t,\tag{22}$$

with  $\gamma > \beta^{-1} - 1$ . The constraint on the dividend policy ensures the bank does not grow out of its no-run constraint. Subject to this set of constraints, the intermediary maximizes the present discounted value of its profits

To solve the intermediary's problem, it is useful to also define the return on equity

$$q_t = r_{t+1} + (r_{t+1} - j_t) \frac{D_t}{N_t} - (r_{t+1} - i_t) \frac{M_t}{N_t}.$$
(23)

With this, the optimal behaviour of a representative intermediary can be summarized in the following proposition.

**Proposition 2.** For  $\frac{r_{t+1}-\rho_t}{1+\rho_t} \in (0, \theta \frac{1-\lambda}{\lambda})$  and  $N_t > 0$ , the intermediary's dividend constraint is binding

$$\Pi_t = \gamma N_t, \tag{24}$$

optimal lending is given by

$$K_t = \frac{1}{1 - \frac{\lambda}{\theta} \left(\frac{j_t - \rho_t}{1 + \rho_t} + \theta\right)} \left( N_t + \frac{1}{\theta} \frac{j_t - \rho_t}{1 + \rho_t} M_t \right), \tag{25}$$

the optimal interest rate on the intermediary's debt is given by

$$\frac{j_t - \rho_t}{1 + \rho_t} = \max\left\{0, \frac{\theta}{\lambda} \left(1 - \lambda - \sqrt{1 - \frac{\lambda}{\theta} \left(\frac{r_{t+1} - \rho_t}{1 + \rho_t} + \theta\right)}\right)\right\},\tag{26}$$

the intermediary's demand for liquid assets is given by

$$\frac{\rho_t - i_t}{1 + \rho_t} = \frac{1}{\theta} \left( \frac{j_t - \rho_t}{1 + \rho_t} \right)^2,\tag{27}$$

and accumulation of net worth follows

$$N_{t+1} = \frac{1+q_t}{1+\gamma} N_t \tag{28}$$

with

$$q_t = r_{t+1} + \frac{\lambda}{1 - \lambda} (r_{t+1} - i_t).$$
(29)

*Proof.* Please refer to the appendix.

**Households.** A unit mass of households inhabit the economy. They are born at time zero and have an infinite horizon. Since households are ex-ante identical as well as ex-post identical given that the intermediary complies with conditions that avoid a run, I suppress the *h*-subscript in this section. Their utility is a function of consumption and labour. As given in (1), it features a constant elasticity of intertemporal substitution  $\sigma$  and a constant Frisch elasticity of labour supply  $\psi$ . The household chooses consumption  $C_t$ , labour  $N_t$  and its holdings of risk-free bonds  $B_t$ . In principle, it also decides whether to hold the intermediary's debt  $D_t$  or not. However, it always decides in equilibrium to hold as much debt as is supplied by the intermediary since the intermediary acts subject to the no-run constraint and the interest rate on bank debt is at least as high as the risk-free rate,  $j_t \ge \rho_t$ . With bank debt effectively risk-free and at least as remunerative as risk-free bonds, it strictly dominates them. With this in mind, the household faces

flow budget constraints

$$C_t + B_t + D_t = w_t L_t + \Pi_t + T_t + (1 + \rho_{t-1})B_{t-1} + (1 + j_{t-1})D_{t-1}.$$
(30)

The boundary condition is given by

$$\lim_{t \to +\infty} \frac{B_{t+1}}{\prod_{s=0}^{t} (1+\rho_s)} = 0.$$
 (31)

We can derive standard household first-order conditions

$$\frac{C_{t+1}}{C_t} = [\beta(1+\rho_t)]^{\sigma}$$
(32)

and

$$\chi L_t^{-\frac{1}{\psi}} = C_t^{-\frac{1}{\sigma}} w_t.$$
(33)

The households' intertemporal budget constraint is given by

$$\sum_{t=0}^{+\infty} \frac{C_t}{\prod_{s=0}^{t-1} (1+\rho_s)} = (1+\rho_{-1})B_{-1} + (1+j_{-1})D_{-1} + \sum_{t=0}^{+\infty} \frac{1}{\prod_{s=0}^{t-1} (1+\rho_s)} \bigg[ w_t L_t + \Pi_t + T_t + \frac{j_t - \rho_t}{1+\rho_t} D_t \bigg].$$
(34)

**Government.** The government chooses how many liquid assets  $M_t^s \ge 0$  to supply. Its budget constraint is closed with lumpsum transfers  $T_t$  according to

$$T_t + (1 + i_{t-1})M_{t-1}^s = M_t^s.$$
(35)

Letting the government invest in the risk-free bond does not change the implications of the model.

**Market clearing.** There are four markets in the economy that clear. Risk-free bonds are in zero net supply. The capital market is cleared by the rental rate  $z_t$ . Demand for liquid assets is set equal to supply by the interest rate  $i_t$ . Finally, wages  $w_t$  clear the labour market. The market for bank debt is rationed. Intermediaries set the interest rate  $j_t$  and quantity  $D_t$ .

A formal definition of the economy's equilibrium follows.

**Definition 1.** Equilibrium consists of quantities  $\{B_t, C_t, D_t, K_{t-1}, L_t, M_t, N_t, T_t, \Pi_{t+1}\}_{t=0}^{+\infty}$  and prices  $\{i_t, j_t, r_t, w_t, z_t, \rho_t\}_{t=0}^{+\infty}$  such that:

- 1. In every period, firms take as given prices  $(r_t, z_t)$  and choose quantities  $(K_{f,t}, L_{f,t})$  to maximize profits (15).
- 2. Intermediaries take as given initial cum-dividend equity  $N_0$  and prices  $\{i_t, r_t, z_t, \rho_t\}_{t=0}^{+\infty}$ and choose  $\{j_t, D_t, N_{t+1}, K_t, M_t, \Pi_{t+1}\}_{t=0}^{+\infty}$  to maximize the present discounted value of dividends (18), subject to the budget constraint (7), the law of motion for net worth (19), the definition of return on capital (20), a constraint on the interest on debt (21), the no-run condition (11) and a constraint on the dividend policy (22).
- 3. Households take as given prices  $\{j_t, r_t, w_t\}_{t=0}^{+\infty}$  as well as debt  $\{D_t\}_{t=0}^{+\infty}$  and choose  $\{B_t, C_t\}_{t=0}^{+\infty}$  to maximize utility (1) subject to budget constraints (30) and boundary condition (31).
- 4. The government chooses a sequence of liquid assets and taxes  $\{M_t^s, T_t\}_{t=0}^{+\infty}$  such that its budget constraint (35) holds.
- 5. In every period, the market for risk-free bonds clears with

$$B_t = 0. (36)$$

The capital market clears according to

$$K_{f,t} = (1 - \xi_t) K_{t-1} \tag{37}$$

with  $K_{-1}$  given. The market for liquid assets clears with

$$M_t = M_t^s. aga{38}$$

The labour market clears with

$$N_{f,t} = N_t. aga{39}$$

### 4 Mechanism

This section studies the equilibrium and provides intuition for the results. The key markets, in which most of the action takes place, are the market for capital and the market for liquid assets. These markets are interrelated and jointly determine the equilibrium spreads.

Aggregate demand for liquidity. Before we study how the economy reacts to disturbances, it is necessary to get a good grasp on the workings of the market for liquid assets. In particular, we want to derive the aggregate demand for liquid assets as a function of the liquidity premium,  $\rho_t - i_t$ . The liquidity premium captures the cost of holding liquidity for intermediaries. First, consider a relatively high level of the liquidity premium. How much liquidity is demanded when the liquidity premium is high? The individual intermediary's optimal holding of liquid assets, given in equation (27) and portrayed in plot (a) of figure 2, does not directly answer the question. It only says that a high liquidity premium, which implies a high cost from holding liquidity for intermediaries, needs to be matched by a high investment wedge in equilibrium. In fact, the investment wedge represents the benefit of holding liquidity because liquidity makes the intermediary's financial constraint slacker and therefore allows it to invest in more capital and reap the wedge. So, if the cost of holding liquidity is high, the benefits, driven by the investment wedge, must also be high in equilibrium. The equilibrium investment wedge is pinned down in the capital market. All else equal, a high investment wedge is sustained in the capital market, depicted in plot (c) of figure 2, only if little liquidity is held by intermediaries. This is because liquidity shifts out the capital-supply schedule, which plots equation (25). If we do this exercise for every possible level of the liquidity premium, we can draw a downward-sloping aggregate demand curve for liquidity in plot (b) of figure 2.

**Shock to net worth.** Having derived an aggregate demand for liquidity, we close the market for liquidity with a supply curve. For simplicity, let us start considering a policy which supplies a given amount of liquidity inelastically. Since the quantity of intermediaries' net worth  $N_t$  is the model's key state variable, it is natural to start exploring the model's dynamics by understanding how it reacts to a change in intermediaries' net worth. We can think of this as a consequence of a one-off TFP shock that changes the return on capital ex post. Similarly, we can think of this as the consequence of a shock that directly destroys some capital. The result of such shock is portrayed in figure 3. Less net worth directly implies an inward shift in the capital-supply curve in plot (c). As a consequence, there is less capital investment and a higher investment wedge. A higher investment wedge gives intermediaries an incentive to invest more and to do so they must find a way to relax their financial constraint. To this end, they try to improve their liquidity ratio. This is captured by an outward shift in demand for liquidity in plot (b). Given that the supply of liquid assets is fixed, the increase in demand translates into an increase in the liquidity premium. The economy ends up with less capital, the



Figure 2: Aggregate demand for liquidity.

same amount of liquidity, a higher liquidity premium and a higher investment wedge. Clearly, liquidity ratios increased after the shock. From equation (26), it is clear that the premium on intermediaries' debt went up in parallel to the increase in the investment wedge. This implies that bank leverage increases.



Figure 3: Effects of change in net worth of intermediaries.

### 5 Steady state

In this section, we analyse the long-run dynamics of the model. In particular, we are interested in the long-run level of the spreads and the effects of the quantity of liquid assets.

**Definition 2.** A steady state is a constant sequence for net worth  $\{N_{ss}\}$ , prices  $\{i_{ss}, j_{ss}, r_{ss}, \rho_{ss}\}$ , quantities  $\{C_{ss}, K_{ss}, N_{ss}\}$  and policy  $\{M_{ss}\}$  that satisfies the model's equilibrium conditions.

**Lemma 4.** Given  $\gamma \leq \beta^{-1} \left(1 + \theta \frac{2-\lambda}{\lambda}\right) - 1$  and  $M_{ss} < -\lambda + \theta(1-\lambda)(1+\rho_{ss})/(j_{ss}-\rho_{ss})$ , the economy has a unique steady state with  $N_{ss} > 0$ .

A restriction on dividend policy  $\gamma$  and one on policy  $M_{ss}$  are necessary to obtain a steady state with a strictly positive level of intermediary net worth. If  $\gamma$  is too high, which implies very large dividend payouts every period, this implies zero net worth in the steady state because the economy cannot generate the high return necessary to maintaining a constant positive level of net worth. An excessively large quantity of liquid assets can take away incentives to accumulate net worth and therefore result in a steady state with zero net worth. Knowing the parameter space of interest, we can zoom in and study the properties of steady states with strictly positive net worth.

**Proposition 3.** For  $\gamma < \beta^{-1}(1+\theta) - 1$ , the steady-state net worth, prices and quantities are independent of policy. Moreover, we have that  $r_{ss} - i_{ss} > 0$  and  $j_{ss} - i_{ss} = 0$ .

As long as the dividend policy is sufficiently prudent, intermediaries are not fragile in the steady state. Hence, there is no TED spread. Given that the intermediary is not fragile, the quantity of liquid assets supplied is completely irrelevant. Notice that no restriction on policy is needed in this case. Also, notice that as  $\gamma$  becomes smaller and tends to  $\beta^{-1} - 1$  the credit spread also goes away.

**Proposition 4.** For  $\beta^{-1}(1+\theta)-1 < \gamma \leq \beta^{-1}\left(1+\theta\frac{2-\lambda}{\lambda}\right)-1$  and  $M_{ss} < -\lambda+\theta(1-\lambda)(1+\rho_{ss})/(j_{ss}-\rho_{ss})$ , the steady-state prices and quantities are independent of policy. Steady-state equity depends on policy according to

$$N_{ss} = \left[1 - \frac{\lambda}{\theta} \left(\frac{1+j_{ss}}{1+\rho_{ss}} - 1 + \theta\right)\right] K_{ss} - \frac{1}{\theta} \left(\frac{1+j_{ss}}{1+\rho_{ss}} - 1\right) M_{ss}.$$
(40)

Moreover, we have that  $r_{ss} - i_{ss} > 0$  and  $j_{ss} - i_{ss} > 0$ .

As dividend payouts become larger, the return on equity must rise in the long-run to maintain a stable level of equity. A high return on equity is obtained with a high marginal product of capital. When the marginal product of capital is high because of low levels of net worth, intermediaries become fragile. They offer higher returns on their debt and demand liquid assets. Thus, there is both a credit spread and a TED spread in steady state. Interestingly, the supply of liquid assets has no impact on the level of the spreads. It only crowds out steady-state net worth.

If the government pursued a policy of supplying extremely large quantities of liquid assets in the long-run, this would lead to a steady state with no equity. In this steady state, intermediaries would rely on holdings of liquid assets and a high interest on debt to coordinate creditors.

### 6 Quantitative analysis

This section quantifies the importance of the bank fragility in the transmission of economic shocks.

#### 6.1 Calibration

The parameters  $\lambda$ ,  $\theta$ , and  $\gamma$  describe the banking sector of the economy. These can be calibrated, along with the discount factor  $\beta$ , using information about the level of interest rates, interest-rate spreads, and bank balance sheets. We set one discrete time period to be a month.

First, in a steady state where the return on bank equity exceeds the risk-free rate, the return on equity q is equal to the minimum fraction  $\gamma$  of equity distributed as dividends. This ensures that the level of equity remains stable. Using data from Bankscope, the average return on bank equity is 13.4% over the period 2000–2006 in nominal terms. Subtracting 2.6% CPI inflation over the same period, we set q and  $\gamma$  to be 10.8% at an annual rate.

We use data on the 3-month Treasury bill rate minus CPI inflation to give a measure of i, the real yield on the liquid asset. From 1960 to 2006, the average nominal T-bill rate is 5.5% and CPI inflation is 4.1% implying a real yield of 1.4% for i at an annual rate.

For the interest rate j on bank debt, we use 3-month LIBOR rates. This is an appropriate measure of unsecured bank funding costs; we do not use interest rates on insured deposits because these offer protection against bank runs. The notion of a bank run in the model aligns with runs in money markets by large investors who are not covered by deposit insurance.<sup>6</sup> The TED spread is the difference between the 3-month LIBOR and the 3-month Treasury bill rate. Using data from St Louis Fed, the average TED spread between 1986 and 2006 is 0.63%. This gives a measure of j - i at an annual rate.

Using information on *i*, *j* – *i*, and *q*, the appendix gives a formula to compute the hypothetical risk-free but illiquid interest rate  $\rho$ . The discount factor  $\beta$  is then given by  $\beta = 1/(1 + \rho)$ . The appendix also shows that there is a mapping from these observations to the banking parameter  $\theta$ , which represents the loss given default for holders of bank debt.

<sup>&</sup>lt;sup>6</sup>Three quarters of US commercial-bank funding is deposits, and in the largest commercial banks, half of deposits are uninsured (Egan et al., 2017).

Other important information comes from commercial banks' balance sheets. Using data on total equity capital and total assets from the Federal Deposit Insurance Corporation, the average bank capital ratio is 7.9% from 1986 to 2006. For liquid assets, we use data from the Federal Reserve Board on commercial banks' holdings of Treasury and agency securities and cash assets (including vault cash and reserves). As a share of total bank assets, liquid assets average to 20.8% over the period 1986–2006. Information on all the calibration targets is collected in Table 1.

Description	Notation	Value
Real Treasury Bill rate	i	1.4%
TED spread	j-i	0.63%
Real return on bank equity	9	10.8%
Bank capital ratio	n	7.9%
Liquidity ratio	m	20.8%

Table 1: Targets used to calibrate the parameters of the model

The appendix shows how information on n and m can be used to calculate the parameter  $\lambda$  that gives the size of the discount on bank assets if they were to be liquidated. The implied value of  $\lambda$  and the other banking parameters are shown in Table 2.

The calibration implies an annualized credit spread r - i of 167 basis points in steady state. This is in the ballpark of the literature. The fragility of banks in steady state is 6.7%. This means that the bank does not fail as long as 6.7% of its debt-holders do not lose faith in it.

The parameters describing the macroeconomic features of the model are set following the literature. The elasticity of intertemporal substitution  $\sigma$  is 1/2 and the Frisch elasticity of labour supply  $\psi$  is 1. The capital elasticity of output  $\alpha$  is set to 1/3 to match the capital share of national income. The depreciation parameter  $\delta$  is chosen to give a 7.5% annualized depreciation rate.

#### 6.2 Results

We simulate the model to show the effects of a one-off capital destruction shock. The impulse response functions are shown in Figure 4 alongside those for an RBC model with the same macroeconomic features but no banking sector. In the RBC model, households can directly invest in capital. To make the models comparable, the RBC

Description	Notation	Value
Bank-asset liquidity relative to T-bills	λ	0.822
Loss given bank default	$\theta$	0.0068
Minimum dividend distribution	γ	0.0090
Subjective discount factor	β	0.9988
Elasticity of intertemporal substitution	σ	0.5
Frisch elasticity of labour supply	$\psi$	1
Capital elasticity of output	α	1/3
Depreciation	δ	0.0063

#### Table 2: Calibrated parameters of the model

model includes an exogenous but time-invariant spread between the return on capital and the marginal rate of substitution. Variables such as interest rates, spreads, and ratios are percentage point deviations from steady state (annualized for interest rates and spreads), with 1 meaning 1 percentage point. All other variables are percentage deviations from steady state, with 1 denoting 1%. To begin with, we assume policy is completely passive and the supply of liquid assets is not changed.

Consider first the effectively frictionless responses of variables in the RBC model. The shock directly reduces the capital stock by 5%, which brings down GDP. Investment rises to equate the marginal product of capital to the interest rate (in the RBC model, all interest rates move in line with the T-bill rate). Interest rates rise because investment demand increases, while GDP has fallen.

In the model with banks, the loss of some of the assets held by banks reduces equity, which tightens the constraint on bank lending. Banks offer higher interest rates on deposits, with the TED spread rising by 38 basis points. Leverage increases, with the bank capital ratio falling by almost 4 percentage points even though bank assets decline. The liquidity ratio rises, but this simply reflects the constant supply of liquid assets. Credit spreads (the difference between the return on illiquid bank assets and the T-bill rate) rise because of the tighter constraint on banks. This results in less investment and a slower recovery of the capital stock compared to the RBC model. Consequently, GDP is lower and returns to its steady state at a slower rate.

#### 6.3 Quantitative easing

The no-run constraint implies that the quantity of liquid assets held by banks has the effect of reducing fragility. We simulate the effects of an exogenous increase



Figure 4: Impulse response functions for a capital destruction shock

in liquidity by considering an unexpected 50% temporary increase, diminishing over time such that the shock has a half-life of 5 years. The impulse response functions are shown in Figure 5. The shock increases banks' liquidity ratio by 8 percentage points. The reduction in fragility allows banks to take on more leverage and pay a lower interest rate on their debt. The bank capital ratio decline by 0.8 of a percentage point, and the

TED spread by 8 basis points. There is a small reduction in credit spreads and a small increase in investment, which raises GDP.



Figure 5: Impulse response functions for an expansion of liquid assets

#### 6.4 Liquidity policy in response to shocks

We can also study the supply of liquid assets as a systematic response to shocks. In this case, the optimal policy is to respond to shocks by supplying enough liquid assets to close the wedge between marginal rate of substitution and the marginal product of capital. This can be implemented by targeting zero variation either in the TED spread or in the credit spread. Impulse responses to a one-off 5% capital destruction shock under optimal policy are represented by the blue dashed line in figure 6. The red dashed line represents the case of an inelastic supply of liquid assets, as already portrayed in figure 4. As we would expect, optimal policy completely stabilizes both the TED and credit spreads. To accomplish this, the quantity of liquid assets must increase massively and extremely persistently. The high persistence is necessary because in the absence of spreads bank equity does not recover. It remains persistently below steady state. The greater supply of liquid assets leads to banks' liquidity ratio going up by 40 percentage points. This reduces bank fragility so much that the bank capital ratio can drop by almost 4 percentage points with no need for the TED spread to increase. The responses of macroeconomic variables are the same as in an RBC model, also reported in figure 4.



Figure 6: Capital destruction shock with and without the optimal policy response

## 7 Conclusion

In this paper, we build a tractable macroeconomic model that allows us to study the implications of bank fragility for the business cycle and the effectiveness of policy. The model nests a canonical real business cycle framework, and can be easily calibrated using observable interest-rate spreads and balance-sheet variables. Fragility is a function of a bank's leverage and its asset-portfolio illiquidity. As a result of a coordination game, creditors demand higher returns to lend to more fragile banks. Hence, fragility is costly for banks. This friction gives banks an incentive to keep their leverage low and to hold liquid assets.

We find that the friction generates significant amplification and propagation of shocks. A bad shock harms the economy further by reducing banks' net worth. With lower net worth, banks lend less in order to keep their fragility in check and limit increases in their funding costs. A relevant example is a shock that destroys a share of the economy's capital stock. This is widely used in the literature as a representation of the fall in housing prices that preceded the Global Financial Crisis. The downturn in GDP that follows such shock is approximately 25% deeper and significantly more persistent in our model relative to a frictionless RBC framework. Credit spreads go up by about 15 basis points and banks' funding costs, as captured by the TED spread between the interest on bank debt and the interest on T-bills, goes up by approximately 40 basis points.

A key feature of the model is that it generates a demand for liquid assets by the banking sector. Since liquid assets mitigate a bank's fragility, banks are willing to hold them even if they offer a lower return than illiquid lending. The presence of a demand for liquid assets implies that there is an important role in the economy for the supply of liquid assets. Indeed, an increase in the supply of liquid assets reduces the fragility of the banking system and allows it to lend more. In turn, this boosts economic activity. The supply of liquid assets can also target a spread, such as the credit spread. This stabilizes the economy by eliminating the amplification via the banking system. However, in our quantitative analysis we find that a huge expansion in the supply of liquid assets is necessary to stabilize interest-rate spreads after a capital-destruction shock. Moreover, the expansion in the supply of liquid assets ends up being practically permanent, since in the absence of interest-rate spreads banks are not profitable and therefore do not accumulate net worth.

Interestingly, there is no role for liquidity regulation in the economy. While a literature has identified a role for liquidity regulation at a micro level, for the banking system as a whole such regulation is ineffective. On the other hand, the supply of liquid assets is important.

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### **A Proofs**

Proof of lemma 1. The household problem's lagrangean can be written as

$$\mathcal{L}_{h} = \max_{\{C_{h,t}, L_{h,t}, H_{h,b,t}\}_{t=0}^{+\infty}} \min_{\mu} E_{0} \sum_{t=0}^{+\infty} \beta^{t} \left[ \frac{C_{h,t}^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} - \chi \frac{L_{h,t}^{1+\frac{1}{\psi}}}{1 + \frac{1}{\psi}} \right] + \mu \sum_{t=0}^{+\infty} \frac{1}{\prod_{s=0}^{t-1} (1 + \rho_{s})} (w_{t}L_{h,t} + T_{t} + \Pi_{t} - C_{h,t}) + \mu \sum_{t=0}^{+\infty} \frac{1}{\prod_{s=0}^{t-1} (1 + \rho_{s})} \int_{0}^{1} D_{b,t}H_{h,b,t} \left[ (1 - \phi_{b,t}) \frac{j_{b,t} - \rho_{t}}{1 + \rho_{t}} - \phi_{b,t} \theta \right] db, \quad (41)$$

with  $\phi_{b,t} = \phi(F_{b,t}, H_{b,t})$  defined in equation (2).

Looking at the lagrangean, we confirm that the net payoff of holding bank b's debt at time t is given by

$$\pi_t(H_{b,t}, F_{b,t}) = \frac{\mu_h^*}{\prod_{s=0}^{t-1} (1+\rho_s)} \left[ (1-\phi_{b,t}) \frac{j_{b,t}-\rho_t}{1+\rho_t} - \phi_{b,t} \theta \right] D_{b,t} \, \mathrm{d}b \,, \tag{5}$$

where  $\mu^*$  is the equilibrium lagrangean multiplier associated with the household's intertemporal budget constraint. Since each bank is infinitesimally sized, an individual  $\phi_{b,t}$  is irrelevant for the household's problem. Hence,  $\mu^*$  is independent of  $(F_{b,t}, H_{b,t})$ .

**Proof of lemma 2**. We use proposition 2.2 in Morris and Shin (2003). The game features the unique perfect Bayesian equilibrium described in the lemma if the following six conditions hold.

- 1. Action monotonicity.  $\pi_t(S_t, F_t)$  is non-decreasing in  $S_t$ .
- 2. State monotonicity.  $\pi_t(S_t, F_t)$  is non-increasing in  $F_t$ .
- 3. Strict Laplacian state monotonicity. There exists a unique  $F_t^*$  that solves

$$\int_0^1 \pi_t(S_t, F_t^*) \, \mathrm{d}S_t = 0. \tag{42}$$

- 4. Uniform limit dominance. There exists  $\underline{F}_t$ ,  $\overline{F}_t$  and  $\nu > 0$  such that (i)  $\pi_t(S_t, F_t) \le -\nu$  for all  $S_t \in [0, 1]$  and  $F_t \ge \underline{F}_t$ , (ii)  $\pi_t(S_t, F_t) > \nu$  for all  $S_t \in [0, 1]$  and  $F_t \le \overline{F}_t$ .
- 5. Continuity. The function  $\int_0^1 g(S_t) \cdot \pi_t(S_t, \hat{F}_t) dS_t$  is continuous with respect to signal  $\hat{F}_t$  and density g.

6. Finite expectations of signals.

As long as  $j_t \ge \rho_t$ ,  $\mu_t^* > 0$  and  $D_t > 0$ , it is easy to show all these conditions apply in our case.

**Proof of lemma 3**. Use the equilibrium early-repayment threshold (6) and aggregate using the fact that

$$H_t = \int_0^1 H_{h,t} \, \mathrm{d}h.$$
 (43)

With the intermediary's flow budget constraint (7) and fragility (9), we can confirm the proposition.  $\Box$ 

**Proof of proposition 2**. Guess that the  $\text{ROE}_t > \rho_t$  in every period. Then, constraint (22) is binding in every period since the return on internal funds is higher than the marginal rate of substitution of households, which own the intermediary. This fixes the intertemporal margin of the intermediary. Its problem is a sequence of static maximizations of next-period cum-dividend equity

$$(1+r_t)K_t + (1+i_t)M_t - (1+j_t)D_t.$$
(44)

The intermediary chooses  $j_t$ ,  $D_t$ ,  $L_t$ ,  $M_t$  to maximize subject to the no-run constraint (11), the lower bound on interest on debt (21) and the flow budget constraint (7). The optimality conditions to this problem can be re-written as equations (25), (26) and (27). By substituting these findings in the definition of return on equity, we find that they imply (29). This confirms our initial guess that the intermediary pays out as few dividends as possible at every point in time.

**Proof of lemma 4.** Any steady state with  $N_{ss} > 0$  requires  $ROE_t = \gamma$  at every point in time, as can be easily verified from the law of motion for equity (28). The equilibrium return on equity is given by equation (29). Together with equations (26) and (27), the return pins down interest rates ( $i_{ss}, j_{ss}, r_{ss}$ ). With the Euler equation (32), we determine  $\rho_{ss}$ . The lending rate  $r_{ss}$  determines all quantities via the RBC equations. Given  $K_{ss}$  and interest rates ( $j_{ss}, \rho_{ss}$ ), supply of capital (25) pins down equity  $N_{ss}$ .

Notice that a return on equity greater than  $\beta^{-1}\left(1+\theta\frac{2-\lambda}{\lambda}\right)-1$  cannot be sustained in a steady state with  $N_{ss} > 0$  because it implies  $(1+r_{t+1})/(1+\rho_t) > 1+\theta(1-\lambda)/\lambda$ . From capital supply (25), this implies infinite K for any strictly positive level of equity. Also notice that capital supply (25) rules out  $M_{ss} \ge \left[\theta - \lambda \left(\frac{1+j_{ss}}{1+\rho_{ss}} - 1 + \theta\right)\right]/\left(\frac{1+j_{ss}}{1+\rho_{ss}} - 1\right)$  in any steady state with strictly positive equity. **Proof of proposition 3.** From the steady-state condition  $ROE_t = \gamma$  and equilibrium return on equity (29) jointly with equations (26) and (27), we determine interest rates  $(i_{ss}, j_{ss}, r_{ss})$ . In the parametric region,  $i_{ss} = j_{ss} = \rho_{ss}$  and  $r_{ss} > \rho_{ss}$ . Hence, there is a positive credit spread and no TED spread. With  $r_{ss}$ , we pin down steady-state quantities from the RBC equations. From capital supply (25), we obtain equity  $N_{ss} = (1 - \lambda)K_{ss}$ . Lastly, we determine  $\rho_{ss} = \beta^{-1} - 1$  from the Euler equation (32). This confirms that policy  $M_{ss}$  plays no role in determining other steady-state variables.

**Proof of proposition** 4. From the steady-state condition  $ROE_t = \gamma$  and equilibrium return on equity (29) jointly with equations (26) and (27), we determine interest rates  $(i_{ss}, j_{ss}, r_{ss})$ . In the parametric region,  $i_{ss} < \rho_{ss}$  and  $(j_{ss}, r_{ss}) > \rho_{ss}$ . Hence, there is a positive credit spread and a positive TED spread. With  $r_{ss}$ , we pin down steady-state quantities from the RBC equations. From capital supply (25), we obtain equity as a function of steady-state capital (determined independently from policy) and policy. The condition on policy ensures that equity is strictly positive in steady state. Lastly, we determine  $\rho_{ss} = \beta^{-1} - 1$  from the Euler equation (32). This confirms that policy  $M_{ss}$  only plays a role in determining steady-state equity.

# **B** Figures

Country	Year
Australia	1989
Austria	2008, 2011
Belgium	2008, 2011
Czechia	1995
Denmark	1877, 1885, 1907, 1992, 2008, 2011
Finland	1990
France	1882, 1889, 1937, 2008
Germany	1891, 1901, 1930, 2008
Hong Kong	1998
Hungary	1995, 2008
Iceland	2008
Ireland	2007
Italy	1992, 2008, 2011, 2016
Japan	1990, 1997, 2001
Korea	1997
Luxembourg	2008
Malaysia	1985, 1997
Norway	1987, 2008
Philippines	1997
Portugal	2008, 2011
Russia	2008
Spain	2008, 2010
Sweden	1991, 2008
Switzerland	1990, 2008
Taiwan	1998
Thailand	1979, 1983, 1997
UK	1878, 1890, 1914, 1991, 2008
US	1890, 1893, 1907, 1930, 1984, 1990, 2007

Table 3: List of banking crises underlying figure 1.



Figure 7: The dynamics of the global financial crisis in the US.

The figure plots the average evolution of bank-equity returns (in log points) and bank-funding spreads (in percentage points) in the US with 0 indicating January 2007. More details are provided in the description of figure 1.

## C Calibration

The calibration procedure is as follows. First, the steady-state value of q is equal to  $\gamma$ , so the average real return on bank equity directly pins down  $\gamma$ . Using the equation for  $\rho_t$ , the steady-state value of  $\rho$  is found from the averages of *i*, *j*, and *q*:

$$\rho = \left(\frac{q-j}{q-i}\right)i + \left(\frac{j-i}{q-i}\right)j$$

Using the  $\rho$  from above and the steady-state Euler equation, we obtain  $\beta = 1/(1 + \rho)$ . To obtain,  $\theta$ , we take the demand for liquid assets equation and divide both sides by  $1 + \rho$ :

$$\theta = \frac{(q-j) - (j-\rho)}{1+\rho}$$

Finally,  $\lambda$  is obtained from the steady-state balance-sheet constraint using the average capitalization and liquidity ratios n and m:

$$\lambda = \frac{\frac{1-n}{1+\frac{(j-\rho)}{\theta(1+\rho)}} - m}{1-m}$$